Så:

$$V = \iint_S f(x,y)\,dx\,dy = \int_0^{2\pi}\int_0^{2} f(2+r\cos\theta,\ -1+r\sin\theta)\, r\,dr\,d\theta$$

Bytte til polarkoordinater

OBS!

M: Hva er:

$$z = f(2+r\cos\theta,\ -1+r\sin\theta) = 4-(r\cos\theta)^2-(r\sin\theta)^2$$

polarkoord.

Setter inn
$x = 2 + r\cos\theta$
$y = -1 + r\sin\theta$

$$= 4 - r^2$$

Så:

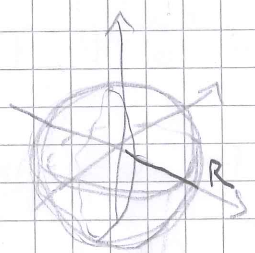
$$V = \int_0^{2\pi}\int_0^{2} (4-r^2)\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\left[2r^2 - \frac{1}{4}r^4\right]_{r=0}^{2} d\theta$$

$$= \int_0^{2\pi}\{8-4-0\}\,d\theta = \int_0^{2\pi} 4\,d\theta$$

$$= [4\theta]_{\theta=0}^{2\pi} = \underline{\underline{8\pi}}$$

4) Overflateareal kule radius R:



Parametrisering:

kulekoord.

$$\vec{r}(u,v) = \big(R\sin(u)\cos(v),\ R\sin u\sin v,\ R\cos u\big)$$

$$v \in [0, 2\pi],\quad u \in [0, \pi]$$

Regner ut alt vi må trenge:

$$\frac{\partial \vec{r}}{\partial u} = (R\cos u\cos v,\ R\cos u\sin v,\ -R\sin u)$$

$$\frac{\partial \vec{r}}{\partial v} = (-R\sin u\sin v,\ R\sin u\cos v,\ 0)$$

$$\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} = \Big(R^2\sin^2 u\cos v,\ R^2\sin^2 u\sin v,$$

$$R^2\big(\underbrace{\cos u\sin u\cos^2 v + \cos u\sin u\sin^2 v}_{=\cos u\sin u}\big)\Big)$$

$$\boxed{\cos^2 v + \sin^2 v = 1}$$

Så:

$$\left|\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right| = \Big\{R^4\sin^4 u\cos^2 v + R^4\sin^4 u\sin^2 v + R^4\cos^2 u\sin^2 u\Big\}^{\frac{1}{2}}$$

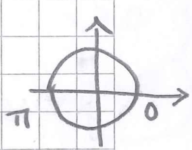
$\sin^2 + \cos^2 = 1$

$$= R^2(\sin^4 u + \cos^2 u\sin^2 u)^{\frac{1}{2}}$$

$$= R^2\big(\sin^2 u\,(\underbrace{\sin^2 u + \cos^2 u}_{1})\big)^{\frac{1}{2}} = R^2(\sin^2 u)^{\frac{1}{2}}$$

$$= \underline{R^2|\sin u|}$$

Rot def. som pos. versjon



Så:

$$A = \iint_R \left|\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right| du\,dv = \int_0^{2\pi}\int_0^{\pi} R^2|\sin u|\,du\,dv$$

se s. 449

$$= R^2 \underbrace{2\pi}_{\text{Ingen } v} \int_0^{\pi} \sin u\,du = 2\pi R^2[-\cos u]_{u=0}^{\pi}$$

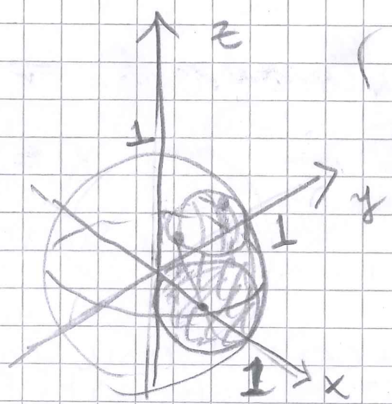
$\sin \geq 0$ på $[0, \pi]$

$$= 2\pi R^2(-(-1)-(-1))$$

$= \underline{\underline{4\pi R^2}}$

FINN AREAL AV DEN DEL AV KULA $x^2+y^2+z^2=1$
SOM ER OVER SIRKELEN $(x-\frac{1}{2})^2+y^2 \leq \frac{1}{4}$:

6.4.7.)



$$(x-\frac{1}{2})^2 - y^2 \leq \frac{1}{4}$$

Radius $\frac{1}{2}$ $\left((\frac{1}{2})^2 = \frac{1}{4}\right)$,

sentrum $(\frac{1}{2}, 0)$.

Flaten er beskrevet v/: $z = f(x,y) = \sqrt{1-x^2-y^2}$

$= \sqrt{1-r^2}$ (polarkoord.)

Enhetskule: $1 = x^2+y^2+z^2$

s. 595 for formel

$$\frac{\partial f}{\partial x} = -\frac{x}{\sqrt{1-x^2-y^2}}$$

$$\frac{\partial f}{\partial y} = -\frac{y}{\sqrt{1-x^2-y^2}}$$

Så: $\sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}$

$$= \sqrt{1+\frac{x^2}{1-x^2-y^2}+\frac{y^2}{1-x^2-y^2}} = \frac{1}{\sqrt{1-x^2-y^2}}$$

Grenser for integrasjonsområde (i polarkoordinater)

Sirkel: $(r\cos\theta - \frac{1}{2})^2 + r^2\sin^2\theta \leq \frac{1}{4}$

Dvs: $r^2\cos^2\theta + r^2\sin^2\theta - r\cos\theta + \frac{1}{4} \leq \frac{1}{4}$