Plenum 4/2-15

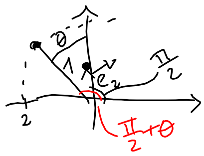
1.9: 6, 10, 11
1.10: 5
2.7: 7, 8, 9
2.8: 2

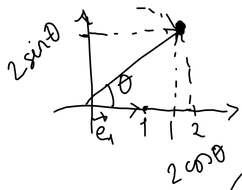
1.9: Lineæravbildninger

6.) Metode for å finne matrisen til en lineæravbildning:

Se hva avbildningen gjør med enhetsvektorene.

$$\vec{T}(\vec{e}_1) = \vec{T}\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \underline{\begin{bmatrix}2\cos\theta\\2\sin\theta\end{bmatrix}}$$





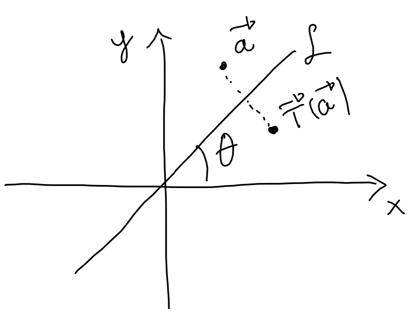
$$\vec{T}(\vec{e}_2) = \vec{T}\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}2\cos(\frac{\pi}{2}+\theta)\\2\sin(\frac{\pi}{2}+\theta)\end{bmatrix}$$

sin/cos sum av vinkler; roter koordinatsys.

$$= \underline{\begin{bmatrix}-2\sin\theta\\2\cos\theta\end{bmatrix}}$$

Matrisen til $\vec{T}$ er $A = \begin{bmatrix}2\cos\theta & -2\sin\theta\\2\sin\theta & 2\cos\theta\end{bmatrix} = 2\underline{\underline{\begin{bmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{bmatrix}}}$

10.)



$\vec{T}: \mathbb{R}^2 \to \mathbb{R}^2$; speiler om L

A_ϕ: dreier ϕ i pos. ret.

B: speiler om x-aksen

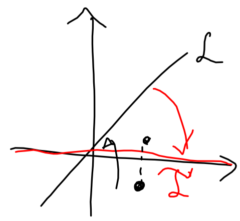
<u>VIS</u>: Matrisen til $\vec{T}$, C, er $C = A_\theta B A_{-\theta}$.

Hvordan speile om $\mathcal{L}$ i flere skritt?

1) Roter $-\theta$ sånn at $\mathcal{L}$ ligger "oppå" x-aksen: $A_{-\theta}$

2) Speil om x-aksen (dvs. roterte $\mathcal{L}$): B

3) Roter θ for å reversere steg 1.): A_θ



Dvs: $C = \underbrace{A_\theta}_{3)}\underbrace{B}_{2)}\underbrace{A_{-\theta}}_{1)}$

$$\vec{T}(\vec{x}) = C\vec{x} = (A_\theta B A_{-\theta})\vec{x} = (A_\theta B) A_{-\theta}\vec{x}$$

$$C = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Eks 3 & Eks. 2

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & \overbrace{\cos\theta\sin\theta + \sin\theta\cos\theta}^{2\sin\theta\cos\theta} \\ \sin\theta\cos\theta + \cos\theta\sin\theta & \underbrace{\sin^2\theta - \cos^2\theta}_{-(\cos^2\theta - \sin^2\theta)} \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

11) $\vec{a} = \begin{bmatrix}-2\\1\end{bmatrix}$, $\vec{b} = \begin{bmatrix}1\\3\end{bmatrix}$

a) $\begin{cases} \vec{e}_1 = \underline{x}\,\vec{a} + \underline{y}\,\vec{b} \\ \vec{e}_2 = \underline{z}\,\vec{a} + \underline{u}\,\vec{b} \end{cases}$

$$\begin{cases} \begin{bmatrix}1\\0\end{bmatrix} = x\begin{bmatrix}-2\\1\end{bmatrix} + y\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}-2x+y\\x+3y\end{bmatrix} \\ \begin{bmatrix}0\\1\end{bmatrix} = z\begin{bmatrix}-2\\1\end{bmatrix} + u\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}-2z+u\\z+3u\end{bmatrix} \end{cases}$$

$$\begin{cases} 1 = -2x+y \\ 0 = x+3y \end{cases} \Rightarrow x = -3y \Rightarrow 6y + y = 1 \Rightarrow \underline{\underline{y = \tfrac{1}{7}}} \Rightarrow \underline{\underline{x = -\tfrac{3}{7}}}$$

$$\begin{cases} 0 = -2z+u \\ 1 = z+3u \end{cases} \Rightarrow u = 2z \Rightarrow z + 6z = 1 \Rightarrow \underline{\underline{z = \tfrac{1}{7}}} \Rightarrow \underline{\underline{u = \tfrac{2}{7}}}$$

b) $\vec{T}: \mathbb{R}^2 \to \mathbb{R}^2$, $\vec{T}(\vec{a}) = \begin{bmatrix}1\\1\end{bmatrix}$, $\vec{T}(\vec{b}) = \begin{bmatrix}1\\-1\end{bmatrix}$

$$\vec{T}(\vec{e}_1) = \vec{T}(x\vec{a} + y\vec{b}) = x\vec{T}(\vec{a}) + y\vec{T}(\vec{b})$$

a)

$\vec{T}$ er lineær

$$= x\begin{bmatrix}1\\1\end{bmatrix} + y\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}-\frac{3}{7}\\-\frac{3}{7}\end{bmatrix} + \begin{bmatrix}\frac{1}{7}\\-\frac{1}{7}\end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix}-\frac{2}{7}\\-\frac{4}{7}\end{bmatrix}}}$$

$$\vec T(\vec e_2) \overset{a)}{=} \vec T(z\vec a + u\vec b) \overset{\vec T \text{ er lineær}}{=} z\vec T(\vec a) + u\vec T(\vec b)$$

$$= \frac{1}{7}\begin{bmatrix}1\\1\end{bmatrix} + \frac{2}{7}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}\frac{1}{7}+\frac{2}{7}\\ \frac{1}{7}-\frac{2}{7}\end{bmatrix} = \begin{bmatrix}\frac{3}{7}\\ -\frac{1}{7}\end{bmatrix}$$

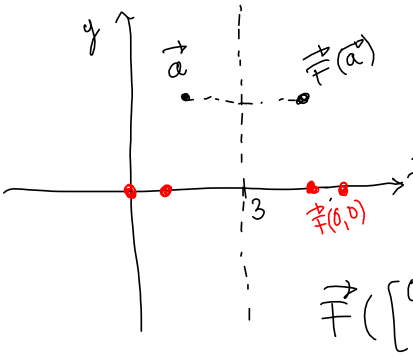
c) Matrisen til $\vec T$: $A = \begin{bmatrix}-\frac{2}{7} & \frac{3}{7}\\ -\frac{4}{7} & -\frac{1}{7}\end{bmatrix}$

1.10: Affinavbildninger

$A\vec x$

$\boxed{A\vec x + \vec b}$

5) a) $\vec F: \mathbb{R}^2 \to \mathbb{R}^2$, affin; Speiler om linja $x=3$



$\vec F$ affin $\Rightarrow$ $\vec F(\vec x) = \underset{\in \mathbb{R}^{2\times 2}}{A}\vec x + \underset{\in \mathbb{R}^2}{\vec b}$

$$\vec F\left(\begin{bmatrix}0\\0\end{bmatrix}\right) = A\begin{bmatrix}0\\0\end{bmatrix} + \vec b = \vec 0 + \vec b = \vec b$$

$$\vec F\left(\begin{bmatrix}0\\0\end{bmatrix}\right) = \begin{bmatrix}6\\0\end{bmatrix} \Rightarrow \vec b = \begin{bmatrix}6\\0\end{bmatrix}.$$

$A = \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$;

$$\vec F\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = A\begin{bmatrix}1\\0\end{bmatrix} + \vec b = \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}6\\0\end{bmatrix}$$

$$= \begin{bmatrix}a_{11}\\a_{21}\end{bmatrix} + \begin{bmatrix}6\\0\end{bmatrix} = \begin{bmatrix}a_{11}+6\\ a_{21}\end{bmatrix}$$

Fra figur:

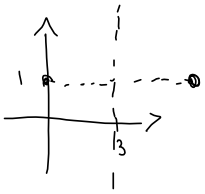
$$\vec{F}\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=\begin{bmatrix}5\\0\end{bmatrix}$$

Fra forrige side:

$$\begin{bmatrix}5\\0\end{bmatrix}=\begin{bmatrix}a_{11}+6\\a_{21}\end{bmatrix}\Rightarrow \begin{matrix}a_{11}+6=5\\ \underline{a_{21}=0}\end{matrix}\Rightarrow \underline{a_{11}=-1}$$

Fra figur:

$$\vec{F}\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=\begin{bmatrix}6\\1\end{bmatrix}$$



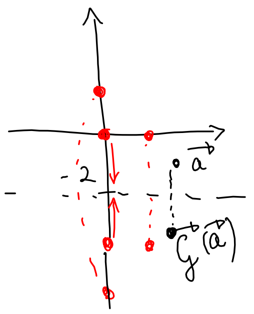
$$\vec{F}\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=A\begin{bmatrix}0\\1\end{bmatrix}+\begin{bmatrix}6\\0\end{bmatrix}=\begin{bmatrix}a_{12}+6\\a_{22}\end{bmatrix}$$

$$\begin{bmatrix}a_{12}+6\\a_{22}\end{bmatrix}=\begin{bmatrix}6\\1\end{bmatrix}\Rightarrow \begin{matrix}a_{12}=\underline{0}\\ a_{22}\underline{=1}\end{matrix}$$

Så:

$$A=\underline{\underline{\begin{bmatrix}-1&0\\0&1\end{bmatrix}}}$$

b)



$$\vec{G}\left(\begin{bmatrix}0\\0\end{bmatrix}\right)=\underline{\underline{\begin{bmatrix}0\\-4\end{bmatrix}\overset{a)}{=}\vec{b}}}\text{; konstantledd}$$

$$\vec{G}\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=\begin{bmatrix}1\\-4\end{bmatrix}$$

$$\vec{G}\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=\begin{bmatrix}0\\-5\end{bmatrix}$$

Samme regning som i a):

$$\begin{bmatrix}a_{11}+0\\a_{21}+(-4)\end{bmatrix}=\begin{bmatrix}1\\-4\end{bmatrix}\qquad \begin{bmatrix}a_{12}+0\\a_{22}+(-4)\end{bmatrix}=\begin{bmatrix}0\\-5\end{bmatrix}\Rightarrow A=\underline{\underline{\begin{bmatrix}1&0\\0&-1\end{bmatrix}}}$$