4.3.2

b)
$$\begin{pmatrix} 2 & 1 & 3 & 4 \\ 1 & 1 & 0 & 3 \end{pmatrix} \overset{I \leftrightarrow II}{\sim} \begin{pmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & 3 & 4 \end{pmatrix}$$

$$\overset{II - 2I}{\sim} \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & -1 & 3 & -2 \end{pmatrix}$$

$$\overset{\substack{I + II \\ (-1) \cdot II}}{\sim} \underline{\underline{\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -3 & 2 \end{pmatrix}}}$$

4.4.6

$$C = \begin{pmatrix} 1 & 1 & -1 & 0 \\ \underline{2} & -1 & -5 & 3 \\ \underline{-1} & 2 & a^2+3a & -3a \end{pmatrix} \underset{\substack{II-2I \\ III+I}}{\overset{2 \text{ radops}}{\sim}} \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & a^2+3a-4 & 3-3a \end{pmatrix}$$

$$a^2+3a-4 = 0 \iff a = \frac{-3 \pm \sqrt{9+16}}{2} \iff a = 1 \text{ eller } a = -4$$

$$a=1: \quad C \sim \underline{\begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}}$$

$$a=-4: \quad C \sim \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 15 \end{pmatrix} \sim \underline{\begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

$$a \neq 1, a \neq -4: \quad C \sim \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & \dfrac{3-3a}{\underbrace{a^2+3a-4}_{(a-1)(a+4)}} \end{pmatrix} \sim \underline{\begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -\dfrac{3}{a+4} \end{pmatrix}}$$

b) For $a=1$ svarer systemet til $\left(\begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right)$

$$\begin{aligned} x+y-z &= 0 \\ y+z &= -1 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} x &= -y+z \\ y &= -z-1 \end{aligned}$$

Så $a=1$ gir uendelig mange løsninger (z kan velges fritt)

$a=-4$: Ingen løsninger (siste søyle er pivotsøyle)

$a \neq 1$, $a \neq -4$: Nøyaktig en løsning (alle søyler unntatt siste er pivotsøyler)

4.5.6

a)

$$\begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccccccc} & & & \multicolumn{3}{c}{\overbrace{\hphantom{0 \quad 0 \quad 0}}^{I}} & \text{HS (b)} \\ \end{array}$$

$$\begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & 0 & 1 & 0 & 3 \\ 0 & -2 & 1 & 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\overset{III+2II}{\sim} \begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 0 & 5 \\ 0 & 1 & \textcircled{1} & 0 & 1 & 0 & 3 \\ 0 & 0 & 3 & 0 & 2 & 1 & 9 \end{pmatrix}$$

$$\frac{1}{3}III \quad \sim \quad II - III \quad \begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 3 \end{pmatrix}$$

$$\overset{I-2II}{\sim} \begin{pmatrix} 1 & 0 & 0 & 1 & -\frac{2}{3} & \frac{2}{3} & 5 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 3 \end{pmatrix}$$

Derfor: $B^{-1} = \begin{pmatrix} 1 & -\frac{2}{3} & \frac{2}{3} \\ 0 & \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$ og løsningen av systemet er $x = 5, y = 0, z = 3$ (siste søyle)

b) utvidet matrise for $x + 2y = 5$

$y + z = 3$ har vi skrevet

$-2y + z = 3$ opp i a)

Vi skal altså løse $B\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = B^{-1}\begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{2}{3} & \frac{2}{3} \\ 0 & \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix},$$

som vi også viste i a)

c) $$\begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & -2 & a+1 & b^2-10 \end{pmatrix} \overset{III+2II}{\sim} \begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & a+3 & b^2-4 \end{pmatrix}$$

$a \neq -3$ $$\sim \begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & \frac{b^2-4}{a+3} \end{pmatrix} \Rightarrow$$ en løsning (alle søyler er pivotsøyler)

$a = -3$ $$\sim \begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & b^2-4 \end{pmatrix}$$

$b^2-4 \neq 0 \Leftrightarrow b \neq 2, b \neq -2$: Ingen løsninger (siste søyle er pivotsøyle)

$b^2-4 = 0 \Leftrightarrow b = 2$ eller $b = -2$: Uendelig mange løsninger (tredje søyle ikke pivotsøyle)

4.4.4

a)
$$\begin{pmatrix} 0 & 1 & 2 & 0 & 0 \\ \textcircled{1} & 0 & -1 & 1 & 1 \\ 6 & 0 & -6 & 7 & h \\ 2 & 1 & 0 & 0 & 0 \end{pmatrix} \overset{I \leftrightarrow II}{\sim} \dots$$

$$\sim \begin{pmatrix} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & h-6 \\ 0 & 0 & 0 & 0 & 2h-14 \end{pmatrix}$$

$2h-14 \neq 0 \Leftrightarrow h \neq 7$: Ingen løsninger (siste søyle pivotsøyle)

$h=7$:

$$\begin{pmatrix} 1 & \textcircled{0} & -1 & \textcircled{1} & 1 \\ 0 & 1 & 2 & \textcircled{0} & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \overset{I-III}{\sim} \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} \Rightarrow x_1 - x_3 = 0 \\ \Rightarrow x_2 + 2x_3 = 0 \\ \Rightarrow x_4 = 1 \end{matrix}$$

Løsninger: $x_1 = x_3$, $x_2 = -2x_3$, x_3 vilkårlig
$x_4 = 1$

$$\left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right)$$

4.5.9

· Nabopunktene til X er A, Z, Y

———— || ———— Y X, Z, B

———— || ———— Z X, Y, C

$$\begin{aligned} x &= \tfrac{1}{3}(a+z+y) \\ y &= \tfrac{1}{3}(x+z+b) \\ z &= \tfrac{1}{3}(x+y+c) \end{aligned} \iff \begin{aligned} 3x - y - z &= a \\ -x + 3y - z &= b \\ -x - y + 3z &= c \end{aligned}$$

$$\underbrace{\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}_{\vec{b}}$$

b)

$$A^{-1}: \left(\begin{array}{ccc|ccc} 3 & -1 & -1 & 1 & 0 & 0 \\ -1 & 3 & -1 & 0 & 1 & 0 \\ -1 & -1 & 3 & 0 & 0 & 1 \end{array}\right) \sim \cdots \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{array}\right)$$

$$\overbrace{\begin{matrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{matrix}}^{A^{-1}}$$

c) Vi setter $a = 1,\ b = 2,\ z = 3$

$$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Leftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{7}{4} \\ 2 \\ \frac{9}{4} \end{pmatrix} \Rightarrow \begin{array}{l} x = \frac{7}{4} \\ y = 2 \\ z = \frac{9}{4} \end{array}$$

d) $x = 1, y = 9, z = 3$:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = A\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 6 \end{pmatrix}$$

$$\Rightarrow \underline{\underline{a = -2,\ b = 2,\ c = 6}}$$