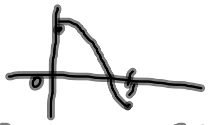
5.5.2 $\vec{F}: \mathbb{R}^n \to \mathbb{R}^n$ $\quad \vec{u}_{n+1} = \vec{F}(\vec{u}_n)$

$\vec{u}_n \to \vec{u}$: Er $\vec{u}$ fikspunkt for $\vec{F}$?

$$\vec{F}(\vec{u}) = \vec{F}\left(\lim_{n\to\infty} \vec{u}_n\right) = \lim_{n\to\infty} \vec{F}(\vec{u}_n) = \lim_{n\to\infty} \vec{u}_{n+1}$$

$$\underset{v=n+1}{=} \lim_{v\to\infty} \vec{u}_v = \vec{u} \Rightarrow \vec{u} \text{ er fikspunkt.}$$

5.5.3



$f: [0,1] \to [0,1]$

$g(x) = f(x) - x$

$g(0) = f(0) - 0 = f(0) \geq 0$

$g(1) = f(1) - 1 \leq 0$

Fra skjæringssetningen: Finnes en $c \in [0,1]$ slik at $g(c) = 0 \Leftrightarrow f(c) - c = 0 \Leftrightarrow f(c) = c$

$\Rightarrow$ c er fikspunkt for f.

5.5.4

A: mengden av alle punkter i terrenget.

$\vec{F}$: punkter i terrenget $\to$ punkter på kartet

punkt i terrenget $\to$ tilhørende punkt på kartet. er plassert over.

$\vec{F}$ kontraksjon fra $A \to A$, og har da et unikt fikspunkt, i dette fikspunktet ligger punktet på kartet direkte over tilsvarende punkt i terrenget.

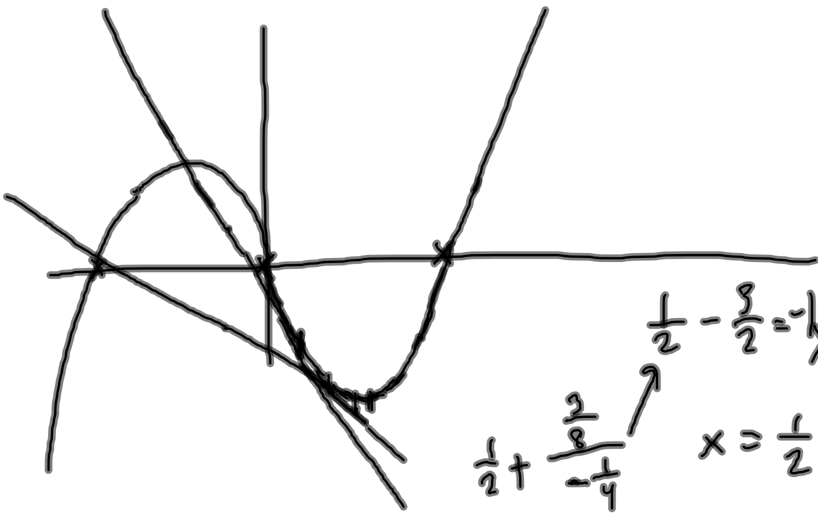
5.6.1

a) $f(x) = x^3 - x = x(x^2-1) = x(x+1)(x-1)$

nullpunkter: $x=0$, $x=-1$, $x=1$

$x^3 - x$

d)



$f'(x)\ 3x^2-1=0$

$x = \frac{\sqrt{3}}{3} \approx 0{,}62$

$\frac{1}{2} - \frac{3}{2} = -1 \quad x - \frac{x^3-x}{3x^2-1}$

$\frac{1}{2} + \frac{\frac{3}{8}}{-\frac{1}{4}}$ $\quad x = \frac{1}{2} \quad = \frac{1}{2} - \frac{\frac{1}{8}-\frac{1}{2}}{\frac{3}{4}-1} =$

5.6.3

$$\vec{F}(x,y) = \begin{pmatrix} \frac{1}{2} \operatorname{sen}(x+y) \\ \frac{1}{2}\cos(x-y) \end{pmatrix} = \vec{0}$$

$\sin(x+y) = 0 \iff x + y = k_1 \pi$ $\quad k_1$ heltall

$\frac{1}{2}\cos(x-y) = 0 \iff x - y = \frac{\pi}{2} + k_2\pi$ $\quad k_2$ heltall.

$$\Downarrow$$

$2x = \frac{\pi}{2} + (k_1 + k_2)\pi \quad 2y = -\frac{\pi}{2} + (k_1 - k_2)\pi$

$$\underline{\underline{x = \frac{\pi}{4} + \frac{k_1+k_2}{2}\pi \quad y = -\frac{\pi}{4} + \frac{k_1-k_2}{2}\pi}}$$

setter $k_1 = k_2 = 0$: $\underline{\underline{x = \frac{\pi}{4} \quad y = -\frac{\pi}{4}}}$

5.7.2 $\vec{F}(x,y) = \begin{pmatrix} e^{x+y^2-1} \\ x-y \end{pmatrix}$

$\vec{F}'(x,y) = \begin{pmatrix} e^{x+y^2-1} & 2ye^{x+y^2-1} \\ 1 & -1 \end{pmatrix}$

$\vec{G}(1,-1) = (0,1)$

$\vec{F}(0,1) = \begin{pmatrix} e^{0+1-1} \\ 0-1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\vec{F}'(0,1) = \begin{pmatrix} e^0 & 2e^0 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$

$\det \vec{F}'(0,1) = -1 - 2 = -3$

$\Rightarrow \vec{F}$ har omvendt funksjon.

$\vec{G}'(1,-1) = \left(\vec{F}'(0,1)\right)^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}^{-1} = -\frac{1}{3}\begin{pmatrix} -1 & -2 \\ -1 & 1 \end{pmatrix} = \underline{\underline{\frac{1}{3}\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}}}$

$$\vec{H}(1,-1) = (-3,-2)$$

$$\vec{F}(-3,-2) = \begin{pmatrix} e^{-3+4-1} \\ -3+2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\vec{F}'(-3,-2) = \begin{pmatrix} 1 & -4 \\ 1 & -1 \end{pmatrix}$$

det $\vec{F}'(-3,-2) = -1+4 = 3$

$\Rightarrow \vec{F}$ har omvendt funksjon, definert på omegn om $(1,-1)$

$$\vec{H}'(1,-1) = \left(\vec{F}'(-3,-2)\right)^{-1}$$

$$= \underline{\underline{\frac{1}{3}\begin{pmatrix} -1 & 4 \\ -1 & 1 \end{pmatrix}}}$$

5.7.3

$$x^3+y^3+y=1 \iff g(x,y)=x^3+y^3+y-1=0$$

anta $g(x_0,y_0)=0$

$$\frac{\partial g}{\partial x}=3x^2$$

$$\frac{\partial g}{\partial y}=3y^2+1>0$$

siden $\frac{\partial g}{\partial y}(x_0,y_0)=3y_0^2+1>0$, så har $g(x,f(x))=0$ en løsning f i omegn om x_0, og

$$f'(x_0)=-\frac{\frac{\partial g}{\partial x}(x_0,y_0)}{\frac{\partial g}{\partial y}(x_0,y_0)}=\underline{\underline{-\frac{3x_0^2}{3y_0^2+1}}}$$

5.7.14

$\phi(X,y,z)$

$\phi(X(y,z),y,z)=0$: $\frac{\partial\phi}{\partial x}\neq 0 \;\therefore\; \rightarrow$ finnes slik X

$$\frac{\partial X}{\partial y}=-\frac{\frac{\partial\phi}{\partial y}}{\frac{\partial\phi}{\partial x}}$$

$\phi(x,Y(x,z),z)=0$ $\frac{\partial\phi}{\partial y}\neq 0 \rightarrow$ finnes slik Y

$$\frac{\partial Y}{\partial z}=-\frac{\frac{\partial\phi}{\partial z}}{\frac{\partial\phi}{\partial y}}$$

$\phi(x,y,Z(x,y))=0$ $\frac{\partial\phi}{\partial z}\neq 0 \rightarrow$ finnes slik Z

$$\frac{\partial Z}{\partial x}=-\frac{\frac{\partial\phi}{\partial x}}{\frac{\partial\phi}{\partial z}}$$

$$\frac{\partial X}{\partial y}\cdot\frac{\partial Y}{\partial z}\cdot\frac{\partial Z}{\partial x}=\left(-\frac{\cancel{\frac{\partial\phi}{\partial y}}}{\cancel{\frac{\partial\phi}{\partial x}}}\right)\cdot\left(-\frac{\cancel{\frac{\partial\phi}{\partial z}}}{\cancel{\frac{\partial\phi}{\partial y}}}\right)\left(-\frac{\cancel{\frac{\partial\phi}{\partial x}}}{\cancel{\frac{\partial\phi}{\partial z}}}\right)=\underline{\underline{-1}}$$