Plenum 4/3-15

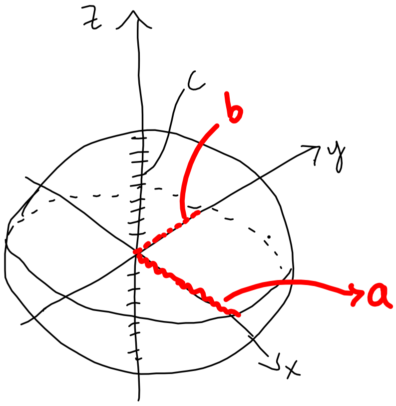
3.9: 6, 8, (14)

6.1: 1efg, 7

6.2: 3

3.9: Parametriserte flater

6) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



Kan omskrives:

$$\underbrace{\left(\frac{x}{a}\right)}_{=\tilde{x}}^2 + \underbrace{\left(\frac{y}{b}\right)}_{=\tilde{y}}^2 + \underbrace{\left(\frac{z}{c}\right)}_{=\tilde{z}}^2 = 1$$

Så: $\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2 = 1$

↳ Kule med sentrum origo, radius 1.

Kulekoordinater: $\tilde{x} = R\sin\phi\cos\theta$

s.195

$\tilde{y} = R\sin\phi\sin\theta$

$\tilde{z} = \underbrace{R}_{=1}\cos\phi \quad , \quad \phi \in [0,\pi], \ \theta \in [0, 2\pi]$

Das:

$$\frac{x}{a} = \tilde{x} = \sin\phi\cos\theta \qquad x = a\sin\phi\cos\theta$$

$$\frac{y}{b} = \tilde{y} = \sin\phi\sin\theta \quad \Rightarrow \quad y = b\sin\phi\sin\theta$$

$$\frac{z}{c} = \tilde{z} = \cos\phi \qquad z = c\cos\phi$$

Parametriseringen er:

$$\vec{r}(\phi,\theta) = (a\sin\phi\cos\theta, \ b\sin\phi\sin\theta, \ c\cos\phi),$$

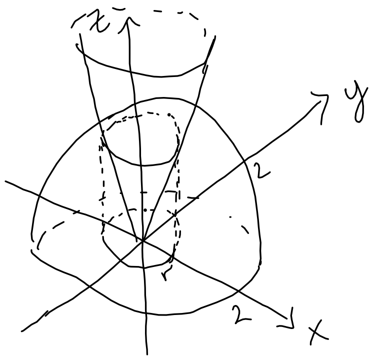
$$\underline{\underline{\phi\in[0,\pi] \ \& \ \theta\in[0,2\pi]}}$$

8) Kule:

$$x^2 + y^2 + z^2 = 4 = 2^2$$

Kjegle

$$z^2 = 3(x^2+y^2)$$



Skjæring kule & kjegle

$$\underbrace{z^2}_{\boxed{\text{kjegle}}} = 3(x^2+y^2) = \underbrace{3(4-z^2)}_{\boxed{\text{kula}}}$$

$$4z^2 = 12$$

$$z = \pm\sqrt{3}$$

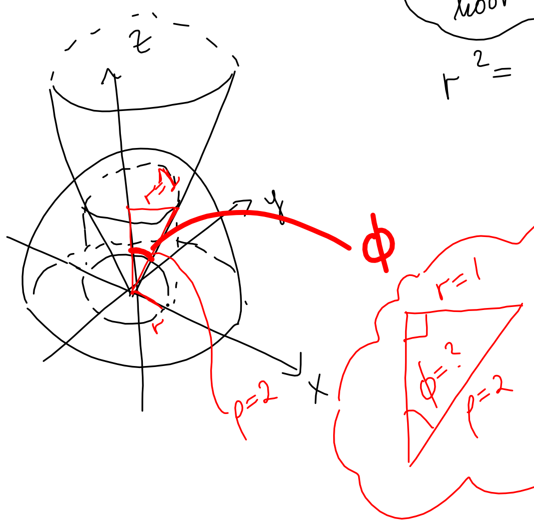
Del over xy-planet $\Rightarrow$ $\underline{z = \sqrt{3}}$

$$\underline{4} = x^2 + y^2 + z^2 = r^2 + z^2 = \underline{r^2 + 3}$$

Kule-koordinater

siden radius $\Rightarrow > 0$

$$r^2 = 1 \Rightarrow r = \pm 1 \Rightarrow r = 1$$



$\sin\phi = \frac{1}{2}$

$\phi = \frac{\pi}{6}$

Vet: $\theta \in [0, 2\pi]$ siden vil ha hele sirkelen.

Kulekoordinater:

$$\vec{r}(\theta, \phi) = (2\cos\theta \sin\phi,\ 2\sin\theta\sin\phi,\ 2\cos\phi)$$

der $\underline{\underline{\phi \in [0, \frac{\pi}{6}]}}$, $\theta \in [0, 2\pi]$