FINN: $\iiint_A z\,dx\,dy\,dz$ der A er området over paraboloiden $z=x^2+y^2$ & under kula $x^2+y^2+z^2=2$

3) a) · Skjæring paraboloide & kule: Når $r=1$, fra figur. Vil bruke sylinderkoordinater ⇒ $r\in[0,1]$ og $\theta\in[0,2\pi]$. Hva med z?

Fra fig: z skal være <u>over</u> paraboloide, dvs:

$$z \geq x^2+y^2 = \underbrace{r^2}_{\text{sylinderkoord}}$$

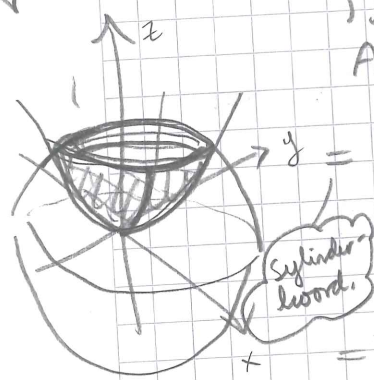
z skal også være <u>under</u> kula:

$$z^2 \leq 2-x^2-y^2$$

$$z \leq \sqrt{2-x^2-y^2} = \sqrt{2-r^2} \Rightarrow z\in[r^2, \sqrt{2-r^2}]$$

+ siden z er positiv fra figur

<u>Så</u>:



$$\iiint_A z\,dx\,dy\,dz$$

$$= \int_0^{2\pi}\int_0^1\int_{r^2}^{\sqrt{2-r^2}} z\,r\,dz\,dr\,d\theta$$

Jacobideterminant

Går <u>fra</u> paraboloide <u>til</u> kule

$$= \int_0^{2\pi}\int_0^1 \left[\frac{1}{2}z^2 r\right]_{z=r^2}^{\sqrt{2-r^2}} dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left(\frac{1}{2}(2-r^2)r - \frac{1}{2}r^5\right) dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left(-\frac{1}{2}r^5 - \frac{1}{2}r^3 + r\right) dr\,d\theta$$

$$= \int_0^{2\pi}\left[-\frac{1}{12}r^6 - \frac{1}{8}r^4 + \frac{1}{2}r^2\right]_{r=0}^{1} d\theta$$

$$= \int_0^{2\pi}\left(-\frac{1}{12} - \frac{1}{8} + \frac{1}{2}\right)d\theta = \int_0^{2\pi}\frac{-2-3+12}{24}\,d\theta$$

$$= \underline{\underline{\frac{7\pi}{12}}}$$