Plenumsregning i MAT1110 21.1.2013

Seksjon 1.9 $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

1. $T(x,y,z) = \begin{pmatrix} 2x - y + z \\ -x + y - 3z \end{pmatrix}$

$$M_T = \left[T(1,0,0) \;\Big|\; T(0,1,0) \;\Big|\; T(0,0,1) \right]$$

$$= \begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & -3 \end{bmatrix}$$

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2. $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^4$ $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$T(e_1) = \begin{bmatrix} -1 \\ 2 \\ -3 \\ 4 \end{bmatrix}$ $T(e_2) = \begin{bmatrix} 0 \\ -2 \\ 4 \\ 7 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$$M_T = \begin{bmatrix} -1 & 0 \\ 2 & -2 \\ -3 & 4 \\ 4 & 7 \end{bmatrix}$$

3. $a, b \in \mathbb{R}^2 \quad T: \mathbb{R}^2 \to \mathbb{R}^2$

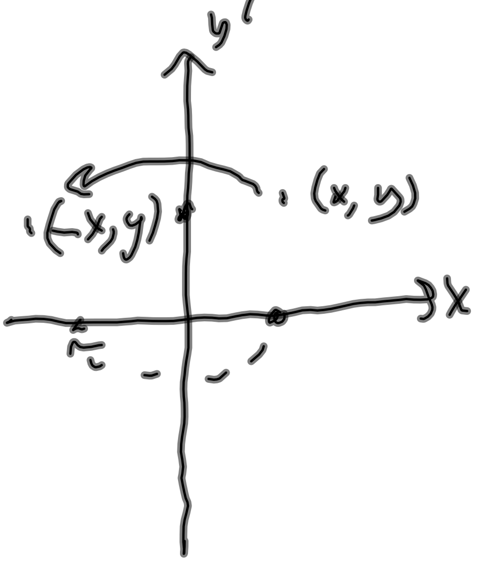
$$T(a) = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad T(b) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$T(3a - 2b) = 3T(a) - 2T(b)$$

$$= 3\begin{pmatrix} -2 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$$

4. $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

speiler om y-aksen



$$(1,0) \longmapsto (-1,0)$$

$$(0,1) \longmapsto (0,1)$$

$$M_T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

5. $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

fordobler 2. komponent

$$M_T = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

6 $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

fordoble lengde, rotere m. θ

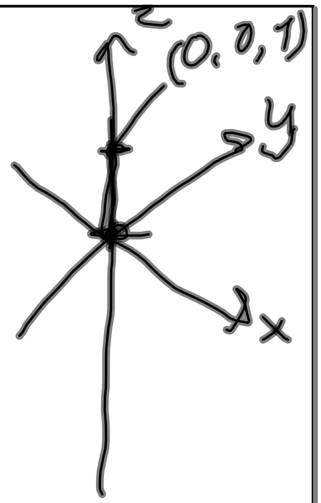
$$T(a) = R_\theta(2a) = 2R_\theta(a)$$

$$M_T = 2 \cdot M_{R_\theta} = 2A_\theta$$

$$= \begin{bmatrix} 2\cos\theta & -2\sin\theta \\ 2\sin\theta & 2\cos\theta \end{bmatrix}$$

7. $T: \mathbb{R}^3 \to \mathbb{R}^3$

proj. på xy-planet



$$M_T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

8. $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

speile om x-aksen og
rotere m. θ

$$M_T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$$

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9. $A_{-\theta}$ er inversen til A_θ fordi kombinasjonen er identiteten

$$A_\theta A_{-\theta} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin(\theta)$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \cos\theta\sin\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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B er inversen til A hvis

$AB = I \qquad ABa = Ia = a$

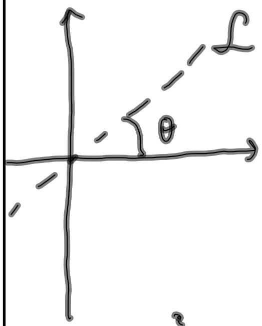
$A_\theta A_{-\theta} a = a$

10.

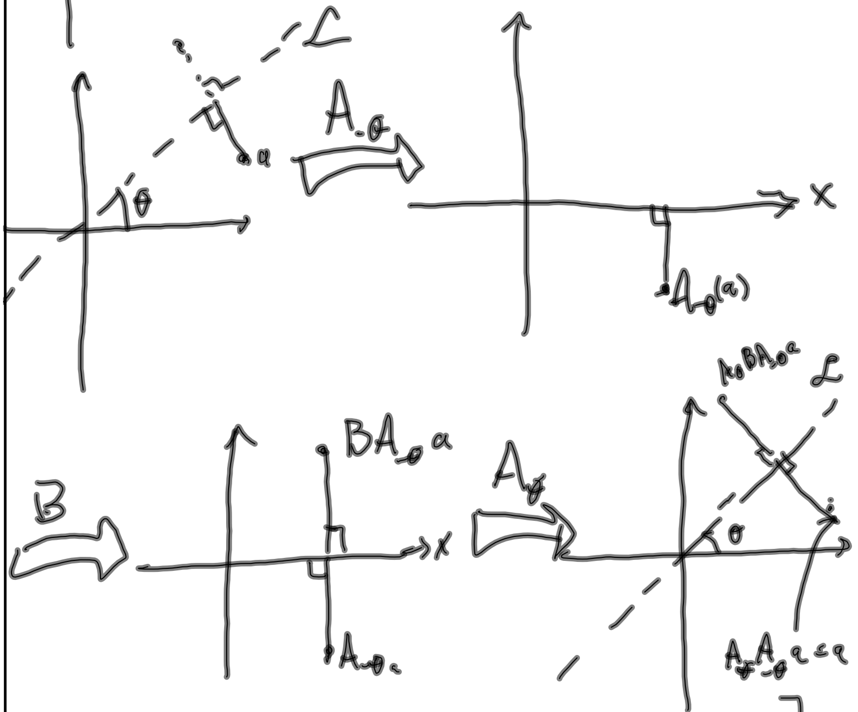
C speiler om $\mathcal{L}$

A_θ rotere med θ

B speile om x-akson



Forklar at $C = A_\theta B A_{-\theta}$



$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & \cos\theta\sin\theta + \sin\theta\cos\theta \\ \sin\theta\cos\theta + \cos\theta\sin\theta & \sin^2\theta - \cos^2\theta \end{bmatrix}$$

11. $a = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

a) Finn x, y, z, u s.a.

$e_1 = xa + yb \quad e_2 = za + ub$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = x \begin{pmatrix} -2 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$x = -3y \quad 6y + y = 1$

$1 = -2x + y$

$0 = x + 3y \quad \boxed{y = \frac{1}{7}, \; x = \frac{-3}{7}}$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = z \begin{pmatrix} -2 \\ 1 \end{pmatrix} + u \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$0 = -2z + u \quad u = 2z \quad 1 = 7z$

$1 = z + 3u \quad \boxed{z = \frac{1}{7} \quad u = \frac{2}{7}}$

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11. b) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$$T(a) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad T(b) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$T(e_1) = T\left(-\frac{3}{7}a + \frac{1}{7}b\right) = \frac{-3}{7}T(a) + \frac{1}{7}T(b)$$

$$= \frac{-3}{7}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{7}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -\frac{3}{7} + \frac{1}{7} \\ -\frac{3}{7} - \frac{1}{7} \end{pmatrix} = \begin{pmatrix} -\frac{2}{7} \\ -\frac{4}{7} \end{pmatrix}$$

$$T(e_2) = T\left(\frac{1}{7}a + \frac{2}{7}b\right) = \frac{1}{7}T(a) + \frac{2}{7}T(b)$$

$$= \begin{pmatrix} \frac{1}{7} + \frac{2}{7} \\ \frac{1}{7} - \frac{2}{7} \end{pmatrix} = \begin{pmatrix} \frac{3}{7} \\ -\frac{1}{7} \end{pmatrix}$$

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11. c)

$$M_T = \left[T(e_1) \,\middle|\, T(e_2) \right] = \frac{1}{7}\begin{bmatrix} -2 & 3 \\ -4 & -1 \end{bmatrix}$$

15. $F: \mathbb{R}^n \to \mathbb{R}^n$

$$F(cx + dy) = cF(x) + dF(y)$$

for alle $c, d \in \mathbb{R}$ og $x, y \in \mathbb{R}^n$

(i) må vise: $F(cx) = cF(x)$ for alle $c \in \mathbb{R}$ og $x \in \mathbb{R}^n$

$$F(cx) = F(cx + 0 \cdot 0) = cF(x) + 0F(0)$$
$$= cF(x)$$

ii) Må vise: $F(x+y) = F(x) + F(y)$ for alle $x, y \in \mathbb{R}^n$

$$F(x+y) = F(1 \cdot x + 1 \cdot y) = 1 \cdot F(x) + 1 \cdot F(y)$$
$$= F(x) + F(y)$$

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Seksjon 1.10

1. $$F(x,y,z) = \begin{bmatrix} -2x - 3y + z - 7 \\ -x + z - 2 \end{bmatrix}$$

$$F(x,y,z) = A\begin{pmatrix} x \\ y \\ z \end{pmatrix} + C$$

$$C = F(0,0,0) = \begin{pmatrix} -7 \\ -2 \end{pmatrix}$$

$$Ae_1 = F(e_1) - C = \begin{bmatrix} -9 \\ -3 \end{bmatrix} - \begin{bmatrix} -7 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$A = \left[Ae_1 \,\vdots\, Ae_2 \,\vdots\, Ae_3 \right] = \begin{bmatrix} -2 & -3 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

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2. $\mathcal{L}$ param. med

$$r(t) = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$F : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$$

$$F(x,y,z) = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$F(r(t)) = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{bmatrix} \left(\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right) + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+1+6 \\ 0-3-6 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1+0+4 \\ 0+0-4 \end{pmatrix}$$

$$= \begin{pmatrix} 11 \\ -10 \end{pmatrix} + t \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

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3. $F: \mathbb{R}^2 \to \mathbb{R}^2$

$F(0,0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad F(1,0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad F(0,1) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$F(x,y) = A\begin{pmatrix} x \\ y \end{pmatrix} + C$

$F(0,0) = C = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = F(1,0) - C = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = F(0,1) - C = \begin{pmatrix} -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$A = \begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix}$

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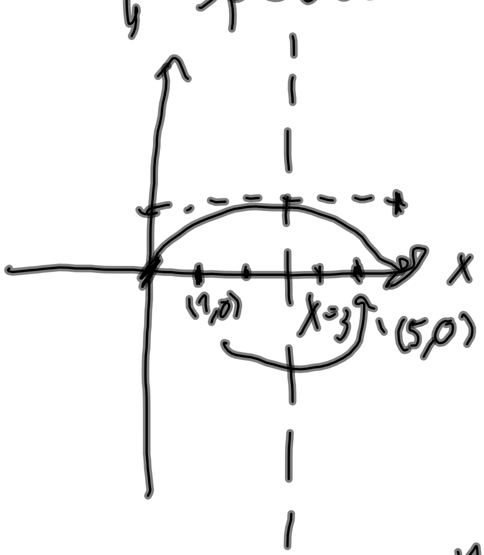
5a) $F: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

speile om $x = 3$



$$F(0,0) = (6,0)$$
$$F(1,0) = (5,0)$$
$$F(0,1) = (6,1)$$

$$F(x,y) = A\begin{pmatrix} x \\ y \end{pmatrix} + c$$

$$c = F(0,0) = (6,0)$$

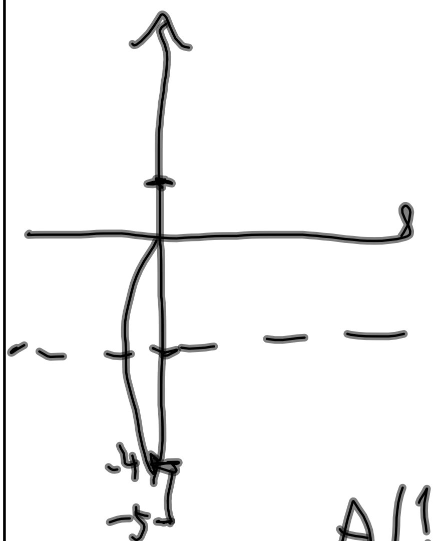
$$A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

5.b) $G : \mathbb{R}^2 \to \mathbb{R}^2$

speiler om $y = -2$



$G(0,0) = (0,-4)$

$G(1,0) = (1,-4)$

$G(0,1) = (0,-5)$

$G(x,y) = A\begin{pmatrix} x \\ y \end{pmatrix} + c$

$c = G(0,0) = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

$A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = G(1,0) - c = \begin{pmatrix} 1 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \end{pmatrix} - \begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

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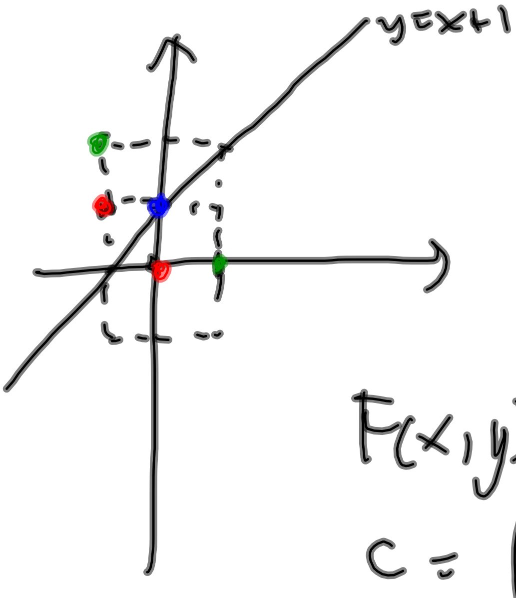
6. $F : \mathbb{R}^2 \to \mathbb{R}^2$ speiler om $y = x+1$



$F(0,0) = (-1,1)$

$F(1,0) = (-1,2)$

$F(0,1) = (0,1)$

$F(x,y) = A\begin{pmatrix} x \\ y \end{pmatrix} + c$

$c = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

8. $f: \mathbb{R}^2 \to \mathbb{R}$

(i) affine

$\Updownarrow$

(ii) grafen til f er et plan

$\left(f(x,y) = ax + by + c \right)$

(i) $\Rightarrow$ (ii): $f(x,y) = A\begin{pmatrix} x \\ y \end{pmatrix} + c$

$A = \begin{bmatrix} a_1 & a_2 \end{bmatrix}$

$f(x,y) = \begin{bmatrix} a_1 & a_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + c = a_1x + a_2y + c$

(ii) $\Rightarrow$ (i): $f(x,y) = ax + by + c$

$= \begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + c$

Definer $A = \begin{bmatrix} a & b \end{bmatrix}$

$f(x,y) = A\begin{bmatrix} x \\ y \end{bmatrix} + c$

1.9.16.

$a_1, a_2 \in \mathbb{R}^2$ ($a_1, a_2 \neq 0$ og ikke parallelle.

$b_1, b_2 \in \mathbb{R}^2$

$T, S : \mathbb{R}^2 \to \mathbb{R}^2$

$T(a_1) = b_1 \quad T(a_2) = b_2$

$S(a_1) = b_1 \quad S(a_2) = b_2$

La $x \in \mathbb{R}^2$ være vilkårlig valgt

Må vise at $T(x) = S(x)$

siden $a_1, a_2 \neq 0$ og ikke parallelle

så finnes $u, v \in \mathbb{R}$ s.a. $x = ua_1 + va_2$

$T(x) = T(ua_1 + va_2) = uT(a_1) + vT(a_2) = ub_1 + vb_2$

$= uS(a_1) + vS(a_2) = S(ua_1 + va_2)$

$= S(x)$ Så $S = T$

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