4.11.8 x_n salgstall avis A etter n år

y_n —— " —— B

z_n —— " —— C

$$x_{n+1} = \underbrace{0.1\,x_n}_{\text{nye}} + \underbrace{0.1\,y_n}_{\text{fra B}} + \underbrace{0.9\,x_n}_{\text{beholder}} = x_n + 0.1\,y_n$$

$$y_{n+1} = \underbrace{0.2\,y_n}_{\text{nye (NB!)}} + \underbrace{0.1\,x_n}_{\text{fra A}} + \underbrace{0.1\,z_n}_{\text{fra C}} + \underbrace{0.8\,y_n}_{\text{beholder}} = 0.1\,x_n + y_n + 0.1\,z_n$$

$$z_{n+1} = \underbrace{0.1\,z_n}_{\text{nye}} + \underbrace{0.1\,y_n}_{\text{fra B}} + \underbrace{0.9\,z_n}_{\text{beholder}} = 0.1\,y_n + z_n$$

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0.1 & 0 \\ 0.1 & 1 & 0.1 \\ 0 & 0.1 & 1 \end{pmatrix}}_{M} \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$$

b) $M = \begin{pmatrix} 1 & 0.1 & 0 \\ 0.1 & 1 & 0.1 \\ 0 & 0.1 & 1 \end{pmatrix}$

$$\det(\lambda I - M) = \begin{vmatrix} \lambda-1 & -0.1 & 0 \\ -0.1 & \lambda-1 & -0.1 \\ 0 & -0.1 & \lambda-1 \end{vmatrix} = (\lambda-1)\begin{vmatrix} \lambda-1 & -0.1 \\ -0.1 & \lambda-1 \end{vmatrix} + 0.1\begin{vmatrix} -0.1 & -0.1 \\ 0 & \lambda-1 \end{vmatrix}$$

$$= (\lambda-1)\left((\lambda-1)^2 - 0.01\right) + 0.1\left(-0.1(\lambda-1)\right) = (\lambda-1)^3 - 0.02(\lambda-1)$$

$$= (\lambda-1)\left((\lambda-1)^2 - 0.02\right) = (\lambda-1)\left(\lambda^2 - 2\lambda + 0.98\right)$$

Løser: $\lambda_1 = 1$, $\lambda_2 = 1 + \frac{\sqrt{2}}{10}$, $\lambda_3 = 1 - \frac{\sqrt{2}}{10}$

$\lambda_1 = 1$: $I - A = \begin{pmatrix} 0 & -0.1 & 0 \\ -0.1 & 0 & -0.1 \\ 0 & -0.1 & 0 \end{pmatrix} \sim \dots \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | set $x_3 = -1$, $x_1 = -x_3$

$$\Rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

b) forts.

$\lambda_2 = 1 + \frac{\sqrt{2}}{10}$

$$\left(1+\frac{\sqrt{2}}{10}\right)I - A = \begin{pmatrix} \frac{\sqrt{2}}{10} & -0.1 & 0 \\ -0.1 & \frac{\sqrt{2}}{10} & -0.1 \\ 0 & -0.1 & \frac{\sqrt{2}}{10} \end{pmatrix} \cdot 10 \sim \begin{pmatrix} \sqrt{2} & -1 & 0 \\ 1 & -\sqrt{2} & 1 \\ 0 & 1 & -\sqrt{2} \end{pmatrix}$$

$$\overset{II \leftrightarrow I}{\sim} \begin{pmatrix} 1 & -\sqrt{2} & 1 \\ 0 & 1 & -\sqrt{2} \\ 0 & 1 & -\sqrt{2} \end{pmatrix} \overset{I+\sqrt{2}II}{\sim} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \to \begin{matrix} x_1 = x_3 \\ x_2 = \sqrt{2}\, x_3 \end{matrix}$$

$$\Rightarrow \vec{v}_2 = \underline{\underline{\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}}} \qquad (\text{sätt } x_3 = 1)$$

$\lambda_3 = 1 - \frac{\sqrt{2}}{10}$: får på samma sätt $\quad \vec{v}_3 = \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$

c) uttrykk $\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ ved hjelp av $\vec{v}_1, \vec{v}_2, \vec{v}_3$

$$\begin{array}{c} \left(\begin{array}{cccc} 1 & 1 & 1 & x_0 \\ 0 & \sqrt{2} & -\sqrt{2} & y_0 \\ -1 & 1 & 1 & z_0 \end{array}\right) \\ \begin{array}{cccc} \vec{v}_1 & \vec{v}_2 & \phantom{-\sqrt{2}} & \end{array} \end{array} \underset{\text{III}+\text{I}}{\overset{\text{II}/\sqrt{2}}{\sim}} \begin{pmatrix} 1 & 1 & 1 & x_0 \\ 0 & 1 & -1 & \frac{y_0}{\sqrt{2}} \\ 0 & 2 & 2 & x_0+z_0 \end{pmatrix}$$

$$\overset{\text{III}-2\text{II}}{\sim} \begin{pmatrix} 1 & 1 & 1 & x_0 \\ 0 & 1 & -1 & y_0/\sqrt{2} \\ 0 & 0 & 4 & x_0-\sqrt{2}y_0+z_0 \end{pmatrix} \overset{\text{3 radops.}}{\sim} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{2}(x_0-z_0) \\ 0 & 1 & 0 & \frac{1}{4}(x_0+\sqrt{2}y_0+z_0) \\ 0 & 0 & 1 & \frac{1}{4}(x_0-\sqrt{2}y_0+z_0) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \frac{1}{2}(x_0-z_0)\vec{v}_1 + \frac{1}{4}(x_0+\sqrt{2}y_0+z_0)\vec{v}_2 + \frac{1}{4}(x_0-\sqrt{2}y_0+z_0)\vec{v}_3$$

$$\begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \frac{1}{2}(x_0-z_0)1^n\vec{v}_1 + \frac{1}{4}(x_0+\sqrt{2}y_0+z_0)\left(1+\frac{\sqrt{2}}{10}\right)^n\vec{v}_2 + \frac{1}{4}(x_0-\sqrt{2}y_0+z_0)\left(1-\frac{\sqrt{2}}{10}\right)^n\vec{v}_3$$

når $n\to\infty$: $\frac{1}{4}(x_0+\sqrt{2}y_0+z_0)\left(1+\frac{\sqrt{2}}{10}\right)^n\vec{v}_2 = \frac{1}{4}(x_0+\sqrt{2}y_0+z_0)\left(1+\frac{\sqrt{2}}{10}\right)^n\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$

vil dominere når $n\to\infty$.

$k_1 = \lim\limits_{n\to\infty}\frac{x_n}{y_n} = \frac{1}{\sqrt{2}}$ $\qquad k_2 = \lim\limits_{n\to\infty}\frac{y_n}{z_n} = \frac{\sqrt{2}}{1} = \sqrt{2}$