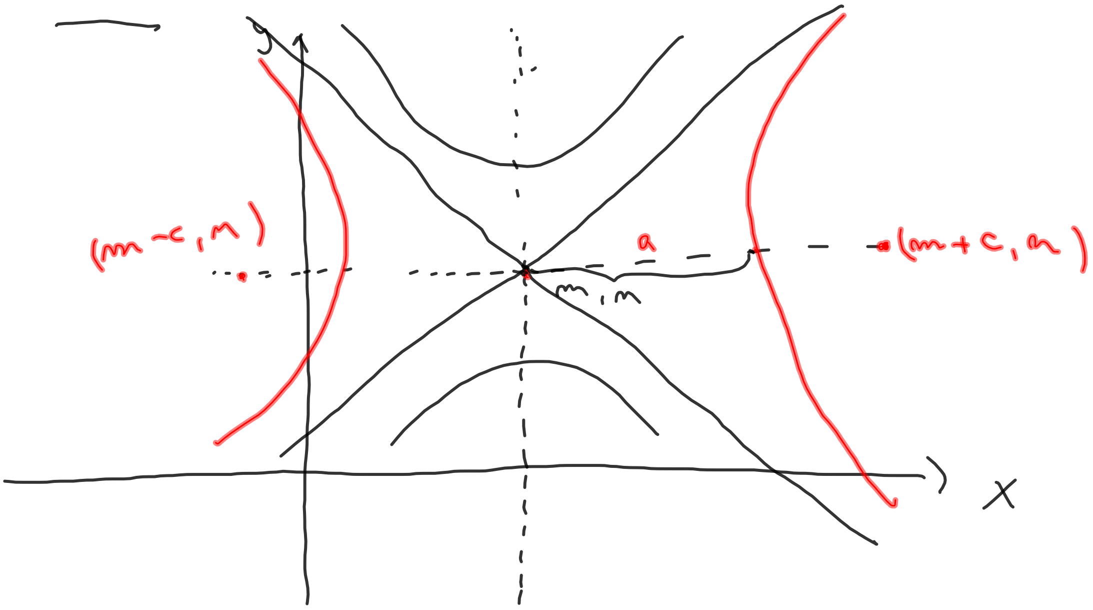
Ex Hyperbler.

$$\frac{(x-m)^2}{a^2} - \frac{(y-n)^2}{b^2} = +1$$

Sentrum i $(m, n)$

Halvakse $a$, brennvidde $c = \sqrt{a^2+b^2}$




Asymptoter $y - n = \pm \frac{b}{a}(x-m)$

Bevis asymptote:

$$\frac{(x-m)^2}{a^2} - \frac{(y-n)^2}{b^2} = 1$$

$$\frac{(y-n)^2}{\cancel{b^2}} = \frac{b^2(x-m)^2}{a^2} - b^2$$

$$y - n = \pm\sqrt{\frac{b^2}{a^2}(x-m)^2 - b^2}$$

$$= \pm\frac{b}{a}\sqrt{(x-m)^2 - a^2}$$

$$\lim_{x\to\infty} \sqrt{(x-m)^2 - a^2} - (x-m) = 0$$

Sjekk dette!

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Eks. Finn sentrum, halvakse, brennpunkt og asymptoter for

$$3x^2 - 12x - y^2 - 6y - 9 = 0$$

$$3(\underbrace{x^2 - 4x}) - (y^2 + 6y) - 9$$

$$= 3(x-2)^2 - 12 - (y+3)^2 + 9 - 9$$

$$= 3(x-2)^2 - (y+3)^2 - 12 = 0$$

$$3(x-2)^2 - (y+3)^2 = 12$$

$$\frac{(x-2)^2}{4} - \frac{(y+3)^2}{12} = 1$$

Sentrum $(2,-3)$

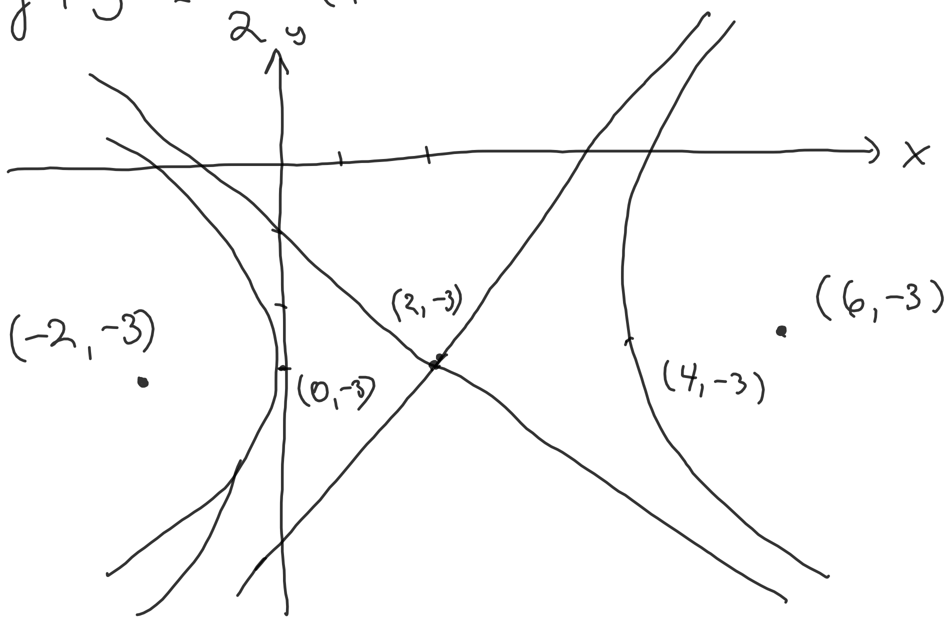
$a^2 = 4$. Halvaksen $a = 2$

$b^2 = 12$, $b = 2\sqrt{3}$

$c^2 = a^2 + b^2 = 16$ Brennvidde $c = 4$

Asymptoter

$$y + 3 = \pm\frac{2\sqrt{3}}{2}(x-2) = \pm\sqrt{3}(x-2)$$

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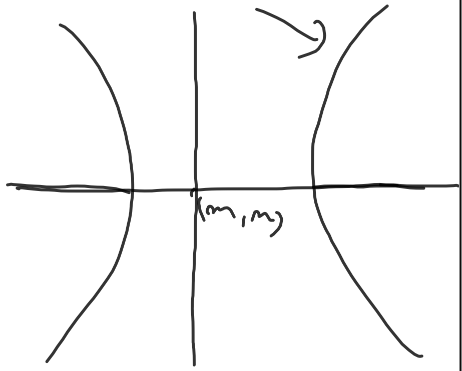
$$\frac{(x-m)^2}{a^2} - \frac{(y-n)^2}{b^2} = 1.$$

$x(t), y(t)$




$x - m = a\cosh t$

$y - n = b\sinh t$

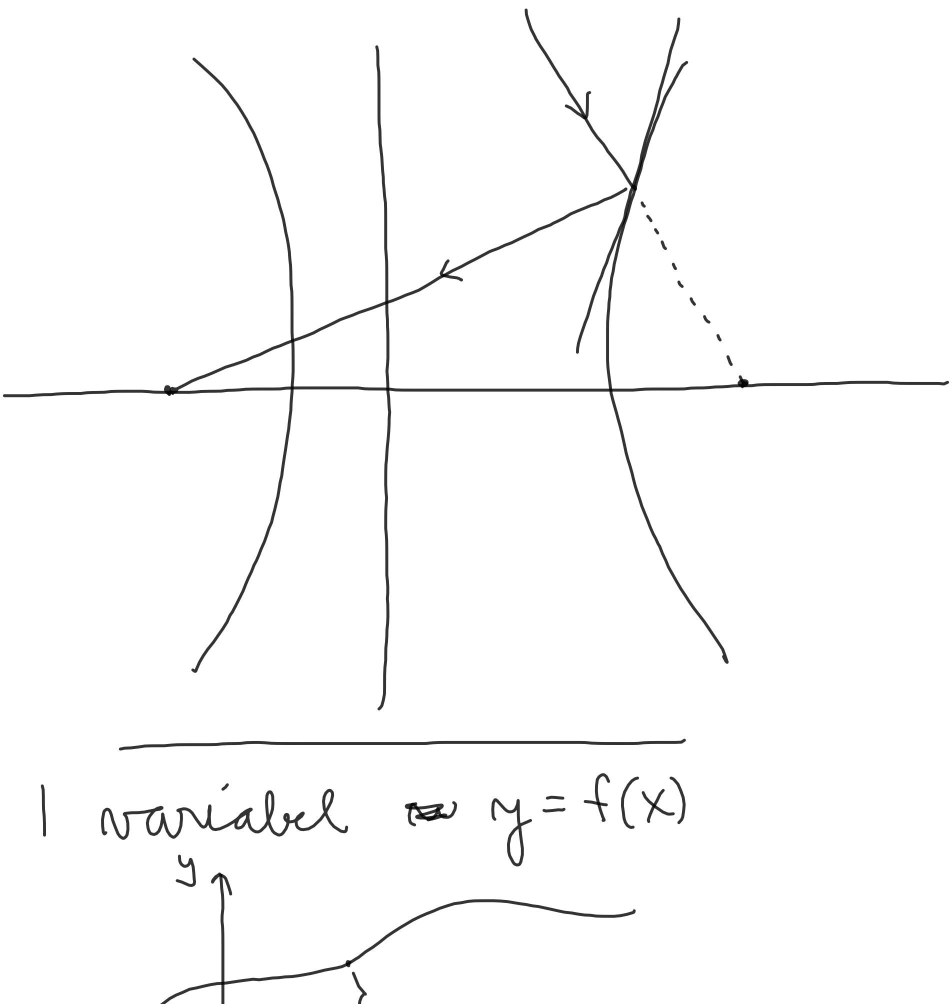
V.S. $\cosh^2 t - \sinh^2 t = 1$

$\cosh t = \frac{e^t + e^{-t}}{2}$,

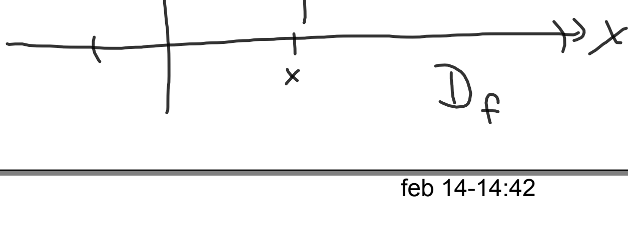
$\sinh t = \frac{e^t - e^{-t}}{2}$

Dette er høyre bit!

$x - m = -a\cosh t$ gir venstre bit.



1 variabel $y = f(x)$

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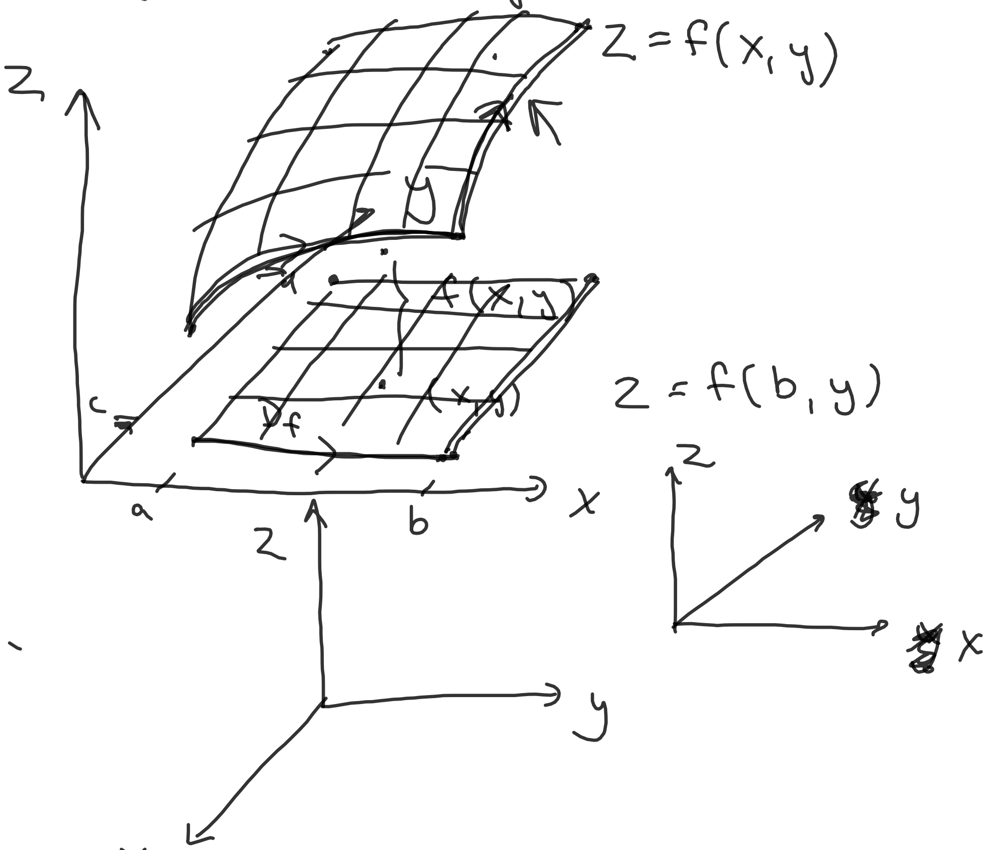
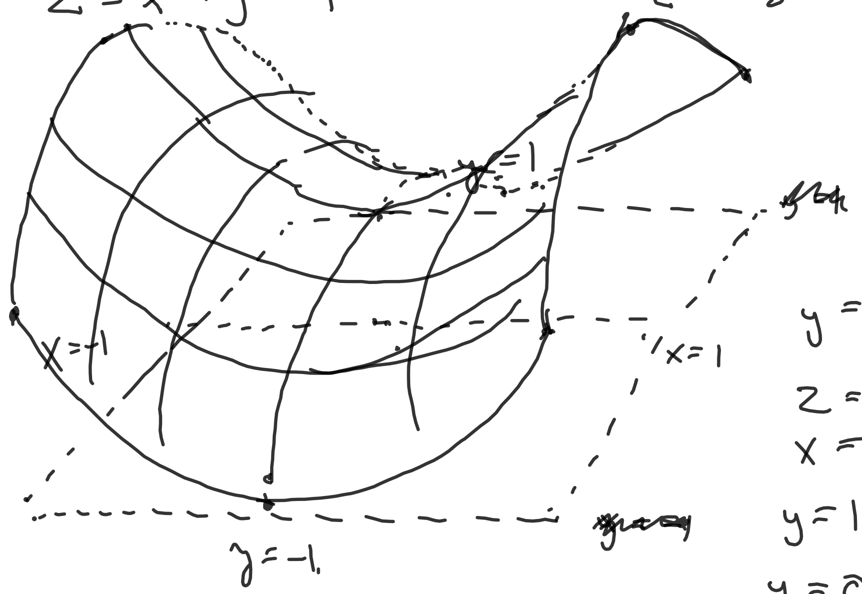
2 variable $z = f(x,y)$




Ex $z = x^2 - y^2 + 1$ $D = [-1,1] \times [-1,1]$




$y = -1$

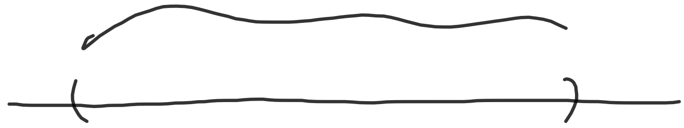
$z = x^2$

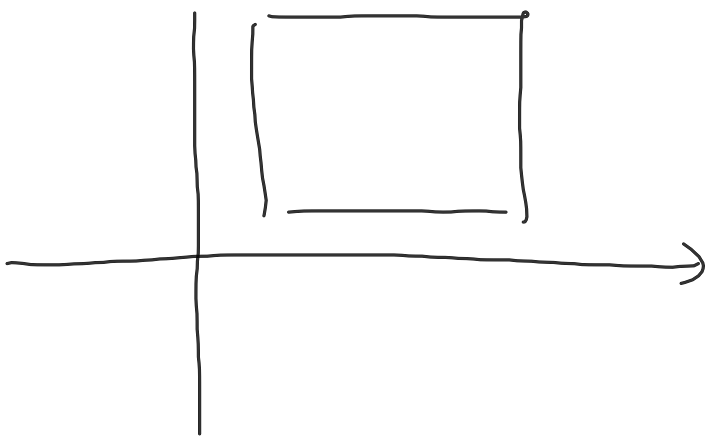
$x = 1, \ z = 2 - y^2$

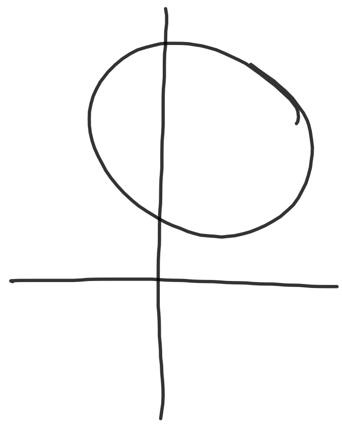
$y = 1, \ z = x^2$

$y = 0, \ z = x^2 + 1$

$x = 0, \ z = 1 - y^2$







Kuleskall med radius R (Sfære med radius R)

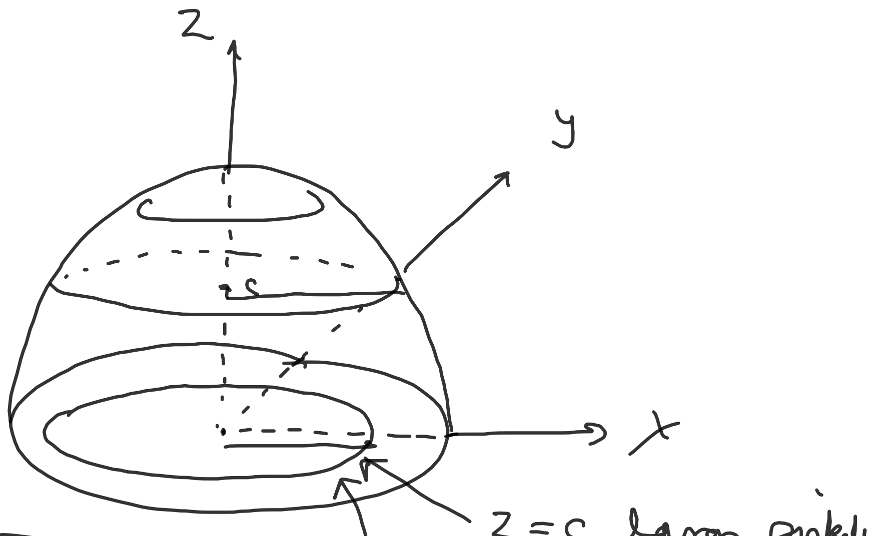
$$x^2+y^2+z^2=R^2$$

$$z=\pm\sqrt{R^2-(x^2+y^2)}$$

Øvre halvsfære $z=\sqrt{R^2-(x^2+y^2)}$

Naturlig def. område

$x^2+y^2 \le R$ : Disk med radius R




$$z=f(x,y)=\sqrt{R^2-(x^2+y^2)}=c.$$

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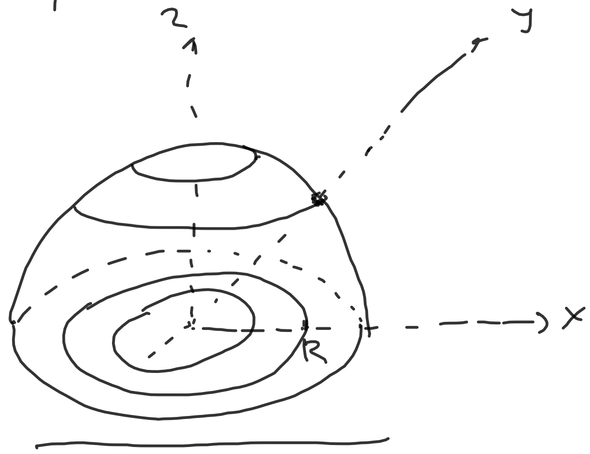
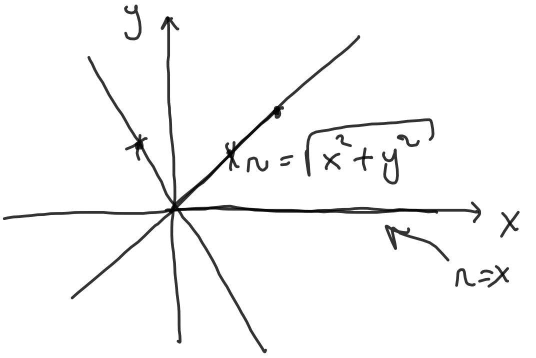
Kombinere: $z = \sqrt{R^2 - (x^2 + y^2)}$

$y = 0$, $z = \sqrt{R^2 - x^2}$

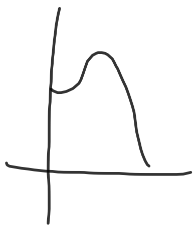


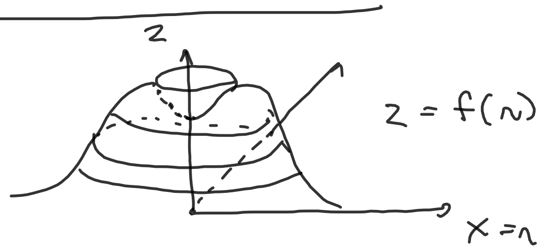

$z = f(x^2 + y^2) = f(r^2)$

Rotasjonsflate



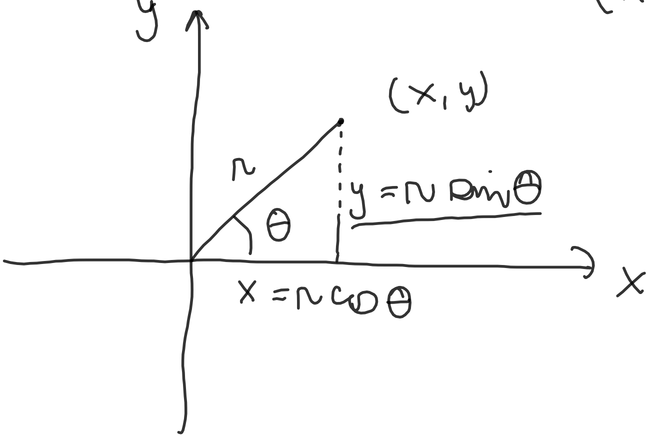

Rotere rundt z-aksen






$z = f(r) = f(\sqrt{x^2 + y^2})$

$(x, y)$ eller $(r, \theta)$



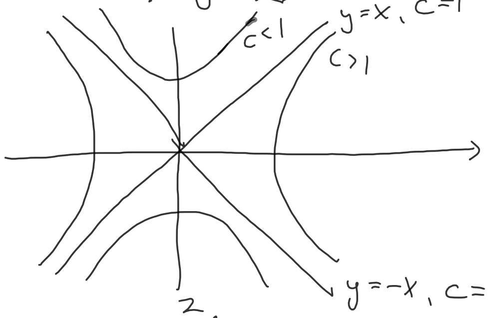

$r = \sqrt{x^2 + y^2}$

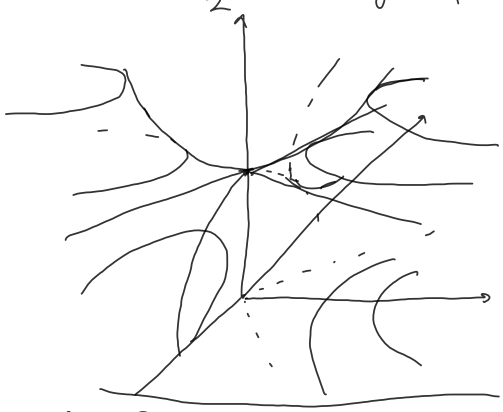
$\sin\theta = \frac{y}{r}$ + kvadrant

Tilbake til sadelflaten: $z = \underbrace{x^2 - y^2 + 1}$

Nivåkurver $x^2 - y^2 + 1 = c$

$x^2 - y^2 = \underbrace{c-1}$




$x^2 - y^2 + 1$

$y = 0$

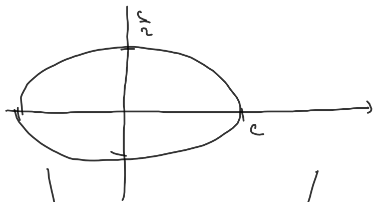
$x^2 + 1$

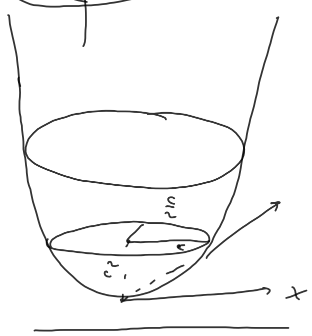
$x = 0$

$1 - y^2$

$z = x^2 + 4y^2$ Parabolsk ellipsoide

Nivåkurver $x^2 + 4y^2 = c^2$ $\quad \frac{x^2}{c^2} + \frac{y^2}{(\frac{c}{2})^2} = 1$




Grafen til $f(x, y, z)$

$w = f(x, y, z)$ Dimensjon 4.

Kan ikke tegne!

Kan se på nivåflaten

$$N_c = \{(x,y,z) \mid f(x,y,z) = c\}.$$

Eks. $f(x,y,z) = x^2 + y^2 + z^2 = c$

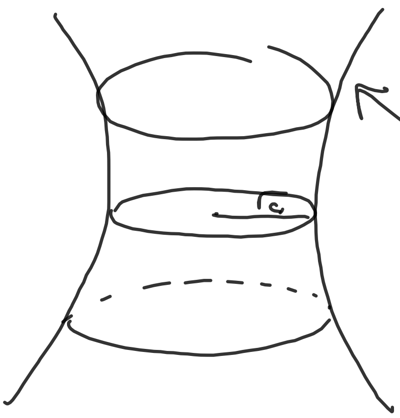
Sfære med radius $\sqrt{c}$, $c \geq 0$.

• $f(x,y,z) = x^2 + y^2 - z^2 = c$

$x^2 + y^2 = c + z^2$

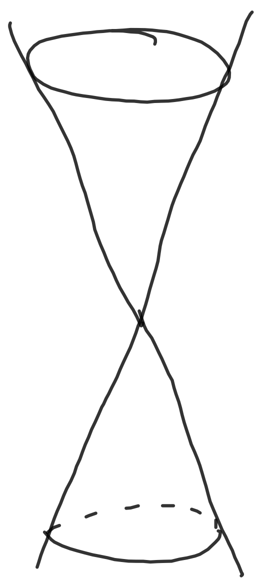
$y = 0$, $\underbrace{x^2 - z^2 = c}$

$c > 0$

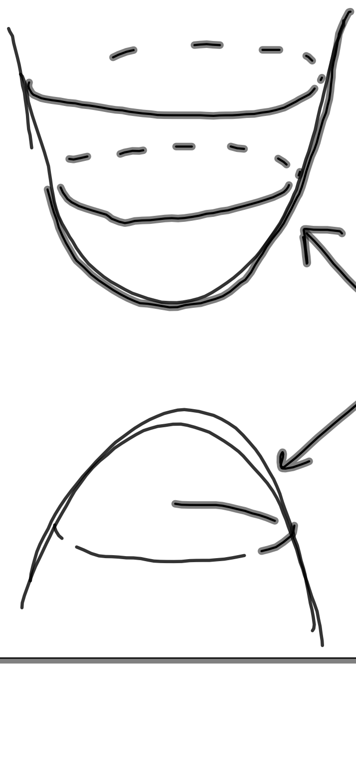


Hyperbel

$c = 0$.



$c < 0$



Hyperbel $x^2 - z^2 = c$

$c < 0$.

md NTNU