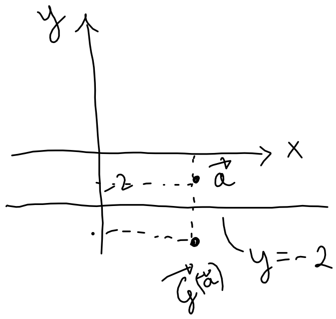
$$\begin{bmatrix}6\\1\end{bmatrix} = \vec{F}(0,1) = A\begin{bmatrix}0\\1\end{bmatrix} + \begin{bmatrix}6\\0\end{bmatrix} = \begin{bmatrix}a_{12}+6\\a_{22}+0\end{bmatrix}$$

Figur affin

$$\begin{matrix} a_{12}+6=6 \\ a_{22}=1 \end{matrix} \Rightarrow \begin{matrix} a_{12}=0 \\ a_{22}=1 \end{matrix}$$

$$\Rightarrow A = \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} \qquad \left(\vec{b} = \begin{bmatrix}6\\0\end{bmatrix}\right)$$

b)



2.7: Kjerneregelen

2) $f(u,v) = ue^{-v}$, $g(x,y,z) = 2xy+z$, $h(x,y,z) = 2y(z+x)$

$$k(x,y,z) = f(\underbrace{g(x,y,z)}_{\text{"spiller rollen som } u\text{"}}, \underbrace{h(x,y,z)}_{\text{"spiller rollen som } v\text{"}})$$

$$\frac{\partial k}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial g}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial h}{\partial x} = e^{-2y(z+x)}2y - (2xy+z)e^{-2y(z+x)}2y$$

Kjerneregel på komponentform

$$= 2ye^{-2y(z+x)}(1-(2xy+z))$$

$$\frac{\partial k}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial g}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial h}{\partial y} = e^{-2y(z+x)} 2x - (2xy+z)e^{-2y(z+x)} \cdot$$

$$\cdot\, 2(z+x)$$

$$= e^{-2y(z+x)}\left(2x - 2(2xy+z)(z+x)\right)$$

8) $T = f(x,y)$, $x = r\cos\theta$, $y = r\sin\theta$

$T(r,\theta) = f(r\cos\theta, r\sin\theta)$

a) $$\frac{\partial T}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x(r,\theta)}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y(r,\theta)}{\partial r}$$

Kjerneregel på komponentform

$$= \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x(r,\theta)}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y(r,\theta)}{\partial \theta} = -\frac{\partial f}{\partial x} r\sin\theta + \frac{\partial f}{\partial y} r\cos\theta$$

b) $r = g(t)$, $\theta = h(t)$;

$$T'(t) = \frac{\partial T}{\partial r}\underbrace{\frac{\partial r}{\partial t}}_{r'(t)} + \frac{\partial T}{\partial \theta}\underbrace{\frac{\partial \theta}{\partial t}}_{\theta'(t)} \overset{a)}{=} \left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)g'(t) + \left(-\frac{\partial f}{\partial x} r\sin\theta + \frac{\partial f}{\partial y} r\cos\theta\right)h'(t)$$

2.8: Linearisering

2) $\vec{F}(x,y) = \begin{bmatrix} x\sin(xy) \\ xe^{y} \\ 2x^3+y \end{bmatrix}$, $\vec{a} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$$\vec{T}_{\vec{a}}\vec{F}(\vec{x}) = \vec{F}(\vec{a}) + \vec{F}'(\vec{a})(\vec{x}-\vec{a}) = \begin{bmatrix} 0 \\ 2 \\ 16 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 1 & 2 \\ 24 & 1 \end{bmatrix}(\vec{x}-\vec{a})$$

$$\vec{F}'(x,y) = \begin{bmatrix} \sin(xy) + x\cos(xy)y & x^2\cos(xy) \\ e^y & xe^y \\ 6x^2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 2 \\ 16 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 1 & 2 \\ 24 & 1 \end{bmatrix}\begin{bmatrix} x-2 \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 16 \end{bmatrix} + \begin{bmatrix} 4y \\ x-2+2y \\ 24x-48+y \end{bmatrix}$$

$$= \begin{bmatrix} 4y \\ x+2y \\ 24x-32+y \end{bmatrix}$$