1.9, nr 1 $\quad T: \mathbb{R}^3 \to \mathbb{R}^2$

Alt 1: $T(x,y,z) = \begin{pmatrix} 2x - y + z \\ -x + y - 3z \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & -3 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Alt 2: Første søyle i A:

$T(\vec{e}_1) = T(1,0,0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Andre søyle i A:

$T(\vec{e}_2) = T(0,1,0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Tredje søyle i A:

$T(\vec{e}_3) = T(0,0,1) = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & -3 \end{pmatrix}$

1.9, nr 3: $\quad \vec{T}: \mathbb{R}^2 \to \mathbb{R}^2 \quad \vec{T}(\vec{a}) = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \ \vec{T}(\vec{b}) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

$\boxed{T: \mathbb{R}^2 \to \mathbb{R}}$

Hva er $\vec{T}(3\vec{a} - 2\vec{b})$?

Siden $\vec{T}$ er lineær, så

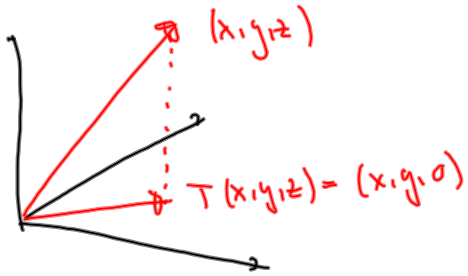
$\vec{T}(3\vec{a} - 2\vec{b}) = \vec{T}(3\vec{a}) - \vec{T}(2\vec{b}) = 3\vec{T}(\vec{a}) - 2\vec{T}(\vec{b})$

$= 3\begin{pmatrix} -2 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -6 \\ -3 \end{pmatrix}}}$

$f(x+y) = f(x) + f(y)$

$f(ax) = af(x)$

1.9, nr 7: $T: \mathbb{R}^3 \to \mathbb{R}^3$



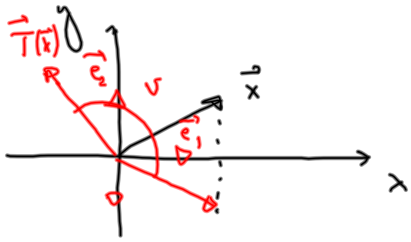
$T(\vec{e}_1) = T(1,0,0) = \begin{pmatrix}1\\0\\0\end{pmatrix}$

$T(\vec{e}_2) = T(0,1,0) = \begin{pmatrix}0\\1\\0\end{pmatrix}$

$T(\vec{e}_3) = T(0,0,1) = \begin{pmatrix}0\\0\\0\end{pmatrix}$

$A = \begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}$, $\quad A\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\y\\0\end{pmatrix}$

1.9, nr 8: $T: \mathbb{R}^2 \to \mathbb{R}^2$ <
1. speile om x-aksen
2. rotere en vinkel ϑ



Operasjon 1: $S(\vec{e}_1) = S\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

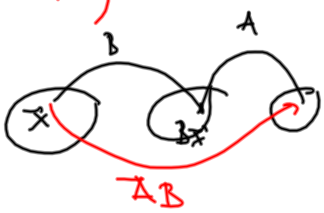
$S(\vec{e}_2) = S\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$

$A_s = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$

Operasjon 2: $A_\vartheta = \begin{pmatrix}\cos\vartheta & -\sin\vartheta\\ \sin\vartheta & \cos\vartheta\end{pmatrix}$

$$A_T = A_\vartheta \cdot A_s = \begin{pmatrix}\cos\vartheta & -\sin\vartheta\\ \sin\vartheta & \cos\vartheta\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$= \underline{\begin{pmatrix}\cos\vartheta & \sin\vartheta\\ \sin\vartheta & -\cos\vartheta\end{pmatrix}}$$



<u>1.10, nr 1:</u> $\vec{F}(x,y,z) = \begin{pmatrix} 2x - 3y + z - 7 \\ -x + z - 2 \end{pmatrix}$ $\quad \vec{b}$

Affinavbildung

$\vec{F}(\vec{x}) = \underbrace{A}_{\text{matrise}}\vec{x} + \underbrace{\vec{b}}_{\text{vektor}}$

$\vec{F}(x,y,z) = \begin{pmatrix} 2x - 3y + z \\ -x + z \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & -3 & 1 \\ -1 & 0 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \underbrace{\begin{pmatrix} 7 \\ 2 \end{pmatrix}}_{\vec{b}}$

<u>1.10, nr 2:</u> $\vec{r}(t) = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2+t \\ -1 \\ 3+2t \end{pmatrix}$

$\vec{F}(x,y,z) = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\vec{F}(\vec{r}(t)) = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix} \begin{pmatrix} 2+t \\ -1 \\ 3+2t \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 2+t+1+6+4t \\ -3-6-4t \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 9+5t \\ -9-4t \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 11+5t \\ -10-4t \end{pmatrix} = \begin{pmatrix} 11 \\ -10 \end{pmatrix} + t\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

1.10, nr 3: Affinavbildung $\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$

$\vec{F}(0,0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\vec{F}(1,0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\vec{F}(0,1) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

Generelt: $\vec{F}(\vec{x}) = A\vec{x} + \vec{b} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

Vel

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \vec{F}(0,0) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \qquad \alpha = 1, \quad \beta = -1$$

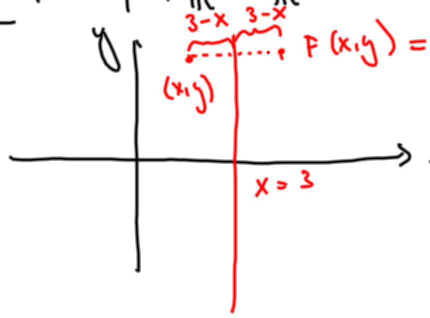
$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \vec{F}(1,0) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \vec{F}(0,1) = \begin{pmatrix} 1 & b \\ 4 & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad a = 1, \; c = 4$$

$$= \begin{pmatrix} b \\ d \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow b = -2, \; d = 1$$

1.10, nr 5: $\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$

a)



$$F(x,y) = (x + 2(3-x), y) = (6-x, y) = \begin{pmatrix} -x \\ y \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}_{A}\begin{pmatrix} x \\ y \end{pmatrix} + \underbrace{\begin{pmatrix} 6 \\ 0 \end{pmatrix}}_{\vec{b}}$$

A3, nr 2: $(1, 2, 4, 8, 16, \dots, 4096) = (2^0, 2^1, 2^2, \dots, 2^{12})$

= 2.^(0,1,2,.., 12)