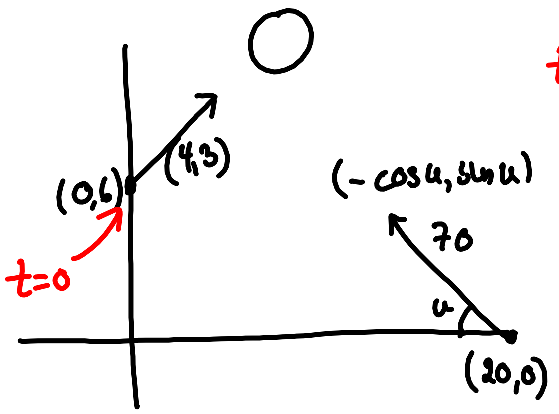
Velkommen til MAT1110

$3^2+4^2=5^2$

15 112 113

$\frac{1\cancel{6}}{\cancel{6}4}$

$\frac{2\cancel{6}}{\cancel{6}5}$



$t=2$

$>$

$$(0,6)+t(4,3)=(0+4t,\ 6+3t)$$
$$=(4t,\ 6+3t)$$

$$(20,0)+70(t-2)(-\cos u, \sin u)$$
$$=(20-70(t-2)\cos u,\ 70(t-2)\sin u)$$

1.4.8

$\bar{a} = (1,1,-1)$

$\bar{b} = (0,-2,-6)$

$\bar{c} = (2,3,3)$

$\bar{b} - \bar{a} = (-1,-3,-5)$

$\bar{c} - \bar{a} = (1, 2, 4)$

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & -3 & -5 \\ 1 & 2 & 4 \end{vmatrix} = (-2,-1,1)$$

$$-2x - y + z = -4 \iff 2x + y - z = 4$$

Sier noe om avst. fra plan til (0,0,0)

Avstand: Likningen må være normert.

$$\frac{2}{\sqrt{6}}x + \frac{1}{\sqrt{6}}y - \frac{1}{\sqrt{6}}z = \frac{4}{\sqrt{6}}$$

Avstand

2.4: 3a) $f'(\vec{a};\vec{r}) = \nabla f(\vec{a})\cdot\vec{r}$

$\boxed{f(x,y) = 3xy + y^2}$ $\nabla f(x,y) = (3y\ ,\ 3x+2y)$

$f'(\vec{a};\vec{r}) = (6,7)\cdot(3,-1)$

$= 18-7 = \underline{\underline{11}}$

4)b) $f(x,y,z) = (x^2-y^2)e^z$

$\nabla f(x,y,z) = \left(2xe^z, -2ye^z, (x^2-y^2)e^z\right)$

$\nabla f(1,-1,3) = (2e^3, 2e^3, 0) = 2e^3(\underline{1,1,0})$

$$\bar{F}(x,y) = (x^2y, x+y^2)$$

$$J = \bar{F}'(x,y) = \begin{pmatrix} \frac{\partial(x^2y)}{\partial x} & \frac{\partial(x^2y)}{\partial y} \\ \frac{\partial(x+y^2)}{\partial x} & \frac{\partial(x+y^2)}{\partial y} \end{pmatrix}$$

$$= \begin{pmatrix} 2xy & x^2 \\ 1 & 2y \end{pmatrix} = \begin{pmatrix} 2xy & x^2 \\ 1 & 2y \end{pmatrix}$$

26.6 $\bar{F} : A \longrightarrow \mathbb{R}^m$ $\quad A \subseteq \mathbb{R}^n$ $\qquad B$

$\bar{a} \in \text{int}(A)$ $\qquad$ $m \times n$ – matrise

$$\hat{\sigma}(\bar{r}) = \bar{F}(\bar{a} + \bar{r}) - \bar{F}(\bar{a}) - B \cdot \bar{r} \qquad \bar{r} \in A$$

slik at $\lim_{r \to 0} \frac{1}{|\bar{r}|} \hat{\sigma}(\bar{r}) = 0$

$\Longrightarrow$ $\bar{F}$ deriverbar i $\bar{a} \in A$ og $\bar{F}'(\bar{a}) = B$.

Kjerneregelen

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

$f: \mathbb{R} \to \mathbb{R}$, $\mathbb{R} \to \mathbb{R}^n \to \mathbb{R}$

$$f(x+t) = f(x) + t f'(x) + \ldots$$

Eks

$T(x,y)$ $\quad T: \mathbb{R}^2 \to \mathbb{R}$

$g(t) = T(\bar{r}(t))$

$\bar{r}(t) = (x(t), y(t))$ $\quad \bar{r}: \mathbb{R} \to \mathbb{R}^2$

Sammensatt: $T(\bar{r}(t)) = (T \circ \bar{r})(t)$ $\quad T \circ \bar{r}: \mathbb{R} \to \mathbb{R}$

$$g'(t) = \frac{\partial T}{\partial x}(\bar{r}(t)) \cdot x'(t) + \frac{\partial T}{\partial y}(\bar{r}(t)) \cdot y'(t)$$

$$= \nabla T \cdot \bar{r}'(t) = \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}\right) \cdot (x'(t), y'(t))$$

$F: \mathbb{R}^n \to \mathbb{R}^m$ $\quad G: \mathbb{R}^k \to \mathbb{R}^n$ $\quad F(G(\bar{x}))$, $\bar{x} \in \mathbb{R}^k$

$(F \circ G: \mathbb{R}^k \to \mathbb{R}^n \to \mathbb{R}^m)$

Hvis $k = m = 1$. $\quad F \circ G : \mathbb{R} \to \mathbb{R}$

$$(F \circ G)'(t) = \frac{\partial F}{\partial x_1} G_1'(t) + \ldots + \frac{\partial F}{\partial x_n} G_n'(t)$$

$$G(t) = (G_1(t), \ldots, G_n(t))$$

$$= \nabla F \cdot G'(t)$$

Polarkoordinater $x = r\cos\theta \quad y = r\cdot\sin\theta$

$$f: \mathbb{R}^2 \to \mathbb{R} \quad f = f(x,y) = f(r\cos\theta, r\sin\theta) = \tilde{f}(r,\theta)$$

Notasjon
$\frac{\partial f}{\partial x} = f_x$

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\cdot\cos\theta + \frac{\partial f}{\partial y}\cdot\sin\theta = f_x\frac{x}{r} + f_y\frac{y}{r} = \frac{1}{r}(xf_x + yf_y)$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}r(-\sin\theta) + \frac{\partial f}{\partial y}r\cos\theta = -y f_x + x f_y$$

$$\frac{\partial}{\partial r}\left(r\cdot\frac{\partial f}{\partial r}\right) = \frac{\partial}{\partial r}\left(f_x\cdot x + f_y\cdot y\right)$$

$$= f_{xx}\cdot\cos\theta\cdot x + f_x\frac{\partial x}{\partial x}\cdot\frac{\partial x}{\partial r} + f_{xy}\sin\theta\cdot x + \cancel{f_x\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial r}}$$

$$+ f_{xy}\cos\theta\cdot y + \cancel{f_y\frac{\partial y}{\partial x}\frac{\partial x}{\partial r}} + f_{yy}\sin\theta\cdot y + f_y\frac{\partial y}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$= f_{xx}\frac{x^2}{r} + f_x\cdot\frac{x}{r} + f_{xy}\frac{xy}{r} + f_{xy}\frac{xy}{r} + f_{yy}\frac{y^2}{r} + f_y\frac{y}{r}$$

$H : \mathbb{R}^k \to \mathbb{R}^m$

Derivert: $H(x_1, \dots, x_k) = (H_1(x_1, \dots x_k), \dots, H_m(x_1, \dots, x_k))$

$$H'(\bar{x}) = \begin{pmatrix} \frac{\partial H_1}{\partial x_1} & \cdots & \frac{\partial H_1}{\partial x_k} \\ \vdots & & \vdots \\ \frac{\partial H_m}{\partial x_1} & \cdots & \frac{\partial H_m}{\partial x_k} \end{pmatrix}$$

Jacobi matrise. $m \times k$

$H = F \circ G \quad , \quad H(\bar{x}) = F(G(\bar{x}))$

$$\mathbb{R}^k \xrightarrow{G} \mathbb{R}^n \xrightarrow{F} \mathbb{R}^m$$

$$H'(\bar{x}) = \begin{pmatrix} \frac{\partial F_1}{\partial x_1}(G(\bar{x})) & \cdots & \frac{\partial F_1}{\partial x_n}(\ ) \\ \vdots & & \\ \frac{\partial F_m}{\partial x_1}(\ ) & \cdots & \frac{\partial F_m}{\partial x_n}(\ ) \end{pmatrix} \begin{pmatrix} \frac{\partial G_1}{\partial x_1}(\bar{x}) & \cdots & \frac{\partial G_1}{\partial x_k}(\bar{x}) \\ \vdots & & \vdots \\ \frac{\partial G_n}{\partial x_1}(\bar{x}) & \cdots & \frac{\partial G_n}{\partial x_k}(\bar{x}) \end{pmatrix} = F'(G(\bar{x})) \cdot G'(\bar{x})$$

$m \times k \qquad m \times n \qquad n \times k$