MAT 1110 Plenum 25.01.17

$\boxed{2.7.1}$ $F(G(x,y))' = \underbrace{F'(G(x,y))}_{\text{matrise}} \cdot \underbrace{G'(x,y)}_{\text{matrise}}$

$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} : n \times 1$

$[x_1 \dots x_m] : 1 \times m$

$f(u,v) = u^2 + v \quad : \mathbb{R}^2 \to \mathbb{R}$

$g(x,y) = 2xy \quad : \mathbb{R}^2 \to \mathbb{R}$

$h(x,y) = x + y^2 \quad : \mathbb{R}^2 \to \mathbb{R}$

$k(x,y) = f(\underbrace{g(x,y), h(x,y)}_{G(x,y)})$, $G(x,y) = (g(x,y), h(x,y))$ $\mathbb{R}^2 \to \mathbb{R}^2$

$k(x,y) = f(G(x,y))$

Finn: $\frac{\partial}{\partial x} k$, $\frac{\partial}{\partial y} k$

$\mathbb{R}^n \to \mathbb{R}^m$

$g(x_1, x_2, \dots, x_n) \to (y_1, y_2, \dots, y_m)$

$m \times n$ - matrise

1) $k'(x,y) = \left(\frac{\partial}{\partial x} k \quad \frac{\partial}{\partial y} k \right)$

2) $\left(f(G(x,y))\right)' = f'(G(x,y)) \cdot G'(x,y)$

$= \left(\frac{\partial}{\partial u} f \quad \frac{\partial}{\partial v} f \right) \cdot \begin{pmatrix} \frac{\partial}{\partial x} g & \frac{\partial}{\partial y} g \\ \frac{\partial}{\partial x} h & \frac{\partial}{\partial y} h \end{pmatrix}$

$$\boxed{\begin{aligned} &\frac{\partial}{\partial u} f = 2u , \quad \frac{\partial}{\partial v} f = 1 \\ &\frac{\partial}{\partial x} g = 2y , \quad \frac{\partial}{\partial y} g = 2x \\ &\frac{\partial}{\partial x} h = 1 , \quad \frac{\partial}{\partial y} h = 2y \end{aligned}}$$

1×2 $2 \times 2 \to 1 \times 2$

$= \left(\underline{2u} \quad 1 \right) \begin{pmatrix} 2y & 2x \\ 1 & 2y \end{pmatrix}$

$= \left(4uy + 1 \quad 2 \cdot 2ux + 2y \right)$

$= \left(4 \cdot (2xy) \cdot y + 1 \quad 2 \cdot 2(2xy)x + 2y \right)$

$= \left(8xy^2 + 1 \quad 8x^2y + 2y \right)$

$\begin{matrix} \parallel & \parallel \\ \frac{\partial}{\partial x} k & \frac{\partial}{\partial y} k \end{matrix}$

$\boxed{2.7.2}$

$$f(u,v) = ue^{-v}$$
$$g(x,y,z) = 2xy + z$$
$$h(x,y,z) = 2y(z+x)$$

$$\frac{\partial}{\partial u} f = e^{-v}, \quad \frac{\partial}{\partial v} f = -ue^{-v}$$
$$\frac{\partial}{\partial x} g = 2y, \quad \frac{\partial}{\partial y} g = 2x, \quad \frac{\partial}{\partial z} g = 1$$
$$\frac{\partial}{\partial x} h = 2y, \quad \frac{\partial}{\partial y} h = 2(z+x), \quad \frac{\partial}{\partial z} h = 2y$$

$$k(x,y,z) = f(\underbrace{(g(x,y,z), h(x,y,z))}_{G(x,y,z)})$$

1) $k'(x,y,z) = \left(\frac{\partial}{\partial x}k \quad \frac{\partial}{\partial y}k \quad \frac{\partial}{\partial z}k\right)$ $\quad 1\times 2 \cdot 2\times 3 \rightarrow 1\times 3$

2) $\left(f(G(x,y,z))\right)' = f'(G(x,y,z)) \cdot G'(x,y,z)$

$$= \left(e^{-v} \quad -ue^{-v}\right)\begin{pmatrix} 2y & 2x & 1 \\ 2y & 2(z+x) & 2y \end{pmatrix}$$

$$= \left(\begin{matrix} e^{-v}(2y - u \cdot 2y) \\ e^{-v}(2x - u \cdot 2(z+x)) \\ e^{-v}(1 - u \cdot 2y) \end{matrix}\right)$$

bytte ut $u = g(x,y,z)$

$v = h(x,y,z)$

$\boxed{2.7.5}$ $G : \mathbb{R}^2 \to \mathbb{R}^3$

$F : \mathbb{R}^3 \to \mathbb{R}^2$

$$H(\vec{x}) = F(G(\vec{x}))$$

$\underline{H'((1,-2)) \text{ er målet}}$

$$H'(\vec{x}) = F'(G(\vec{x})) \cdot G'(\vec{x})$$

$$H'((1,-2)) = F'(G((1,-2))) \cdot G'((1,-2))$$

1) $G(1,-2) = (1,2,3)$

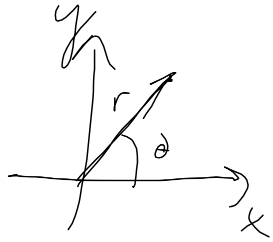
2) $G'(1,-2) = \begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix}$

3) $F'(1,2,3) = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix}$

$$H'(1,-2) = \underset{2\times 3}{\begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix}} \cdot \overset{3\times 2}{\begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix}}$$

$$= \underline{\underline{\begin{pmatrix} 13 & -7 \\ 10 & 0 \end{pmatrix}}}$$

$\boxed{2.7.8}$ a) $T(x,y) = f(x,y)$



$$x = r \cdot \cos\theta$$

$$y = r \cdot \sin\theta$$

$$T(r,\theta) = f(\overbrace{(x(r,\theta), y(r,\theta))}^{G(r,\theta)})$$

$$\frac{\partial T}{\partial r} = \frac{\partial f}{\partial x} \cdot \cos\theta + \frac{\partial f}{\partial y} \cdot \sin\theta$$

$$\frac{\partial T}{\partial \theta} = -\frac{\partial f}{\partial x} \cdot r\sin\theta + \frac{\partial f}{\partial y} \cdot r\cos\theta$$

VS: $T'(r,\theta) = \left(\frac{\partial}{\partial r} T \quad \frac{\partial}{\partial \theta} T \right)$

HS: $f'(G(r,\theta)) \cdot G'(r,\theta)$

$$= \left(\frac{\partial}{\partial x} f \quad \frac{\partial}{\partial y} f \right) \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$$

$$= \left(\frac{\partial}{\partial x} f \cos\theta + \frac{\partial}{\partial y} f \sin\theta \quad -\frac{\partial}{\partial x} f\, r\sin\theta + \frac{\partial}{\partial y} f\, r\cos\theta \right)$$

$H(t) = (g(t), h(t))$

$\boxed{2.7.8}$ b) $r(t)$, $\theta(t)$

$H(t) : \mathbb{R} \to \mathbb{R}^2$

$$T_1(t) = T(\overbrace{r(t), \theta(t)}^{H(t)})$$

$r(t) = g(t)$

$\theta(t) = h(t)$

$$T_1'(t) = \underbrace{T'(r(t), \theta(t))} \cdot H'(t)$$

$$\begin{pmatrix} \frac{\partial}{\partial t} g \\ \frac{\partial}{\partial t} h \end{pmatrix} = \begin{pmatrix} g'(t) \\ h'(t) \end{pmatrix}$$

$$= \left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta \quad -\frac{\partial f}{\partial x} r\sin\theta + \frac{\partial f}{\partial y} r\cos\theta \right) \begin{pmatrix} g' \\ h' \end{pmatrix}$$

$1 \times \cancel{2} \quad \cdot \quad \cancel{2} \times 1 \to 1 \times 1$

$$= \left[\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right] g'(t) + \left[-\frac{\partial f}{\partial x} r\sin\theta + \frac{\partial f}{\partial y} r\cos\theta\right] h'(t)$$

$\boxed{2.7.9}$ $f: \mathbb{R}^{n+1} \to \mathbb{R}$, $\vec{x} = (x_1, x_2, \dots, x_n)$

$g: \mathbb{R}^n \to \mathbb{R}$

$$f(x_1, x_2, \dots, x_n, g(x_1, x_2, \dots, x_n)) = 0 \quad \forall \vec{x}$$

V(i): $\dfrac{\partial g}{\partial x_i} = - \dfrac{\frac{\partial f}{\partial x_i}}{\frac{\partial f}{\partial x_{n+1}}}$

$$G(x_1, x_2, \dots, x_n) = (x_1, x_2, \dots, x_n, g(x_1, x_2, \dots, x_n))$$

$$G(\vec{x}) = (\vec{x}, g(\vec{x}))$$

$$f(G(\vec{x})) = 0 \quad \forall \vec{x}$$

$$\underset{1\times(n+1)}{f'(G(\vec{x}))} \cdot \underset{(n+1)\times n}{G'(\vec{x})} = \underset{1\times n}{\vec{0}} = (\overbrace{0, 0, 0, \dots, 0}^{n})$$

$$\left(\frac{\partial f}{\partial x_1} \quad \frac{\partial f}{\partial x_2} \quad \dots \quad \frac{\partial f}{\partial x_n} \quad \frac{\partial f}{\partial x_{n+1}} \right) \leftarrow$$

$$\cdot \overbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ & & \vdots & & & \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ \frac{\partial g}{\partial x_1} & \frac{\partial g}{\partial x_2} & \cdots & & & \frac{\partial g}{\partial x_n} \end{pmatrix}}^{n} = \vec{0}$$

$$\left(\frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_{n+1}} \cdot \frac{\partial g}{\partial x_1}, \frac{\partial f}{\partial x_2} + \frac{\partial f}{\partial x_{n+1}} \cdot \frac{\partial g}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n} + \frac{\partial f}{\partial x_{n+1}} \cdot \frac{\partial g}{\partial x_n} \right) = \vec{0}$$

$$\frac{\partial f}{\partial x_i} + \frac{\partial f}{\partial x_{n+1}} \cdot \frac{\partial g}{\partial x_i} = 0 \quad \forall i \in \{0, \dots, n\}$$

$$\frac{\partial g}{\partial x_i} = - \frac{\frac{\partial f}{\partial x_i}}{\frac{\partial f}{\partial x_{n+1}}}$$

$\boxed{2.7.9}$ b) $f(x,y) = x^2+y^2-R^2$, $R>0$

anta $y = g(x)$ deriverbar slik at

$$f(x,g(x)) = 0 \quad \forall x \in \mathbb{R}$$

Vis at $g'(x) = -\frac{x}{g(x)}$

<u>n=1</u>

$$f(x,g(x)) = f(G(x))$$

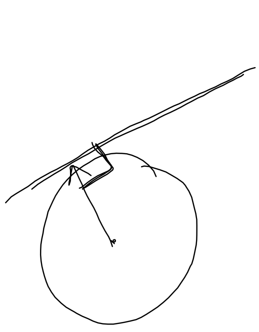
$$G(x) = (x, g(x))$$

$$g'(x) = \frac{\partial g}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{2x}{2y} = -\frac{\cancel{2}x}{\cancel{2}g(x)}$$

$$g'(x) = -\frac{x}{g(x)}$$

$$g'(x)\cdot g(x) + x\cdot 1 = 0$$

$$\underbrace{(x, g(x))}_{\text{posisjon}}\cdot\underbrace{(1, g'(x))}_{\text{fart}} = 0$$



$$x^2+y^2-R^2 = 0$$

$$\underbrace{x^2+y^2 = R^2}_{\text{lik. for sirkel}}$$