2.7.2

$$f(u,v) = ue^{-v}$$

$$g(x,y,z) = 2xy + z$$

$$h(x,y,z) = 2y(z+x)$$

$$k(x,y,z) = f(g(x,y,z), h(x,y,z))$$

$$\frac{\partial k}{\partial x}, \frac{\partial k}{\partial y}, \frac{\partial k}{\partial z} = ?$$

$$\frac{\partial f}{\partial u} = e^{-v}, \quad \frac{\partial f}{\partial v} = -ue^{-v}$$

$$\frac{\partial k}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial g}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial h}{\partial x}$$

$$= e^{-2y(z+x)}\,2y + \left(-(2xy+z)\right)e^{-2y(z+x)}\,2y$$

$$= (2y - 4xy^2 - 2yz)\,e^{-2y(z+x)}$$

$$\frac{\partial k}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial g}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial h}{\partial y}$$

$$\frac{\partial k}{\partial z} = \frac{\partial f}{\partial u}\frac{\partial g}{\partial z} + \frac{\partial f}{\partial v}\frac{\partial h}{\partial z}$$ regnes ut på samme måte

2.7.5

$G : \mathbb{R}^2 \to \mathbb{R}^3, \quad F : \mathbb{R}^3 \to \mathbb{R}^2$

$H(x) = F(G(\vec{x})) \quad \vec{x} \in \mathbb{R}^2,$

$H : \mathbb{R}^2 \to \mathbb{R}^2, \quad G(1,-2) = (1,2,3)$

$$G'(1,-2) = \begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix}, \quad F'(1,2,3) = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix}$$

$$H'(1,-2) = F'(1,2,3)\, G'(1,-2)$$

$$= \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 13 & -7 \\ 10 & 0 \end{pmatrix}$$

2.7.8

a) $T = f(x,y)$ temperatur-funksjon

(x,y) posisjon

Innfører polarkoordinater $x = r\cos\theta$
$y = r\sin\theta$

$T(r,\theta) = f(r\cos\theta, r\sin\theta)$ Fra kjerneregelen

$$\frac{\partial T}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} =$$

$$= \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta} =$$

$$= \frac{\partial f}{\partial x}(-r\sin\theta) + \frac{\partial f}{\partial y} r\cos\theta$$

b) fugl som följer en

kurve $(r(t), \theta(t))$

$r(t) = g(t), \quad \theta(t) = h(t)$

$T(t) = T(r(t), \theta(t))$

$$T'(t) = \frac{\partial T}{\partial r} r'(t) + \frac{\partial T}{\partial \theta} \theta'(t)$$

$$= \left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right) g'(t)$$

$$+ \left(\frac{\partial f}{\partial x}(-r\sin\theta) + \frac{\partial f}{\partial y} r\cos\theta\right) h'(t)$$

2.8.2

$A \subset \mathbb{R}^n$, $F: A \to \mathbb{R}^m$

$\bar{a} \in A$, $T_a F_{(\bar{x})} = F(\bar{a}) + F'(\bar{a})(\bar{x} - \bar{a})$

Her $F(x,y) =$

$$= \begin{pmatrix} x \sin(xy) \\ x e^y \\ 2x^3 + y \end{pmatrix} \qquad \bar{a} = (2,0)$$

$$F'(x,y) = \begin{pmatrix} \sin(xy) + xy\cos(xy), & x^2\cos(xy) \\ e^y, & xe^y \\ 6x^2, & 1 \end{pmatrix}$$

$$F(\bar{a}) = \begin{pmatrix} 0 \\ 2 \\ 16 \end{pmatrix}, \quad F'(\bar{a}) = \begin{pmatrix} 0 & 4 \\ 1 & 2 \\ 24 & 1 \end{pmatrix}$$

$$T_a F(x,y) = \begin{pmatrix} 0 \\ 2 \\ 16 \end{pmatrix} + \begin{pmatrix} 0 & 4 \\ 1 & 2 \\ 24 & 1 \end{pmatrix} \begin{pmatrix} x-2 \\ y \end{pmatrix}$$

$$= \begin{pmatrix} 4y \\ x+2y \\ 24x+y-32 \end{pmatrix}$$