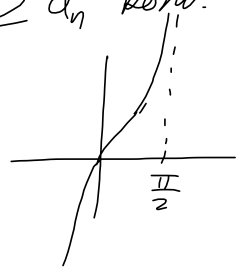
Kap. 12

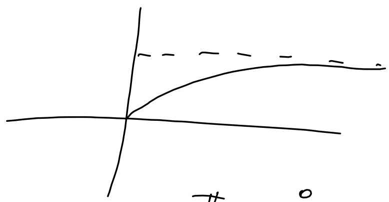
12.1.4 Divergenstesten: $\sum a_n$ konv. $\Rightarrow a_n \to 0$



$x \to \frac{\pi}{2} \Rightarrow \tan x \to \infty$

a) $\sum_{n=0}^{\infty} \arctan n$

divergerer siden $\arctan n \to \frac{\pi}{2} \neq 0$ når $n \to \infty$.



$\arctan x \to \frac{\pi}{2}$ når $x \to \infty$

b) $\sum_{n=1}^{\infty} \cos(\underbrace{\tfrac{1}{n}}_{\downarrow\, 0})$. Ser at $\cos(\frac{1}{n}) \to 1$, $n \to \infty$ $\Rightarrow$ divergerer.

$\left(e^{x} = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^{n}\right)$

c) $\sum_{n=1}^{\infty} \left(1-\sin\frac{1}{n}\right)^{n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1-\sin\frac{1}{n}\right)^{n} = \lim_{n\to\infty} e^{\ln\left(1-\sin\frac{1}{n}\right)^{n}} = \lim_{n\to\infty} e^{n\ln\left(1-\sin\frac{1}{n}\right)}$$

$$= e^{\lim_{n\to\infty}\frac{\ln\left(1-\sin\frac{1}{n}\right)}{\frac{1}{n}}} = e^{\lim_{x\to 0}\frac{\ln(1-\sin x)}{x}} \overset{LH}{=} e^{\lim_{x\to 0}\frac{\frac{-\cos x}{1-\sin x}}{1}}$$

$$= e^{-1} \Rightarrow \text{divergent.}$$

Integraltesten:

Anta at $f: [1,\infty) \to \mathbb{R}$ som er positiv, kontinuerlig, og avtagende.

Da har vi at $\sum_{n=1}^{\infty} f(n)$ konv. $\Leftrightarrow \int_1^{\infty} f(x)\,dx$ konv.

12.2.2 Når konvergerer $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$? For hvilke p.

Sett $f(x) = \frac{1}{x(\ln x)^p}$. Denne er positiv, kont., og avtagende.

Vi må regne ut $\int_2^{\infty} \frac{dx}{x(\ln x)^p}$ sett $u = \ln x \Rightarrow du = \frac{1}{x}dx$

$$u^{-p} \qquad \int_{\ln 2}^{\infty} \frac{du}{u^p} = \begin{cases} p=1 \to \lim_{t\to\infty} [\ln u]_{\ln 2}^{t} \to \infty \\ p \neq 1 \to \lim_{t\to\infty} \left[\frac{u^{-p+1}}{-p+1}\right]_{\ln 2}^{t} \end{cases}$$

Hvis $-p+1 > 0$ så vil $u^{-p+1} \to \infty$ når $u \to \infty$

Hvis $-p+1 < 0$ så vil $u^{-p+1} \to 0$ når $u \to \infty$

$\Rightarrow$ konvergerer hvis og bare hvis $-p+1 < 0 \Leftrightarrow \underline{\underline{p > 1}}$

Forholdstesten: $\sum a_n$ konv. hvis $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$

— " — div. ——— " ——— > 1

Sier ikke noe $= 1$

Rottesten: $\sum a_n$ konv. hvis $\lim \sqrt[n]{a_n} < 1$

— " — div ——— " ——— > 1

Sier ikke noe $= 1$

12.2.5

a) $\sum_{n=0}^{\infty} n \cdot \frac{1}{3^n}$ $\qquad \frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)}{3^{n+1}}}{\frac{n}{3^n}} = \frac{n+1}{3n} \to \frac{1}{3} < 1$ når $n \to \infty$ $\qquad \frac{n+1}{n} = \frac{1+\frac{1}{n}}{1} \to 1$

konvergent.

b) $\sum_{n=1}^{\infty} \frac{3^n}{n^{10}}$ $\qquad \frac{a_{n+1}}{a_n} = \frac{3n^{10}}{(n+1)^{10}} \to 3$ divergent.

c) $\sum_{n=1}^{\infty} (1-\frac{1}{n})^{n^2}$ $\qquad \sqrt[n]{a_n} = \left(\left(1-\frac{1}{n}\right)^{n^2}\right)^{\frac{1}{n}} = \left(1-\frac{1}{n}\right)^n \xrightarrow[n\to\infty]{} e^{-1} < 1$

$\Rightarrow$ konv.

d) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$ $\qquad \frac{a_{n+1}}{a_n} = \frac{\frac{e^{n+1}}{(n+1)!}}{\frac{e^n}{n!}} = \frac{e}{n+1} \to 0$

$\Rightarrow$ konv.

g) $\sum_{n=1}^{\infty} \frac{n!\,4^n}{n^n}$ $\qquad \frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!\,4^{n+1}}{(n+1)^{n+1}}}{\frac{\cancel{n!}\,\cancel{4^n}}{n^n}} = \frac{4\cancel{(n+1)}\,n^n}{(n+1)^{\cancel{n+1}}}$

$= \frac{4n^n}{(n+1)^n} = 4\left(\frac{n}{n+1}\right)^n = \frac{4}{\left(\frac{n+1}{n}\right)^n} = \frac{4}{(1+\frac{1}{n})^n}$

$\to \frac{4}{e} > 1$

divergent.

12.3

Hvis a_n har alternerende fortegn, $|a_n| \to 0$ og avtagende
så er $\sum a_n$ konv.

1.a) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$ alternerende, $|a_n|$ avtagende og $|a_n| \to 0$

$\Rightarrow \sum a_n$ konv.

b) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n+7n^2}$ alternerende

$$\lim_{n\to\infty} \frac{n^2+1}{n+7n^2} = \lim_{n\to\infty} \frac{1+\frac{1}{n^2}}{\frac{1}{n}+7} = \frac{1}{7}$$

$\Rightarrow$ ikke konv (divergenstesten)

c $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}}$ alternerende, $|a_n|$ avtagende, og $\to 0$

$\Rightarrow$ konvergent.

12.4

1.a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ alternerende, $|a_n| \to 0$, avtagende $\Rightarrow$ konv.

$\sum |a_n| = \sum_{n=0}^{\infty} \frac{1}{n+1}$ div, fordi $\sum_{n=\cancel{0}}^{\infty} \frac{1}{n}$ div.

Sier da at $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ er betinget konvergent

b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+4}$ alterende, ... $\Rightarrow$ konv.

$\sum |a_n| = \sum_{n=0}^{\infty} \frac{1}{n^2+4}$ konv (siden $\sum \frac{1}{n^2}$ konv (integraltesten.))

Sier da at $\sum \frac{(-1)^n}{n^2+4}$ er absolutt konvergent