3.2 : 3, 8

3.3 6, 10, 12

3.4 5, 8, 12

3.2, 3 $f(x,y,z) = x^2 z - y\sin(yz)$

$\vec{r}(t) = e^t \vec{i} + t\vec{j} + \cos(t^2)\vec{k}$

$g(t) = f(\vec{r}(t))$, $g'(t) = ?$

$g'(t) = \frac{\partial f}{\partial x}(\vec{r}(t))\, x'(t) + \frac{\partial f}{\partial y}(\vec{r}(t))\, y'(t)$

$+ \frac{\partial f}{\partial z}(\vec{r}(t))\, z'(t)$

$\frac{\partial f}{\partial x} = 2xz$, $\frac{\partial f}{\partial x}(\vec{r}(t)) = 2e^t \cos t^2$

$\frac{\partial f}{\partial y} = -\sin yz - yz\cos(yz)$

$\frac{\partial f}{\partial y}(\vec{r}(t)) = -\sin(t\cos t^2) - t\cos(t^2)\cos(t\cos t^2)$

$\frac{\partial f}{\partial z} = x^2 - y^2\cos(yz)$

$\frac{\partial f}{\partial z}(\vec{r}(t)) = e^{2t} - t^2\cos(t\cos t^2)$

$x'(t) = e^t$, $y'(t) = 1$, $z'(t) = -2t\sin(t^2)$

$$g'(t) = \left(2e^t \cos t^2\right) e^t +$$

$$+ \left(-\sin(t\cos t^2) - (\cos t^2) t \cos(t \cos t^2)\right) \cdot 1$$

$$+ \left(e^{2t} - t^2 \cos(t \cos t^2)\right)\left(-2t \sin t^2\right)$$

(ordner dere fin dere fasitsvaret)

3.2.8

$f: \mathbb{R}^2 \to \mathbb{R}$ har

kontinuerlige partielt deriverte

av orden 2. Gitt

$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$

$g(t) = f(\vec{r}(t))$, $g''(t) = ?$

$$g'(t) = \frac{\partial f}{\partial x}(\vec{r}(t))\, x'(t) + \frac{\partial f}{\partial y}(\vec{r}(t))\, y'(t)$$

$$g''(t) = \left(\frac{\partial^2 f}{\partial x^2}(\vec{r}(t))\, x'(t) + \frac{\partial^2 f}{\partial y \partial x}(\vec{r}(t))\, y'(t)\right) x'(t)$$

$$+ \left(\frac{\partial^2 f}{\partial x \partial y}(\vec{r}(t))\, x'(t) + \frac{\partial^2 f}{\partial y^2}(\vec{r}(t))\, y'(t)\right) y'(t)$$

$$+ \frac{\partial f}{\partial x}(\vec{r}(t))\, x''(t) + \frac{\partial f}{\partial y}(\vec{r}(t))\, y''(t)$$

$$= \frac{\partial^2 f}{\partial x^2}(\vec{r}(t))\left(x'(t)\right)^2 + 2\frac{\partial^2 f}{\partial x \partial y}(\vec{r}(t))\, x'(t)\, y'(t)$$

$$+ \frac{\partial^2 f}{\partial y^2}(\vec{r}(t))\left(y'(t)\right)^2 + \frac{\partial f}{\partial x}(\vec{r}(t))\, x''(t)$$

$$+ \frac{\partial f}{\partial y}(\vec{r}(t))\, y''(t)$$

3.3.6

$$\vec{r}(t) = e^{t}\vec{i} - e^{-t}\vec{j} + \sqrt{2}t\vec{k},\ t \in [0,1]$$

$$f(x,y,z) = xyz$$

$$\int_C f\,ds = \int_0^1 f(\vec{r}(t))\,v(t)\,dt =$$

$$= \int_0^1 e^{t}(-e^{-t})\sqrt{2}t\sqrt{\underbrace{(e^{t})^2+(e^{-t})^2+2}_{(e^{t}+e^{-t})^2}}\,dt$$

$$= \int_0^1 -\sqrt{2}t(e^{t}+e^{-t})\,dt$$

$$\int_0^1 -\sqrt{2}\,\underset{v}{t}\,(\underset{u'}{e^t}+e^{-t})\,dt =$$

$$= -\sqrt{2}\left(\left(\Big|_0^1 te^t - \int_0^1 e^t\right)dt\right.$$

$$\left. + \Big|_0^1 \left(-te^{-t} + \int_0^1 e^{-t}dt\right)\right)$$

$$= -\sqrt{2}\left(\left(e - e + 1\right) + \left(-e^{-1} - e^{-1} + 1\right)\right)$$

$$= \underline{\underline{2\sqrt{2}\,(e^{-1} - 1)}}$$

10) $\vec{r}(t) = (2t - t^2)\vec{i} + \frac{8}{3}t^{3/2}\vec{j}, \ t \in [0,1]$

Utbyggingskostanden pr. kilometer

$p(x,y) = K(10+y)$

Totale utbyggingskostnaden:

$$T = \int_0^1 p(\vec{r}(t))v(t)\,dt =$$

$$= \int_0^1 K\left(10 + \frac{8}{3}t^{3/2}\right)\sqrt{(2-2t)^2 + (4t^{1/2})^2}\,dt$$

$$T = \int_0^1 K\left(10 + \frac{8}{3}t^{3/2}\right)\sqrt{4-8t+4t^2+16t}\,dt$$

$$= \int_0^1 K\left(10+\frac{8}{3}t^{3/2}\right)\sqrt{\underbrace{4t^2+8t+4}_{(2t+2)^2}}\,dt$$

$$= \int_0^1 K\left(10+\frac{8}{3}t^{3/2}\right)(2t+2)\,dt$$

$$= 2K\int_0^1 \left(10+\frac{8}{3}t^{3/2}+10t+\frac{8}{3}t^{5/2}\right)dt$$

$$= 2K\left[10t+\frac{2}{5}\frac{8}{3}t^{5/2}+5t^2+\frac{2}{7}\frac{8}{3}t^{7/2}\right]_0^1$$

$$= 2K\left[10+\frac{16}{15}+5+\frac{16}{21}\right] =$$

$$= 2K\frac{589}{35} \approx 33{,}7\,K$$

3.3.12

Gitt kurve i polarkoordinater

a) der $r = f(\theta)$. Har

$(x, y) = (r\cos\theta, r\sin\theta)$

$\vec{r}(\theta) = f(\theta)\cos\theta\,\vec{i} + f(\theta)\sin\theta\,\vec{j}$

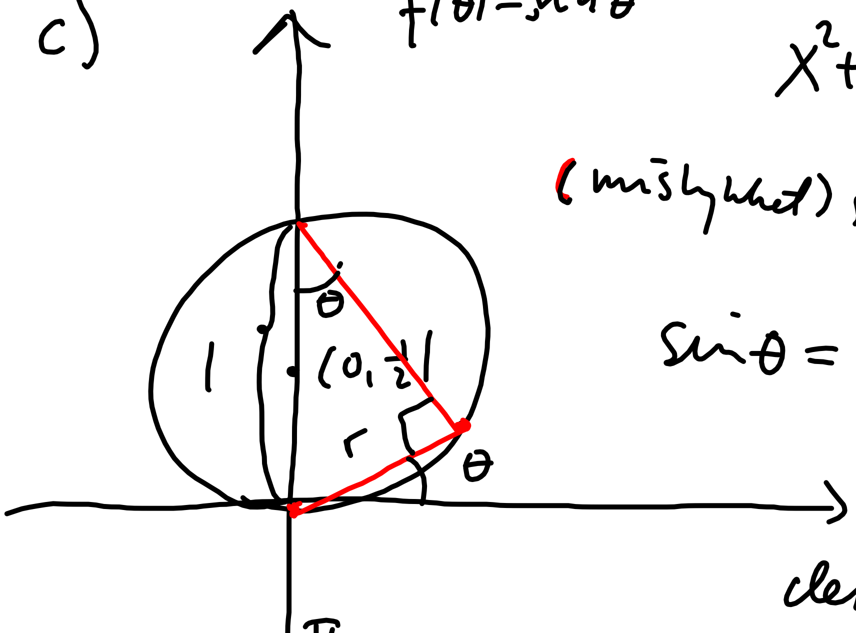
b) $\vec{r}\,'(\theta) = (f'(\theta)\cos\theta + f(\theta)(-\sin\theta))\,\vec{i}$
$+ (f'(\theta)\sin\theta + f(\theta)\cos\theta)\,\vec{j}$

$v(\theta) = \sqrt{(f'(\theta)\cos\theta - f(\theta)\sin\theta)^2 + (f'(\theta)\sin\theta + f(\theta)\cos\theta)^2}$

$= \sqrt{(f')^2\cos^2\theta - 2f'f\cos\theta\sin\theta + f^2\sin^2\theta}$
$\overline{+ (f')^2\sin^2\theta + 2f'f\cos\theta\sin\theta + f^2\cos^2\theta}$

$= \sqrt{(f')^2 + f^2}$

c) $f(\theta)=\sin\theta$



$$x^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{4}$$

(mislykket) sirkel

$\sin\theta = r$

dvs. vår kurve er denne sirkelen.

$$l = \int_0^{\pi} \sqrt{(f')^2+(f)^2}\, d\theta =$$

$$= \int_0^{\pi} \sqrt{(\cos\theta)^2+(\sin\theta)^2}\, d\theta = \int_0^{\pi} d\theta = \pi$$

(bue lengde av sirkel med radius $\frac{1}{2}$)

d) $g(x,y) = xy$, $\vec{r}(\theta) = \sin\theta\cos\theta\,\vec{i} + \sin\theta\sin\theta\,\vec{j}$

C er kurven fra c)

$$\int_C g(x,y)\,ds = \int_0^{\pi} \sin^3\theta\cos\theta\,d\theta =$$

$u = \sin\theta$, $du = \cos\theta\,d\theta$

$$= \left[\frac{1}{4}\sin^4\theta\right]_0^{\pi} = 0 - 0 = 0.$$

3.4.5

$$F(x,y,z) = yz\vec{i} + x\vec{j} + xy\vec{k}$$

$$\vec{r}(t) = t\vec{i} + (\arctan t)\vec{j} + t\vec{k},\ C$$

$$t \in [0,1) \quad \int_C F \cdot d\vec{r} = ?$$

$$I = \int_C \vec{F} \cdot d\vec{r} = \int_0^1 \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

$$= \int_0^1 (t\arctan t\,\vec{i} + t\vec{j} + t\arctan t\,\vec{k}) \cdot (\vec{i} + \frac{1}{1+t^2}\vec{j} + \vec{k})\,dt =$$

$$= \int_0^1 (2t\arctan t + \frac{t}{1+t^2})\,dt$$

$$\int_0^1 (\underset{\substack{\parallel \\ u'}}{t}\ \underset{\substack{\parallel \\ v}}{\arctan t})\, dt =$$

$$= \left[\frac{1}{2}t^2 \arctan t\right]_0^1 - \int_0^1 \frac{1}{2}t^2 \frac{1}{1+t^2}\, dt$$

$$= \frac{\pi}{8} - \frac{1}{2}\int_0^1 \left(1 - \frac{1}{1+t^2}\right) dt =$$

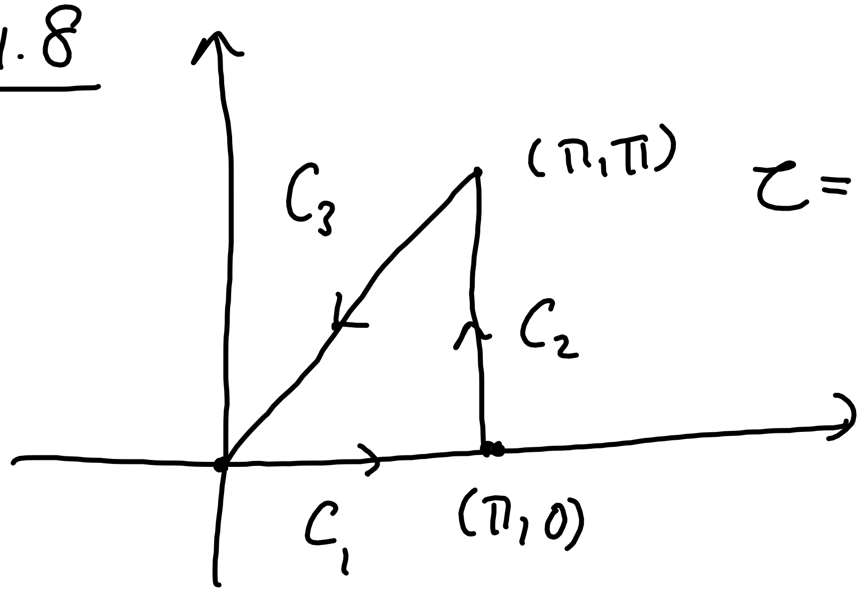
$$= \frac{\pi}{8} - \frac{1}{2}\left[t - \arctan t\right]_0^1 = \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$$

$$= \frac{\pi}{4} - \frac{1}{2}, \quad I = \frac{\pi}{2} - 1 + \int_0^1 \frac{t}{1+t^2}\, dt$$

$$= \frac{\pi}{2} - 1 + \left[\frac{1}{2}\ln(1+t^2)\right]_0^1 =$$

$$= \underline{\underline{\frac{\pi}{2} - 1 + \frac{1}{2}\ln 2}}$$

3.4.8



$C = C_1 \cup C_2 \cup C_3$

$$F(x,y) = \cos x \sin y \,\vec{i} + x\vec{j}$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_{C_1} \vec{F}\cdot d\vec{r} + \int_{C_2} \vec{F}\cdot d\vec{r} + \int_{C_3} \vec{F}\cdot d\vec{r}$$

C_1 er gitt ved $\vec{r}_1(t) = t\vec{i} \quad t \in [0,\pi]$

C_2 —"— $\vec{r}_2(t) = \pi\vec{i} + t\vec{j}, \; t \in [0,\pi]$

C_3 —"— $\vec{r}_3(t) = (\pi - t)\vec{i} + (\pi - t)\vec{j}$
$t \in [0,\pi]$

Bruk $\vec{r}_i(t)$, $i = 1, 2, 3$ til å regne hver av de tre integralene