

4.1.1

$$x + 2y - z = 3$$

$$2x + 3y - 3z = -1$$

$$-x + 2y + 3z = 1$$

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 3 & -3 & -1 \\ -1 & 2 & 3 & 1 \end{pmatrix} \leftarrow$$

$$A \vec{x} = \vec{b}$$

$$\begin{pmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 12 \end{pmatrix}$$

$$\begin{array}{l} \text{II} - 2\text{I} \\ \sim \\ \text{III} + \text{I} \end{array} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & -1 & -1 & -7 \\ 0 & 4 & 2 & 4 \end{pmatrix}$$

$$\begin{array}{l} \text{II} \cdot (-1) \\ \sim \\ \text{III} - 4\text{II} \end{array} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & -2 & -24 \end{pmatrix}$$

$$\begin{array}{l} \text{III} \cdot (-\frac{1}{2}) \\ \sim \end{array} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 1 & 12 \end{pmatrix}$$

$$\begin{array}{l} \text{II} - \text{III} \\ \sim \end{array} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 12 \end{pmatrix} \leftarrow$$

$$3 - 2(-5) + 12 = 3 + 10 + 12 = 25$$

$$\begin{array}{l} \text{I} - 2\text{II} + \text{III} \\ \sim \end{array} \begin{pmatrix} 1 & 0 & 0 & 25 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 12 \end{pmatrix}$$

$$x = 25$$

$$y = -5$$

$$z = 12$$

A.1.2

$$\begin{aligned}x - y + 2z &= 3 \\ 2x - 2y &= 4 \\ -3x + 2y + z &= 0\end{aligned}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & -1 & 2 & 3 \\ 2 & -2 & 0 & 4 \\ -3 & 2 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned} & \text{II} \cdot \frac{1}{2} \\ \sim & \\ \text{II} - \text{I} & \end{aligned} \begin{pmatrix} 1 & -1 & 2 & 3 \\ 0 & 0 & -2 & -1 \\ -3 & 2 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned} & \text{III} + 3\text{I} \\ \sim & \end{aligned} \begin{pmatrix} 1 & -1 & 2 & 3 \\ 0 & 0 & -2 & -1 \\ 0 & -1 & 7 & 9 \end{pmatrix}$$

$$\begin{aligned} & \text{II} \leftrightarrow \text{III} \\ \sim & \end{aligned} \begin{pmatrix} 1 & -1 & 2 & 3 \\ 0 & -1 & 7 & 9 \\ 0 & 0 & -2 & -1 \end{pmatrix}$$

$$\begin{aligned} & \text{II} \cdot (-1) \\ \sim & \\ \text{II} \cdot (-\frac{1}{2}) & \end{aligned} \begin{pmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & -7 & -9 \\ 0 & 0 & 1 & \frac{1}{2} \end{pmatrix}$$

$$\begin{aligned} & \text{II} + 7 \cdot \text{III} \\ \sim & \\ \text{I} - 2 \cdot \text{III} & \end{aligned} \begin{pmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 0 & -\frac{11}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{pmatrix}$$

$$\frac{7}{2} - \frac{11}{2} = -\frac{11}{2}$$

$$\begin{aligned} & \text{I} + \text{II} \\ \sim & \end{aligned} \begin{pmatrix} 1 & 0 & 0 & -\frac{7}{2} \\ 0 & 1 & 0 & -\frac{11}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{pmatrix}$$

$$x = -\frac{7}{2}, \quad y = -\frac{11}{2}, \quad z = \frac{1}{2}$$

$$\boxed{4.1.4} \quad \left. \begin{array}{l} x - 2y + 3z = 1 \\ -x + y - 2z = 0 \\ -3x + 5y - 8z = 2 \end{array} \right\} A \vec{x} = \vec{b}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 3 & 1 \\ -1 & 1 & -2 & 0 \\ -3 & 5 & -8 & 2 \end{pmatrix}$$

A

$$\begin{array}{l} \text{II} + \text{I} \\ \text{III} + 3\text{I} \end{array} \quad \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 5 \end{pmatrix}$$

$$\text{III} - \text{II} \quad \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

$$x - 2y + 3z = 1$$

$$-y + z = 1$$

$$0 = 4$$

ihke möglich!

Lösungsschritt hat keine Lösung!

$$\boxed{4.1.3} \quad \begin{aligned} 2x - 4y + 6z &= -2 \\ -3x + 2y - z &= 8 \\ x - 6y + 11z &= 4 \end{aligned}$$

$$\Updownarrow$$

$$\begin{pmatrix} 2 & -4 & 6 & -2 \\ -3 & 2 & -1 & 8 \\ 1 & -6 & 11 & 4 \end{pmatrix}$$

$$\begin{array}{l} \text{I} \leftrightarrow \text{III} \\ \text{III} - 2\text{I} \\ \text{II} + 3\text{I} \end{array} \quad \begin{pmatrix} 1 & -6 & 11 & 4 \\ 0 & 8 & -16 & -10 \\ 0 & -10 & 17 & 2 \end{pmatrix}$$

$$\begin{array}{l} \text{II} \cdot \frac{1}{8} \\ \text{IV} \cdot \left(-\frac{1}{10}\right) \end{array} \quad \begin{pmatrix} 1 & -6 & 11 & 4 \\ 0 & 1 & -2 & -\frac{5}{4} \\ 0 & 1 & -\frac{17}{10} & -\frac{1}{5} \end{pmatrix}$$

$$\begin{array}{l} \text{III} - \text{II} \end{array} \quad \begin{pmatrix} 1 & -6 & 11 & 4 \\ 0 & 1 & -2 & -\frac{5}{4} \\ 0 & 0 & \frac{3}{10} & \frac{21}{20} \end{pmatrix} \quad \begin{aligned} &-\frac{1}{5} + \frac{5}{4} \\ &= \frac{-4 + 25}{20} = \frac{21}{20} \end{aligned}$$

$$\begin{pmatrix} 1 & -6 & 11 & 4 \\ 0 & 1 & -2 & -\frac{5}{4} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$z = z$$

$$y - 2z = -\frac{5}{4} \rightarrow y = 2z - \frac{5}{4}$$

$$x - 6y + 11z = 4$$

$$\begin{aligned} x &= 6\left(2z - \frac{5}{4}\right) - 11z + 4 \\ &= 2z - \frac{15}{2} + 4 = 2z - \frac{7}{2} \end{aligned}$$

$$x = 2z - \frac{7}{2}$$

$$y = 2z - \frac{5}{4}$$

$$z = z$$
