12.1.3:

Hvis $|x|<1$, $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

a) $1-x+x^2-x^3+\ldots = \sum_{n=0}^{\infty}(-x)^n = \frac{1}{1-(-x)} = \frac{1}{1+x}$

siden $|-x| = |x| < 1$

5. etasje NHA hus

b) $\sum_{n=0}^{\infty} x^{2n} = \sum_{n=0}^{\infty}(x^2)^n = \frac{1}{1-x^2}$

$|x|<1 \Rightarrow x^2<1$

$(|x^2|<1)$

c) $a^2-4a^4+16a^6-\ldots = \frac{1}{4}\left(4a^2-(4a^2)^2+(4a^2)^3-\ldots\right)$

$= -\frac{1}{4}\left(-4a^2+(4a^2)^2-(4a^2)^3+\ldots\right) = -\frac{1}{4}\sum_{n=1}^{\infty}(-4a^2)^n = -\frac{1}{4}\left(\sum_{n=0}^{\infty}(-4a^2)^n - 1\right)$

$\left(|a|<\frac{1}{2} \Rightarrow a^2<\frac{1}{4} \Rightarrow |-4a^2|<1\right)$

$= -\frac{1}{4}\left(\frac{1}{1-(-4a^2)} - 1\right) = -\frac{1}{4}\left(\frac{1}{1+4a^2} - \frac{1+4a^2}{1+4a^2}\right) = \frac{a^2}{1+4a^2}$

d) $\sum_{n=0}^{\infty} e^{-n/2} = \sum_{n=0}^{\infty}\left(e^{-\frac{1}{2}}\right)^n = \frac{1}{1-e^{-\frac{1}{2}}} = \frac{\sqrt{e}}{\sqrt{e}-1}$

$1<e$. $1<\sqrt{e} = e^{\frac{1}{2}}$

$0<e^{-\frac{1}{2}}<1$.