Plenumsregning MAT1110 28.1

Seksjon 2.7

2. $f(u,v) = ue^{-v}$ $\quad g(x,y,z) = 2xy + z$

$h(x,y,z) = 2y(z+x)$

$k(x,y,z) = f(g(x,y,z), h(x,y,z))$

$G(x,y,z) = (g(x,y,z), h(x,y,z))$

$a = (x,y,z)$

$k(a) = f(G(a))$

Kan bruke teorem 2.7.2

$G_1 = g \quad G_2 = h$

$$\frac{\partial k}{\partial x}(a) = \frac{\partial f}{\partial u}\underbrace{(G(a))}_{(g(a),h(a))}\underbrace{\frac{\partial G_1}{\partial x}(a)}_{\frac{\partial g}{\partial x}(a)}$$

$$+ \frac{\partial f}{\partial v}(g(a), h(a))\frac{\partial h}{\partial x}(a)$$

$\frac{\partial f}{\partial u} = e^{-v} \quad \frac{\partial f}{\partial v} = -ue^{-v}$

$\frac{\partial g}{\partial x} = 2y \quad \frac{\partial h}{\partial x} = 2y$

$$\frac{\partial f}{\partial u}(g(a),h(a)) = e^{-h(a)} = e^{-2y(x+z)}$$

$$\frac{\partial f}{\partial v}(g(a),h(a)) = -(2xy+z)e^{-2y(x+z)}$$

$$\frac{\partial k}{\partial x} = 2ye^{-2y(x+z)}(1-(2xy+z))$$

$\frac{\partial g}{\partial y} = 2x \quad \frac{\partial h}{\partial y} = 2(z+x)$

$$\frac{\partial k}{\partial y} = 2xe^{-2y(x+z)}$$
$$+ 2(z+x)\cdot(-(2xy+z)e^{-2y(x+z)})$$

$\frac{\partial g}{\partial z} = 1 \quad \frac{\partial h}{\partial z} = 2y$

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5. $G: \mathbb{R}^2 \longrightarrow \mathbb{R}^3 \quad F: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

$G(1,-2) = (1,2,3)$

$$G'(1,-2) = \begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix}$$

$$F'(1,2,3) = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix}$$

$H(x) = F(G(x))$

Bruker Teorem 2.7.1

$$H'(x) = F'(G(x))\,G'(x)$$

$$H'(1,-2) = F'(G(1,-2))\,G'(1,-2)$$

$$= F'(1,2,3)\,G'(1,-2)$$

$$= \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+3+8 & -4+1-4 \\ 6+4 & 2-2 \end{pmatrix}$$

$$= \begin{pmatrix} 13 & -7 \\ 10 & 0 \end{pmatrix}$$

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7. $E_1 = E_1(p_1, p_2)$

$$p_1 = p_1(t) \qquad p_2 = p_2(t)$$

$$p = (p_1, p_2)$$

$$E_1(t) = E_1(p(t))$$

$$\frac{dE_1}{dt} = \frac{\partial E_1}{\partial p_1}(p_1(t))\frac{\partial p_1}{\partial t}(t) + \frac{\partial E_1}{\partial p_2}(p_2(t))\frac{\partial p_2}{\partial t}(t)$$

$$= \frac{\partial E_1}{\partial p_1} p_1'(t) + \frac{\partial E_1}{\partial p_2} p_2'(t)$$

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8. a) $T = f(x, y)$

$x = r\cos\theta \quad y = r\sin\theta$

$T = f(r\cos\theta, r\sin\theta)$

$$\frac{\partial T}{\partial r} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$= \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \theta}$$

$$= -\frac{\partial f}{\partial x}r\sin\theta + \frac{\partial f}{\partial y}r\cos\theta$$

b) $r = g(t) \quad \theta = h(t)$

$$\frac{\partial r}{\partial t} = g'(t) \quad \frac{\partial \theta}{\partial t} = h'(t)$$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial T}{\partial \theta}\frac{\partial \theta}{\partial t}$$

$$\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)g'(t)$$

$$+\left(-\frac{\partial f}{\partial x}r\sin\theta + \frac{\partial f}{\partial y}r\cos\theta\right)h'(t)$$

Seksjon 2.8

$F : \mathbb{R}^2 \to \mathbb{R}^2$

$$F(x,y) = \begin{pmatrix} x^2 y \\ xy + x \end{pmatrix} \qquad a = (-2,1)$$

$$F(a) = \begin{pmatrix} (-2)^2 1 \\ -2 \cdot 1 - 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$$

$$F'(a) = \begin{bmatrix} \frac{\partial F_1}{\partial x}(a) & \frac{\partial F_1}{\partial y}(a) \\ \frac{\partial F_2}{\partial x}(a) & \frac{\partial F_2}{\partial y}(a) \end{bmatrix}$$

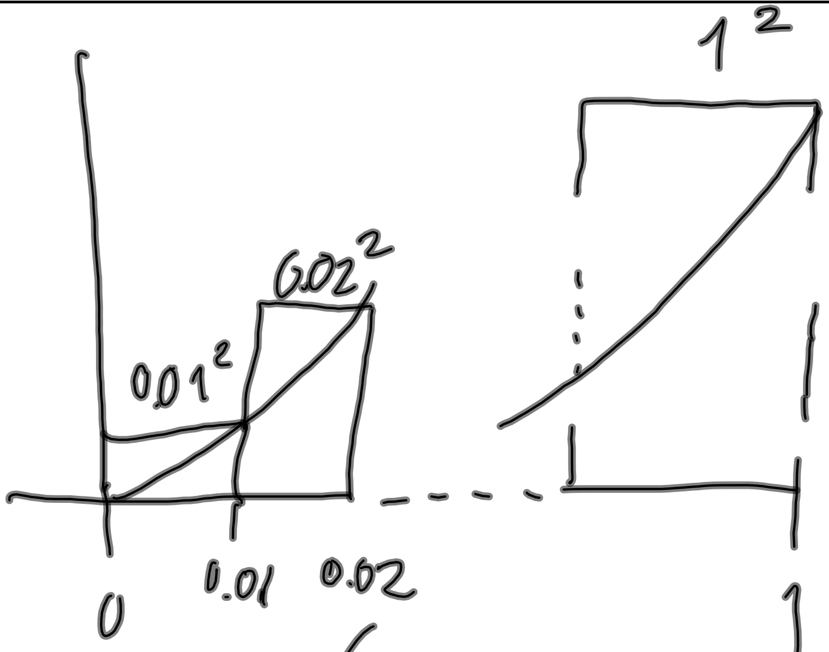
$$= \begin{bmatrix} 2xy & x^2 \\ y+1 & x \end{bmatrix}_{(x,y)=(-2,1)}$$

$$= \begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix} \qquad \vec{X} = (X, y)$$

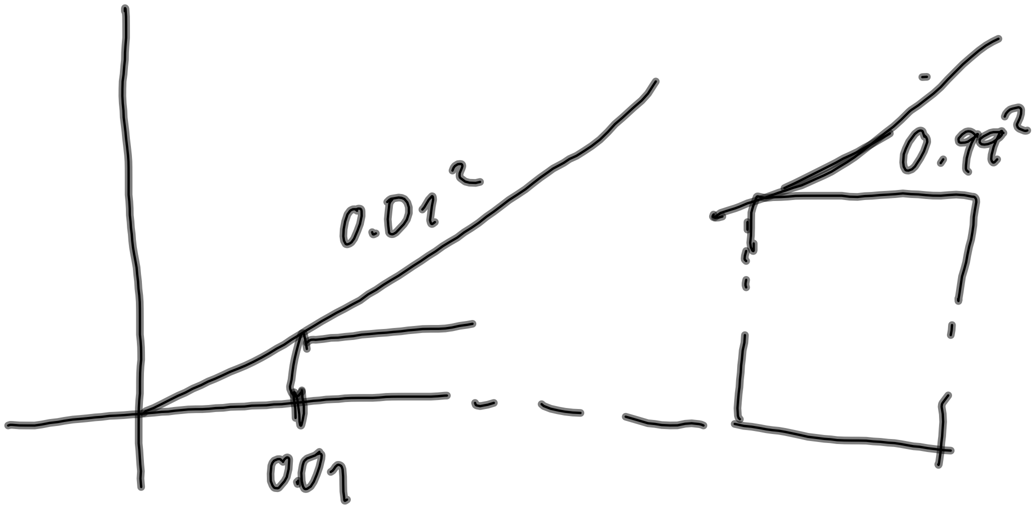
$$T_a F(\vec{x}) = F(a) + F'(a)(\vec{X} - a)$$

$$\begin{bmatrix} 4 \\ -4 \end{bmatrix} + \begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x+2 \\ y-1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 4x + 4y - 8 - 4 \\ -4 + 2x - 2y + 4 + 2 \end{bmatrix} = \begin{bmatrix} -8 - 4x + 4y \\ 2 + 2x - 2y \end{bmatrix}$$



$$\left(0.01^2 + \cdots \; 1^2\right) \cdot 0.01$$



$$\left(0.01^2 + \cdots \; 0.99^2\right) 0.01$$