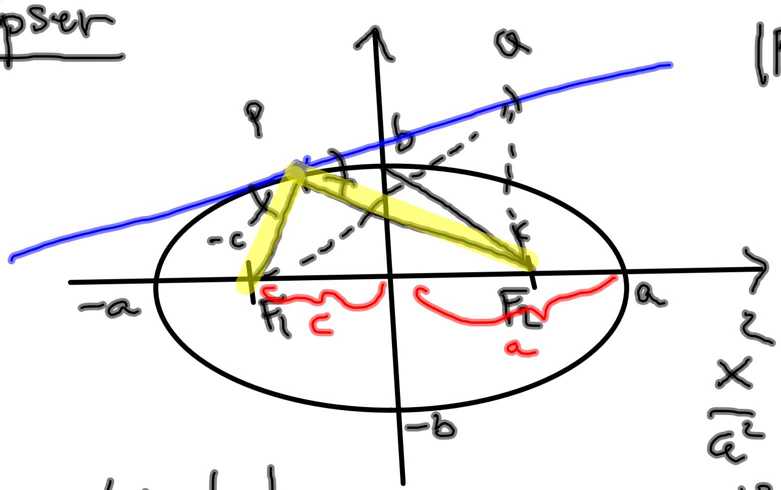
Ellipser



$|PF_1| + |PF_2| = 2a$

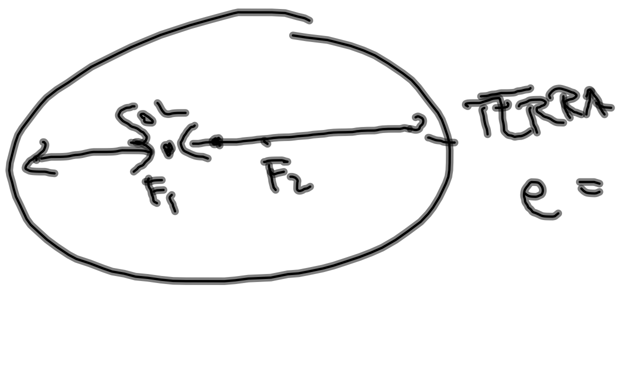
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2 + c^2 = a^2$$

Eksentrisitet:

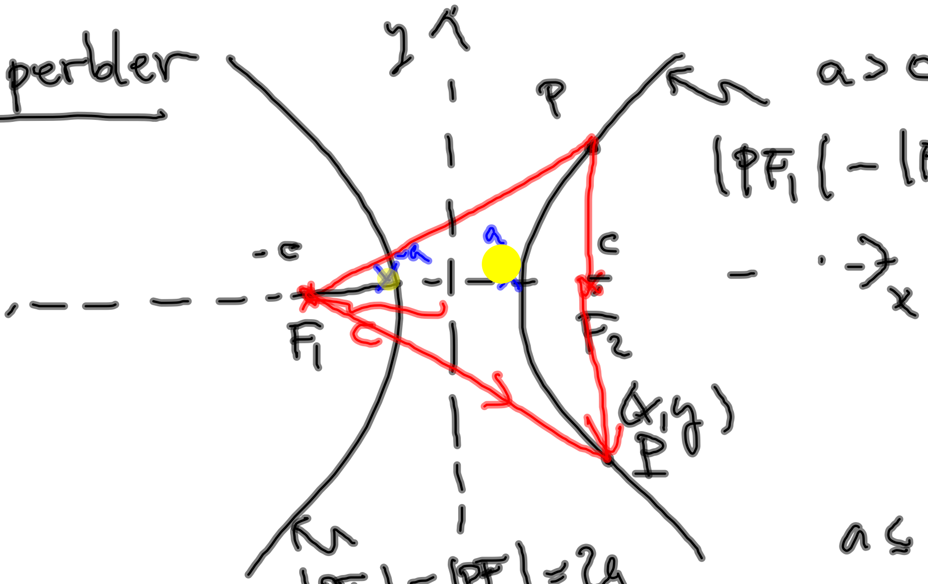
$$e = \frac{c}{a} = \frac{\text{brennvidde}}{\text{store halvakse}}$$

$e = 0$ for en sirkel $e \to 1$ når ellipsen blir langstrakt



$e = \frac{1}{60} \approx 1{,}67\,\%$

Hyperbler



$a > 0$

$|PF_1| - |PF_2| = 2a$

$a < c$

$|PF_2| - |PF_1| = 2a$

$$\sqrt{(x+c)^2+y^2} - \sqrt{(x-c)^2+y^2} = \pm 2a$$

$$\Updownarrow$$

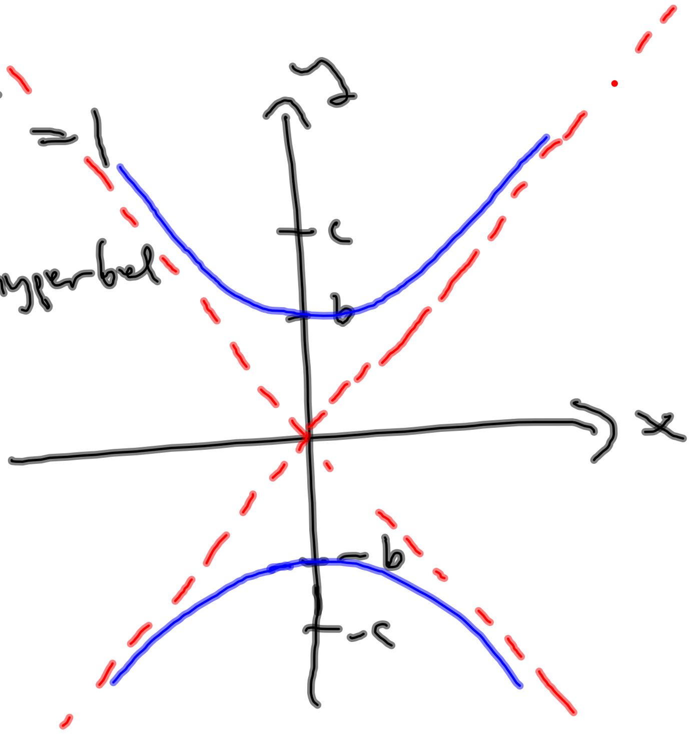
$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

$b^2 = c^2 - a^2$

Kan bytte om på x og y

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

gir en „liggende" hyperbel

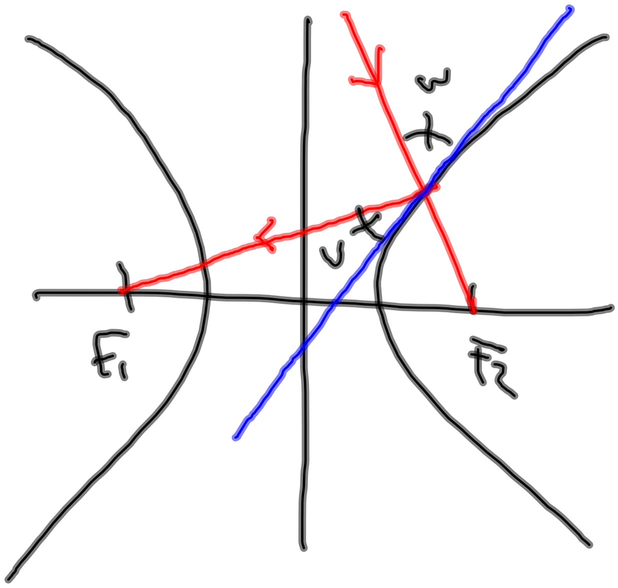


Asymptotene har likninger

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=0$$

$$\frac{x}{a}=\pm\frac{y}{b}$$

Refleksjons-
egenskapen
$v = w$



Parametrisering

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

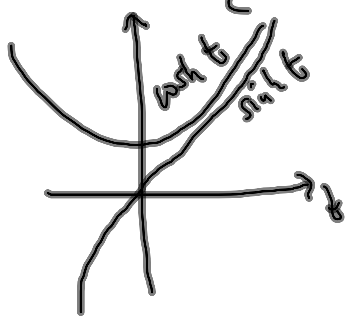
hyperbolske
trig. funk.

$$\cosh t = \frac{e^t + e^{-t}}{2}$$

$$\cosh^2 t - \sinh^2 t = 1$$

$$\sinh t = \frac{e^t - e^{-t}}{2}$$

$\vec{r}(t) =$
$(a \cosh t, b \sinh t)$



Sentrum i (m, n)

$$\frac{(x-m)^2}{a^2} - \frac{(y-n)^2}{b^2} = 1$$

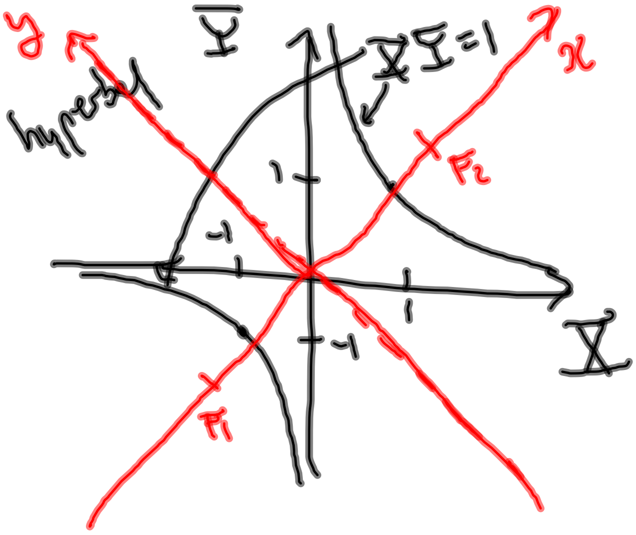
$$\vec{r}(t) = (m + a \cosh t, n + b \sinh t)$$

En ting til:

$$\bar{X}\bar{Y} = 1$$

$$\bar{X} = \frac{x-y}{\sqrt{2}}$$

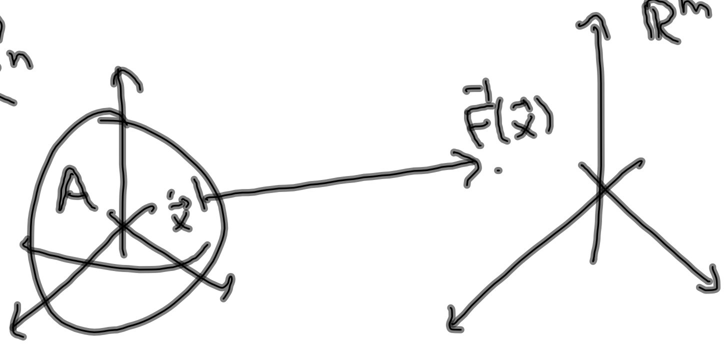
$$\bar{Y} = \frac{x+y}{\sqrt{2}}$$



$$\frac{x^2}{2} - \frac{y^2}{2} = 1$$

LH 3.7 Grafisk fremstilling av skalarfelt

$$\vec{F} : A \longrightarrow \mathbb{R}^m, \quad A \subset \mathbb{R}^n$$



$n = 2,\ m = 1$

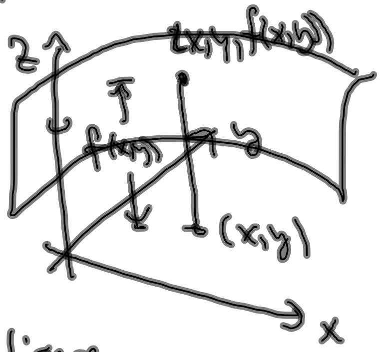
$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$$

graf

$(x,y,z) \in \mathbb{R}^3$

$z = f(x,y)$

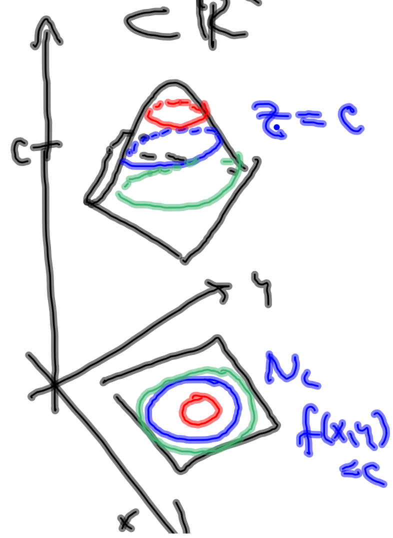
$\{(x,y,f(x,y)) \mid (x,y) \in \mathbb{R}^2\} \subset \mathbb{R}^3$



Hvordan visualisere flater gitt som grafen til en $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$?

<u>Nivåkurver</u>: $c \in \mathbb{R}$

$$N_c = \{(x,y) \mid f(x,y) = c\} = f^{-1}(c) \subset \mathbb{R}^2$$



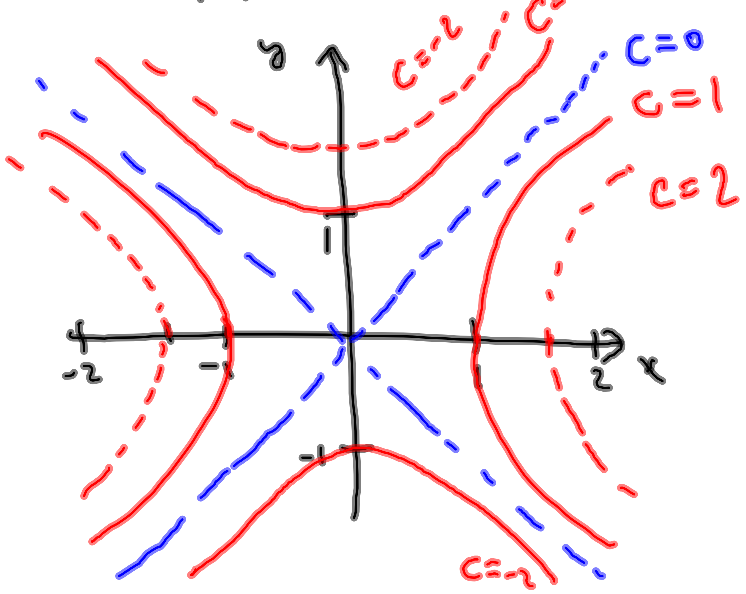
Eks $f(x,y) = x^2 - y^2$

nivåkurver: N_c $x^2 - y^2 = c$

$c > 0$: $\frac{x^2}{c} - \frac{y^2}{c} = 1$ stående hyperbel $a = \sqrt{c}$, $b = \sqrt{c}$

$c = 0$ $\pm x = \pm y$ to linjer

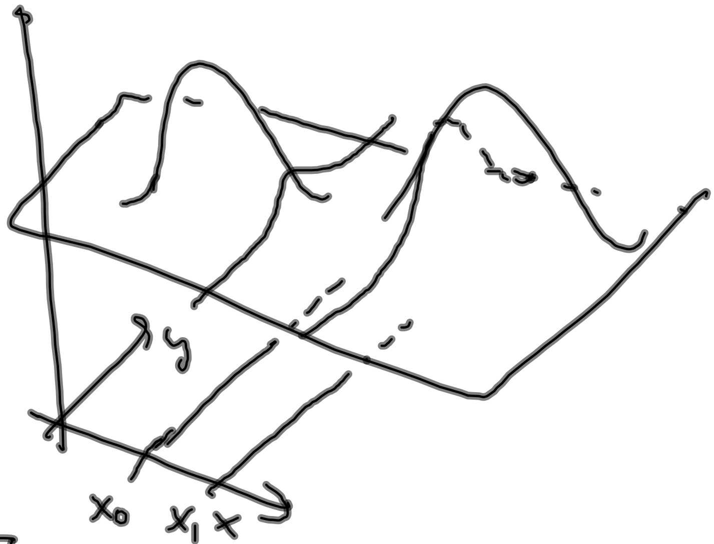
$c < 0$ $-\frac{x^2}{|c|} + \frac{y^2}{|c|} = 1$ liggende hyp. $a = \sqrt{|c|}$ $b = \sqrt{|c|}$



$f(x,y) = x^2 - y^2$

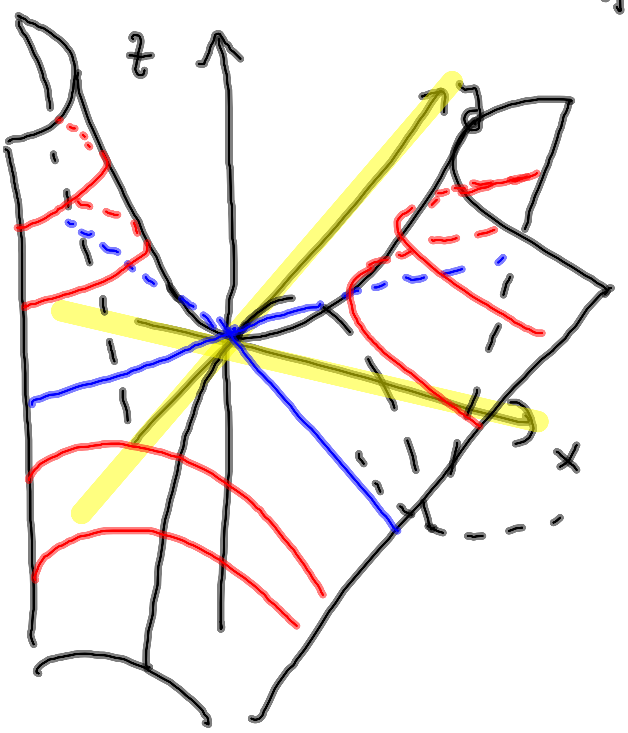
$x = 0$

$f(0,y) = -y^2$

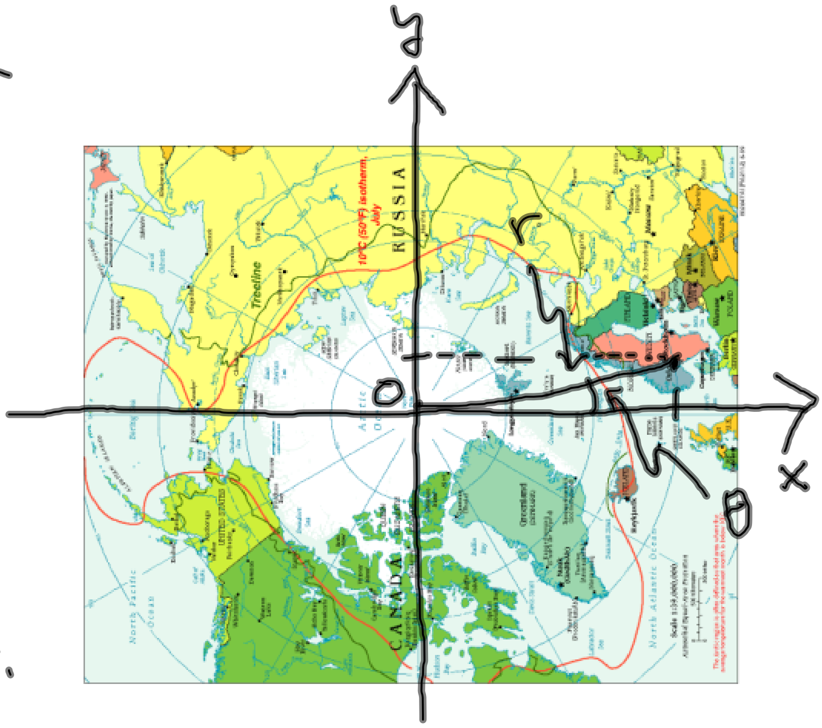


$y = 0$

$f(x,0) = x^2$



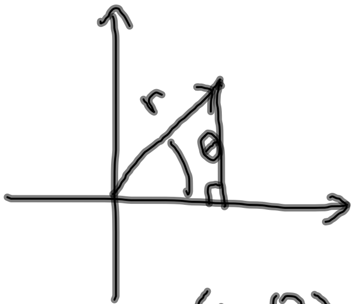
Polarkoordinater



Et punkt kan bestemmes av:

r = avstanden til origo

θ = vinkelen fra x-aksen til strålen fra O til punktet.



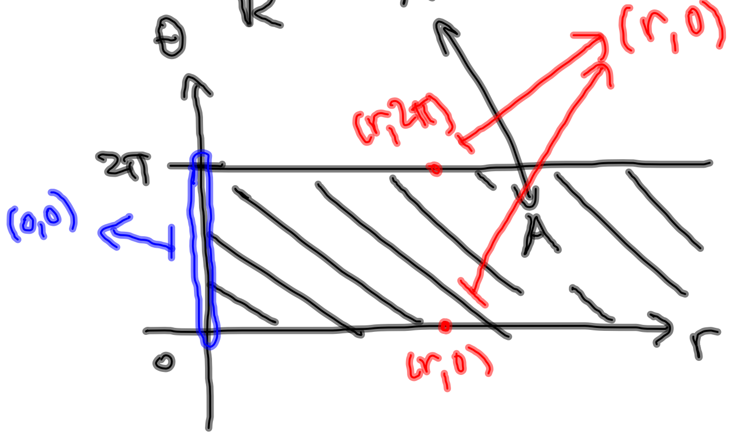
$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$(r,\theta) \longmapsto (x,y)$$

$r \geq 0$, $0 \leq \theta \leq 2\pi$ (evt. $0 \leq \theta < 2\pi$)

$$(r,\theta) \longmapsto (r\cos\theta, r\sin\theta)$$

$$\underset{\subset\ \mathbb{R}^2}{[0,\infty)} \times \underset{=\ A}{[0,2\pi]} \xrightarrow[\vec{F}]{} \mathbb{R}^2$$

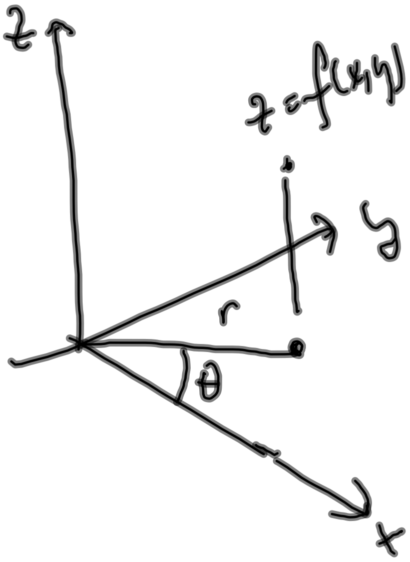


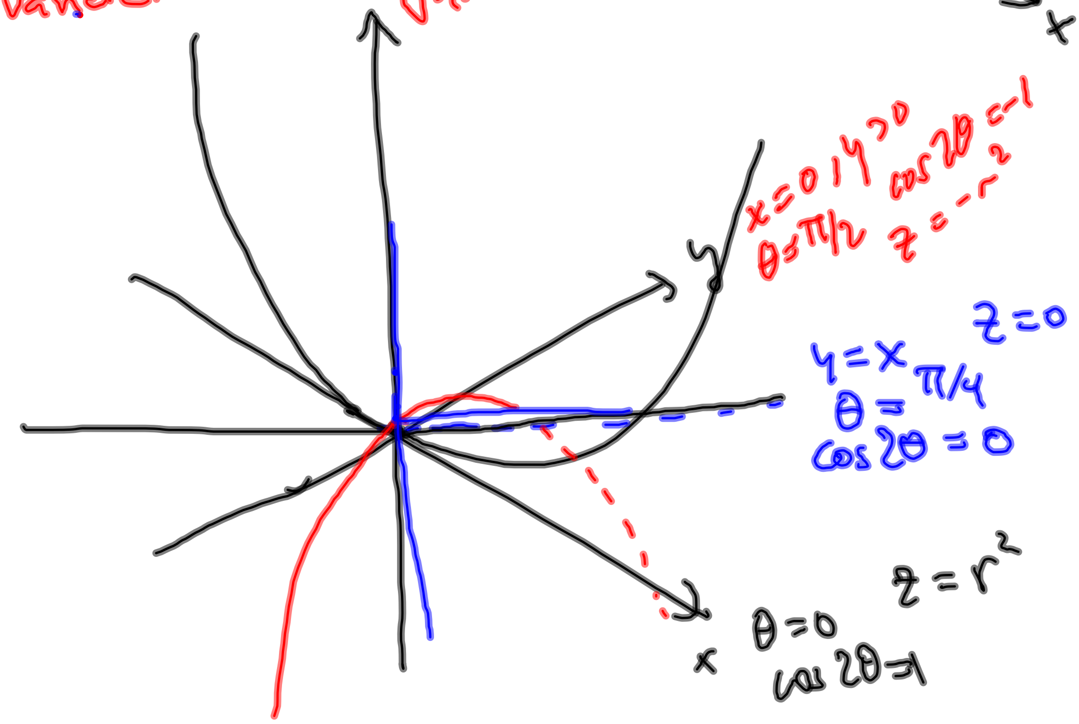
Eks $z = f(x,y) = x^2 - y^2$

$= (r\cos\theta)^2 - (r\sin\theta)^2$

$= r^2(\cos^2\theta - \sin^2\theta)$

$= \underbrace{r^2}_{\text{variere}} \cos \underbrace{2\theta}_{\text{fast}}$





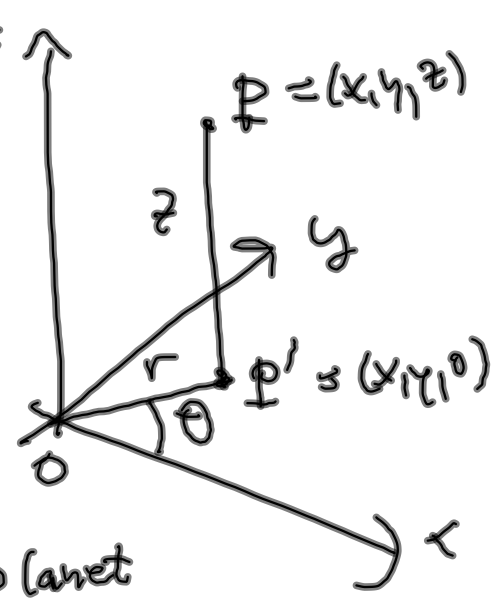
Sylinderkoordinater

Et punkt gis ved

r = avstanden $|OP'|$

θ = vinkelen fra pos. x-akse til OP'

z = høyden over xy-planet

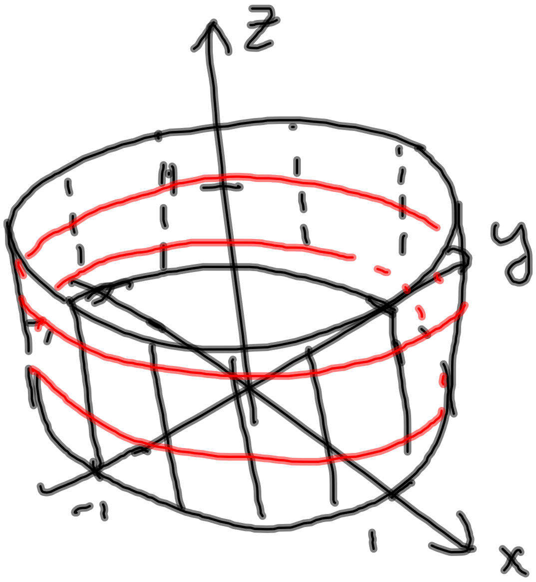


$$r \geq 0, \quad \theta \in [0, 2\pi], \quad z \in \mathbb{R}$$

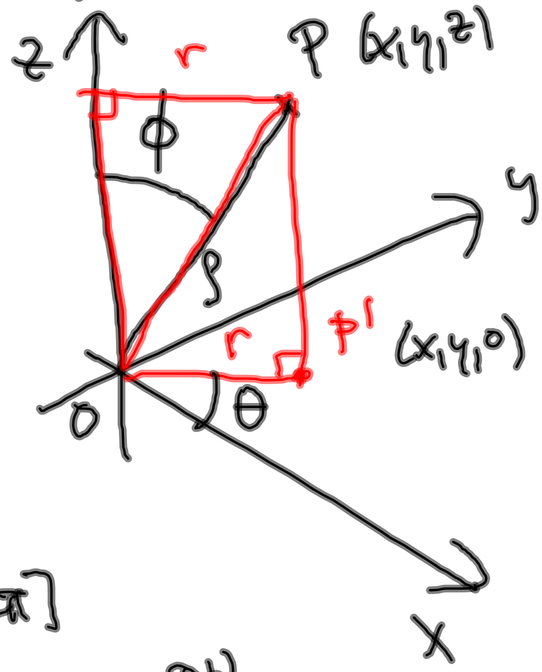
$$(r, \theta, z) \longmapsto (x, y, z)$$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

Eks $r = 1, \ \theta \in [0, 2\pi], \ z \in [0, 1]$



Kulekoordinater / sfæriske koordinater



ρ (rho) $\rho \geq 0$

avstanden $|OP|$

ϕ (phi) $\phi \in [0, \pi]$

vinkelen fra pos. z-akse til OP.

= inklinasjon

θ (theta) $\theta \in [0, 2\pi]$

vinkelen fra pos. x-akse til OP'

der P' er projeksjonen av P på xy-planet.

= asimut (azimuth)

$r = \rho \sin\phi$ (se på OP, ϕ og z-aksen)

$$\boxed{\begin{aligned} x &= r\cos\theta = \rho\sin\phi\cos\theta \\ y &= r\sin\theta = \rho\sin\phi\sin\theta \\ z &= \rho\cos\phi \end{aligned}}$$

Nivåflater

$$f : \mathbb{R}^n \longrightarrow \mathbb{R}, \quad \vec{x} \mapsto y, \quad c$$

$$N_c = \{(x_1, \ldots, x_n) \in \mathbb{R}^n \mid f(\vec{x}) = c\}$$

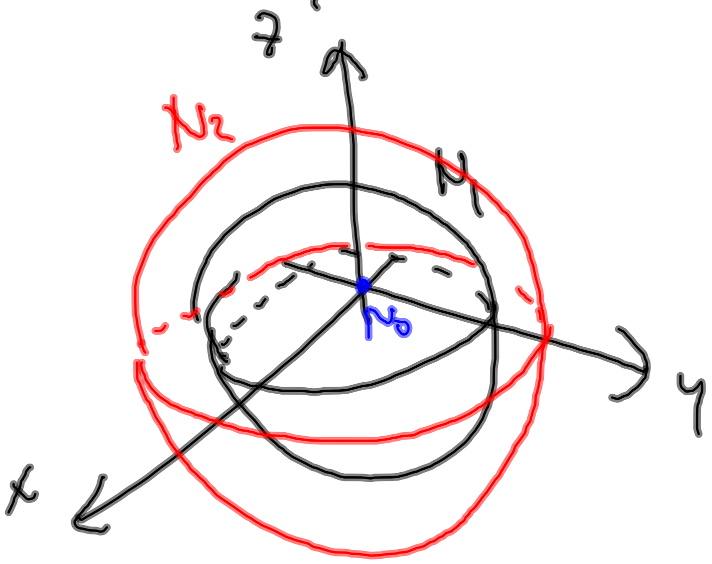
$$= \{\vec{x} \in \mathbb{R} : f(\vec{x}) = c\}$$

$$f(x,y,z) = x^2 + y^2 + z^2$$

N_c $c > 0$: $x^2 + y^2 + z^2 = c$
kuleflate radius $\sqrt{c}$

$c = 0$ $(x,y,z) = (0,0,0)$

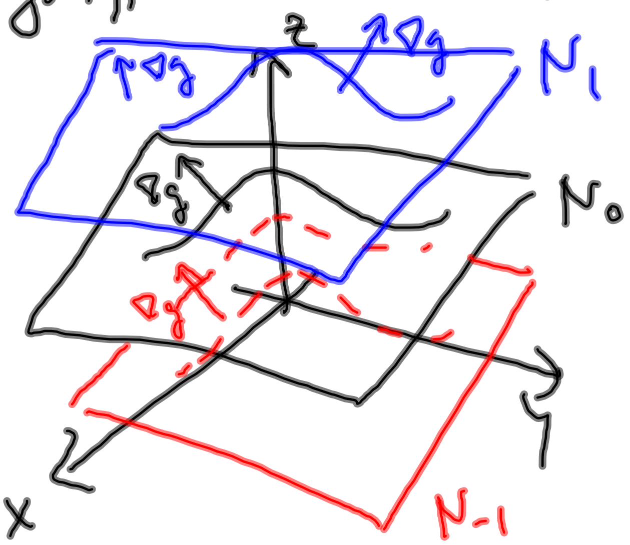
$c < 0$ $\emptyset$ (den tomme mengde)



Grafen til $f: \mathbb{R}^2 \to \mathbb{R}$

er nivåflaten N_0 til $g: \mathbb{R}^3 \to \mathbb{R}$

gitt ved $g(x,y,z) = z - f(x,y)$



Setning 3.7.2

$A \subset \mathbb{R}^n$, $g : A \to \mathbb{R}$ deriverbar i $\vec{a}$

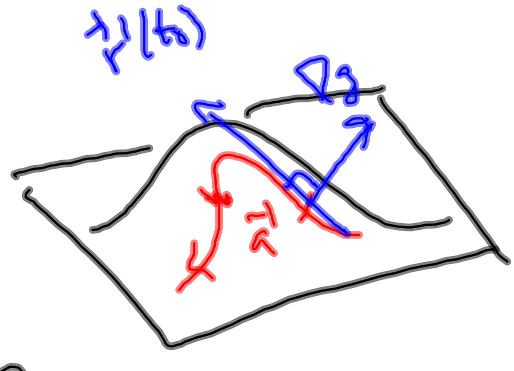
$\vec{a} \in A$

Dersom $c = \vec{g}(\vec{a})$ står $\nabla g(\vec{a})$ normalt på nivåflaten N_c; hvis $\vec{r} : [a,b) \longrightarrow A$ er en kurve på N_c

($g(\vec{r}(t)) = c$ så $\vec{r}(t) \in N_c$)

og $\vec{r}(t_0) = \vec{a}$, så er

$$\nabla g(\vec{a}) \cdot \vec{r}\,'(t_0) = 0$$



Bevis:

$u(t) = g(\vec{r}(t)) = c$

$u'(t) = \nabla g(\vec{r}(t)) \cdot \vec{r}\,'(t) = 0$

$t = t_0$

$$\nabla g(\vec{a}) \cdot \vec{r}\,'(t_0) = 0$$