$$= \pi \left[z \ln\left(1+\frac{1}{z^2}\right) + 2\int \frac{\frac{1}{z^2}}{1+\frac{1}{z^2}}\,dz \right]_{z=0}^{1}$$

Delvis int:

$v' = 1$

$v = z$

$u = \ln\left(1+\frac{1}{z^2}\right)$

$u' = \frac{1}{1+\frac{1}{z^2}} \left(-2\frac{1}{z^3}\right)$

$$= \pi \ln 2 + 2\pi \int_0^1 \frac{1}{1+z^2}\,dz$$

$$= \pi \ln 2 + 2\pi \left[\arctan z\right]_{z=0}^{1}$$

$$= \pi \ln 2 + \frac{\pi^2}{2}$$

4.1: Gauss-eliminasjon

4) $\begin{aligned} x - 2y + 3z &= 1 \\ -x + y - 2z &= 0 \\ -3x + 5y - 8z &= 2 \end{aligned}$

$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ -1 & 1 & -2 & 0 \\ -3 & 5 & -8 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & -1 & 1 & 1 \\ -3 & 5 & -8 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Siste ligning: $0 = 4$ ↙ USANT! $\Rightarrow$ Systemet har ingen løsninger.

4.2 : Trappeform

4.) $\begin{aligned} 3x - 4y + z &= 2 \\ x - 2y &= 1 \\ -2x + 2y - z &= -1 \end{aligned}$

Utvidet matrise:

$$\begin{bmatrix} 3 & -4 & 1 & 2 \\ 1 & -2 & 0 & 1 \\ -2 & 2 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 3 & -4 & 1 & 2 \\ -2 & 2 & -1 & -1 \end{bmatrix}$$

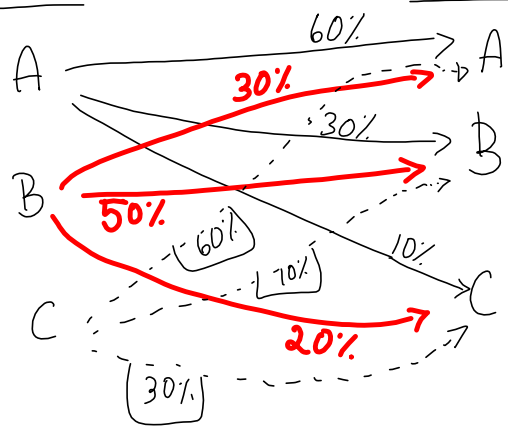
$$\sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & -2 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x - 2y &= 1 \\ y + \tfrac{1}{2}z &= -\tfrac{1}{2} \end{aligned} \Rightarrow \begin{aligned} x &= 1 + 2y \\ 2y &= -1 - z \end{aligned}$$

$$\Rightarrow \begin{aligned} x &= 1 - 1 - z = -z \\ y &= -\tfrac{1}{2} - \tfrac{1}{2}z \end{aligned} \quad \text{og } z \text{ er fri}$$

10.) Leies Returneres



a, b, c = # biler i A, B, C.

$$\Downarrow$$

$$\begin{aligned} a+b+c &= 120 \\ 0{,}6a+0{,}3b+0{,}6c &= a \\ 0{,}3a+0{,}5b+0{,}1c &= b \\ 0{,}1a+0{,}2b+0{,}3c &= c \end{aligned}$$

$$\Rightarrow \quad \begin{aligned} a+b+c &= 120 \\ -0{,}4a+0{,}3b+0{,}6c &= 0 \\ 0{,}3a-0{,}5b+0{,}1c &= 0 \\ 0{,}1a+0{,}2b-0{,}7c &= 0 \end{aligned}$$

Matriseform:

$$\begin{bmatrix} 1 & 1 & 1 & 120 \\ -0{,}4 & 0{,}3 & 0{,}6 & 0 \\ 0{,}3 & -0{,}5 & 0{,}1 & 0 \\ 0{,}1 & 0{,}2 & -0{,}7 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ -4 & 3 & 6 & 0 \\ 3 & 5 & 1 & 0 \\ 1 & 2 & 7 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ 0 & 7 & 10 & 480 \\ 0 & -8 & -2 & -360 \\ 0 & 1 & -8 & -120 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & -8 & -2 & -360 \\ 0 & 7 & 10 & 480 \end{bmatrix}$$

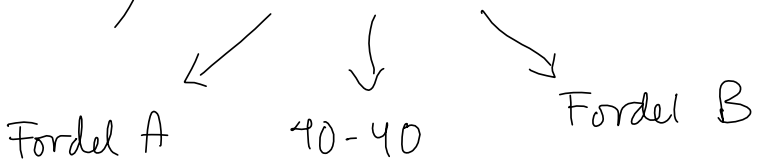
$$\sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & 0 & -66 & -1320 \\ 0 & 0 & 66 & 1320 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & 0 & -66 & 1320 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \quad c = \underline{20}, \quad b = 8c - 120 = \underline{40}$$

$$a = 120 - b - c = \underline{60}$$

4.1:

6) 40 - 40



a) $X = 0{,}6y + 0{,}4z$

- X: sannsynlighet for å vinne fra 40-40
- y: sannsynlighet for å vinne fra fordel A
- z: sannsynlighet for å vinne fra fordel B
- $0{,}6$: sannsynlighet for fordel A fra 40-40
- $0{,}4$: sannsynlighet for fordel B fra 40-40

$0{,}6y$: sannsynlighet for at blir fordel A og deretter vinner A

$0{,}4z$: sannsynlighet for at blir fordel B og så vinner A