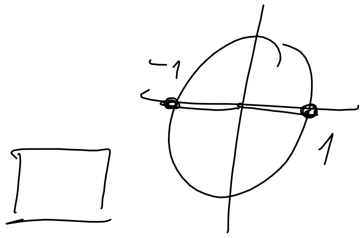
$\boxed{6.1.1}$ d) $\iint_R x\cos(xy)\,dx\,y$ $\qquad R = [1,2]\times[\pi, 2\pi]$

$$= \int_1^2 \int_\pi^{2\pi} x \cos(xy)\,dy\,dx \qquad \int \cos(xy)\,dy = \frac{1}{x}\sin(xy)$$

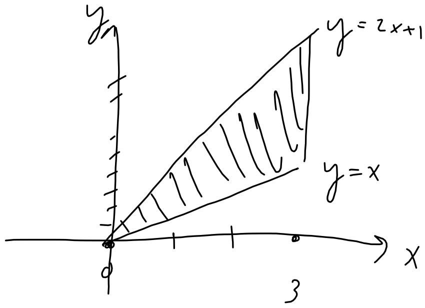
$$= \int_1^2 \Big[\sin(xy)\Big]_\pi^{2\pi} dx = \int_1^2 \sin(2\pi x) - \sin(\pi x)\,dx \qquad \int \sin(kx) = -\frac{1}{k}\cos(kx)$$

$$= \int_1^2 \sin(2\pi x)\,dx - \int_1^2 \sin(\pi x)\,dx$$

$$= \Big[-\frac{\cos(2\pi x)}{2\pi}\Big]_1^2 - \Big[-\frac{\cos(\pi x)}{\pi}\Big]_1^2 = \underline{\underline{\frac{2}{\pi}}}$$



$\boxed{6.2.11}$ b) $\iint_R (x+2xy)\,dx\,dy$ $\qquad R = \{(x,y):\ 0 \le x \le 3,\ \ x \le y \le 2x+1$



$$\int_0^3 \int_x^{2x+1} x + 2xy\ dy\,dx$$

$$= \int_0^3 \Big[xy + xy^2\Big]_{y=x}^{y=2x+1} dx$$

$$= \int_0^3 x(2x+1)(2x+2) - x^2(1+x)\,dx = \dots = \underline{\underline{\frac{459}{4}}}$$

Eksamen 2010

1 a)

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 3 & 1 & 2 \\ 1 & 4 & 0 & 3 \end{pmatrix} \underset{\mathrm{IV}\cdot\frac{1}{3}}{\overset{\mathrm{IV}-\mathrm{I}}{\sim}} \begin{pmatrix} 1 & \boxed{\begin{matrix}1\\2\\3\end{matrix}} & \begin{matrix}0\\1\\1\end{matrix} & \begin{matrix}0\\1\\2\end{matrix} \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\underset{\substack{\mathrm{II}-2\mathrm{IV} \\ \mathrm{III}-3\mathrm{IV}}}{\overset{\mathrm{I}-\mathrm{IV}}{\sim}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \underset{\mathrm{III}-\mathrm{IV}}{\overset{\mathrm{IV}\leftrightarrow\mathrm{II}}{\sim}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

b) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \right\}$ Lin. uavh. ? Nei!

c)

$$\begin{aligned} x + y &= 0 \\ 2y + z &= 1 \\ 3y + z &= 2 \\ x + 4y &= 3 \end{aligned} \iff \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 3 & 1 & 2 \\ 1 & 4 & 0 & 3 \end{array}\right) = A$$

$$\underline{\underline{x = -1, \; y = 1, \; z = -1}}$$

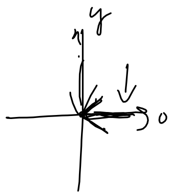
② $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0) \end{cases}$$

a) $\underline{(x,y) \neq (0,0):}$

$$\frac{\partial f}{\partial x}(x,y) = \frac{\overbrace{y\cdot(x^2+y^2)}^{x^2y} - \overbrace{xy\cdot(2x)}^{2x^2y}}{(x^2+y^2)^2} = \frac{y^3 - x^2y}{(x^2+y^2)^2} = \frac{-y(x^2-y^2)}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{x(x^2+y^2) - xy\cdot 2y}{(x^2+y^2)^2} = \frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

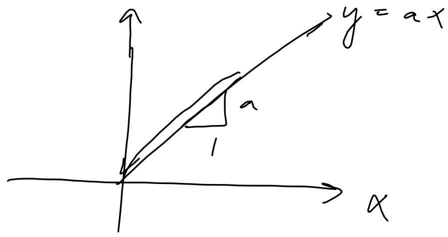


$\underline{(x,y) = (0,0)}$

$$\frac{\partial f}{\partial x} = \lim_{x\to 0} \left.\frac{xy}{x^2+y^2}\right|_{y=0} = \lim_{x \to 0} \frac{0}{x^2} = 0$$

$$\frac{\partial f}{\partial y} = \lim_{y\to 0} \left.\frac{xy}{x^2+y^2}\right|_{x=0} = \lim_{y \to 0} \frac{0}{y^2} = 0$$

ⓑ $y = ax$



$$f(x,y) = f(x, ax) = \frac{x\cdot ax}{x^2+(ax)^2}$$

$$= \frac{x^2\cdot a}{x^2 + x^2\cdot a^2} = \frac{a}{1+a^2} \neq 0 \quad \text{når } a \neq 0$$

f er <u>ikke</u> kont. i origo bortsett fra langs x-aksen.

ⓒ