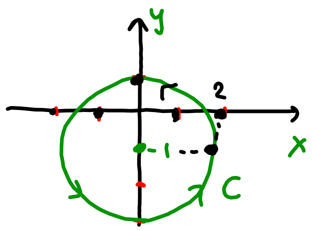
Løsning/gjennomgang prøveeksamen Mat 1110

Lørdag 17. mars 2018

NB: Disse notatene er kortfattede. Se opptak/podcast for gjennomgang.

1. C

2. 

$$(x-a)^2+(y-b)^2=r^2$$
$$(x-0)^2+(y-(-1))^2=2^2$$

E: $x = 2\cos t$
$y = 2\sin t - 1$

Innsatt: $x^2+(y+1)^2 = 4\cos^2 t + 4\sin^2 t = 4$

$t=0$: $(x,y) = (2,-1)$
$t=\frac{\pi}{2}$: $(x,y) = (0,1)$

3. $H(x,y) = F(G(x,y))$ $\qquad (x,y) = (1,1)$

$H'(x,y) = F'(G(x,y)) \cdot G'(x,y)$ $\qquad G(x,y) = G(1,1) = (2,2)$

$$F' = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$$

$$G' = \begin{pmatrix} 1 & 2y \\ 2x & 1 \end{pmatrix} \overset{(1,1)}{=} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$H'(1,1) = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = \begin{array}{cc|cc} & & 1 & 2 \\ & & 2 & 1 \\ \hline 2 & 4 & 10 & 8 \\ 3 & 5 & 13 & 11 \end{array}$$

C

4

$$\vec{T}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix} \qquad \vec{T}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}3\\7\end{pmatrix}$$

Matrisen til $\vec{T}$: $\begin{pmatrix}-1 & 3\\1 & 7\end{pmatrix}$

A

5

$$\begin{vmatrix}0 & 1 & 1\\2 & -1 & 8\\1 & 1 & 0\end{vmatrix} = -1\begin{vmatrix}2 & 8\\1 & 0\end{vmatrix} + 1\cdot\begin{vmatrix}2 & -1\\1 & 1\end{vmatrix} = 8 + 3$$

$\det A \neq 0$

E

6

A
$$\begin{vmatrix}1-\lambda & k\\0 & k-\lambda\end{vmatrix} = (1-\lambda)(k-\lambda) = 0$$

B
$$\begin{vmatrix}1-\lambda & k\\k & 1-\lambda\end{vmatrix} = (1-\lambda)^2 - k^2 = 0$$
$$(\lambda-1)^2 = k^2$$

C
$$\begin{vmatrix}-\lambda & k\\k & -\lambda\end{vmatrix} = \lambda^2 - k^2 = 0 \qquad \lambda = \pm k$$

D
$$\begin{vmatrix}0-\lambda & -k\\k & 1-\lambda\end{vmatrix} = -\lambda(1-\lambda) + k^2 = 0$$
$$\lambda^2 - \lambda + k^2 = 0$$
$$\lambda = \frac{1 \pm \sqrt{1-4k^2}}{2}$$

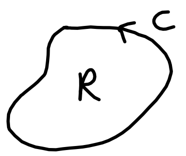
D

7

$\nabla g = \vec{F}$, dvs.
$$\frac{\partial g}{\partial x} = e^{x+2y} \Rightarrow g = e^{x+2y} + \phi(y)$$
$$\frac{\partial g}{\partial y} = 2e^{x+2y} \Rightarrow g = e^{x+2y}$$

D

8



$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \int_C P\,dx + \int_C Q\,dy$$

$$\left.\begin{matrix} Q = x \\ P = 0 \end{matrix}\right\} \text{ gir } \underbrace{\iint_R 1\,dx\,dy}_{\text{areal } R} = \int_C x\,dy$$

D

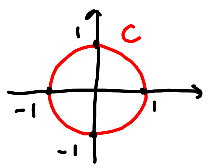
9

$$\vec{r}\,'(t) = \cos t\,\vec{i} + (-\sin t)\,\vec{j}$$

$$\vec{a}(t) = \vec{r}\,''(t) = (-\sin t)\,\vec{i} + (-\cos t)\,\vec{j}$$

A

10



$$\vec{r}(t) = (\sin 2t,\ \cos 2t)$$

$$t \in [0, \pi]$$

$$\vec{F} = (P, Q) \text{ gir } \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1 - 1 = 0$$

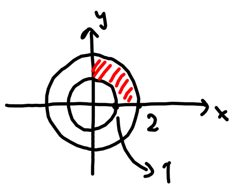
A

11

$$\iint_R \frac{x}{(x^2+y^2)^{3/2}}\,dx\,dy$$

$$1 \leq x^2+y^2 \leq 4$$

og $x \geq 0$, $y \geq 0$



$$r \in [1, 2]$$

$$\theta \in [0, \tfrac{\pi}{2}]$$

B

$$\text{Integralet} = \int_0^{\pi/2} \left[\int_1^2 \frac{r\cos\theta}{r^3} \cdot r\,dr \right] d\theta$$

12

$$16x^2 - 4y^2 + 32x - 4y - 1 = 0$$

$$16(x^2+2x) - 4(y^2+1) = 1$$

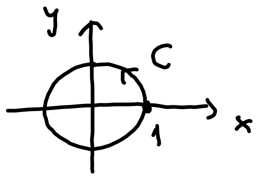
$$16(x^2+2x+1) - 4(y^2+y+\tfrac{1}{4}) = 1 + 16 - 1$$

$$16(x+1)^2 - 4(y+\tfrac{1}{2})^2 = 16$$

B

13 $\begin{pmatrix}0\\1\\2\\2\end{pmatrix}$ og $\begin{pmatrix}0\\2\\-1\\0\end{pmatrix}$ C

14 $\vec{F}(x,y) = (2x+y)\vec{i} + (2y+x)\vec{j}$



ϕ pot. funk.

$$\frac{\partial \phi}{\partial x} = 2x+y \Rightarrow \phi(x,y) = x^2 + xy + g(y)$$

$$\frac{\partial \phi}{\partial y} = 2y+x \Rightarrow \phi(x,y) = y^2 + xy + h(x)$$

$$\phi(x,y) = x^2 + xy + y^2$$

B

15 $$\begin{pmatrix}1 & 7 & 1\\0 & 1 & 0\\1 & 0 & 1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}9\\1\\2\end{pmatrix}$$

$$\begin{pmatrix}1 & 7 & 1\\0 & 1 & 0\\1 & 0 & 1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}2\\0\\2\end{pmatrix} = 2\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}$$

E

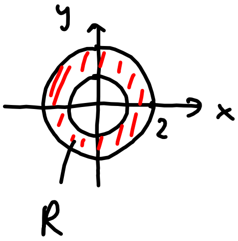
16 $$f(x,y) = 1 + 2x + x^2 + 4y^2 = c$$

$$(x+1)^2 + 4y^2 = c \qquad c = 0 : (x,y) = (-1,0)$$

$$c > 0 : \frac{(x+1)^2}{c} + \frac{4y^2}{c} = 1$$

E

(17) $1 \leq x^2+y^2 \leq 4$



$$\iint_R x^3 \, dx\,dy$$

(D)

$r \in [1, 2]$

$\theta \in [0, 2\pi]$

$$\text{Integral} = \int_0^{2\pi} \left[\int_1^2 (r\cos\theta)^3 \cdot r \, dr \right] d\theta$$

$$= \int_0^{2\pi} \left[\frac{1}{5} r^5 \cdot \cos^3\theta \right]_{r=1}^{r=2} d\theta = \int_0^{2\pi} \left[\frac{32}{5}\cos^3\theta - \frac{1}{5}\cos^3\theta \right] d\theta$$

$$= \frac{31}{5} \int_0^{2\pi} \cos^3\theta \, d\theta = 0.$$

(18) (C)

(19)

$$L\begin{pmatrix} x \\ y \\ z \end{pmatrix} = F\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + F'(3,0,0) \cdot \begin{pmatrix} x-3 \\ y-0 \\ z-0 \end{pmatrix}$$

$$F\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$F' = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} \end{pmatrix} = \begin{pmatrix} 5(x-3)^4 & 0 & 0 \\ 0 & 5y^4z^5 & 5y^5z^4 \end{pmatrix}$$

$$\overset{(3,0,0)}{=} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(C)

20 $\iint_{\mathbb{R}^2} x^2 e^{y^2} \, dx \, dy$ A

Integralet $\geq \iint_{\mathbb{R}^2} x^2 \cdot 1 \, dx \, dy$

$$= \lim_{R \to \infty} \int_{-R}^{R} \left[\int_{-R}^{R} x^2 \, dx \right] dy = +\infty$$

21 B fordi $\det(AB) = \det(A) \cdot \det(B)$

$$\det(A^2) = \det(A) \cdot \det(A)$$
$$= (\det A)^2$$

22 $\begin{pmatrix} 1 & 7 & 0 \\ 0 & 8 & 4 \\ 0 & 0 & 1 \end{pmatrix}$ $\begin{vmatrix} 1-\lambda & 7 & 0 \\ 0 & 8-\lambda & 4 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)(8-\lambda)(1-\lambda)$

$$= 0$$

C

23 $4 \leq x^2 + y^2 + z^2 \leq 9$

området mellom kule med radius 2 og kule med radius 3.

$$\frac{4}{3}\pi \cdot 3^3 - \frac{4}{3}\pi \cdot 2^3 = \frac{4}{3}\pi(27-8) = \frac{76\pi}{3}$$

B

Kulekoord:

$\rho \in [2,3]$

$\phi \in [0,\pi]$

$\theta \in [0,2\pi]$

Integral

$$= \int_0^{2\pi} \int_0^{\pi} \int_2^3 1 \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

(24) $f(x,y) = (x+1)^2 + y^2$ Tangentplan i $(0,1,2)$

$f(0,1) = 1^2 + 1^2 = 2$

$$z = f(0,1) + \frac{\partial f}{\partial x}(0,1)\cdot(x-0) + \frac{\partial f}{\partial y}(0,1)\cdot(y-1)$$

$$= 2 + 2(x+1)\Big|_{(x,y)=(0,1)} \cdot x + 2y\Big|_{(x,y)=(0,1)} \cdot (y-1)$$

$$= 2 + 2x + 2(y-1)$$

$$= 2x + 2y$$

(A)

(25) $\left.\begin{array}{l} f: \mathbb{R}^2 \to \mathbb{R} \\ G: \mathbb{R} \to \mathbb{R}^2 \end{array}\right\}$ $h: \mathbb{R} \to \mathbb{R}$ ved $h(t) = f(G(t))$

$G(t) = (G_x(t), G_y(t))$

$$h'(t) = \frac{\partial f}{\partial x}(G(t)) \cdot G_x'(t) + \frac{\partial f}{\partial y}(G(t)) \cdot G_y'(t)$$

$$= \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} \cdot \begin{bmatrix} G_x'(t) \\ G_y'(t) \end{bmatrix}$$

(D)