4.) V heller saft til E $\Rightarrow$ Har igjen $\frac{8}{9}x_n$
og Emil har $y_n + \frac{1}{9}x_n$.

E heller tilbake $\frac{1}{10}$ av saften han nå har.

$$\left(\underset{\substack{\downarrow \\ = \text{mengde saft i glassene}}}{x} - \frac{1}{9}x = \frac{8}{9}x, \quad x + \frac{1}{9}x = \frac{10}{9}x \Rightarrow \frac{\frac{1}{9}\cancel{x}}{\frac{10}{\cancel{9}}\cancel{x}} = \frac{1}{10} \right)$$

$\Rightarrow$ Viktoria får igjen $\frac{1}{10}\left(y_n + \frac{1}{9}x_n\right)$

$$\Downarrow$$

$$x_{n+1} = \frac{8}{9}x_n + \frac{1}{10}\left(y_n + \frac{1}{9}x_n\right) = 0{,}9x_n + 0{,}1y_n$$

$$y_{n+1} = \frac{9}{10}\left(y_n + \frac{1}{9}x_n\right) = 0{,}1x_n + 0{,}9y_n$$

b) $M = \begin{bmatrix} 0{,}9 & 0{,}1 \\ 0{,}1 & 0{,}9 \end{bmatrix}$

$$\det(\lambda I - M) = \begin{vmatrix} \lambda - 0{,}9 & -0{,}1 \\ -0{,}1 & \lambda - 0{,}9 \end{vmatrix} = (\lambda - 0{,}9)^2 - 0{,}1^2$$

$$= \lambda^2 - 1{,}8\lambda + 0{,}8$$

2. grad formel: $\lambda = \dfrac{1{,}8 \pm \sqrt{3{,}24 - 3{,}2}}{2} = 0{,}9 \pm 0{,}1$

$$\Downarrow$$

$$\lambda_1 = 0{,}8 \quad , \lambda_2 = 1$$

$\underbrace{\vec{v}_1}$: $\lambda_1 I - M = \begin{bmatrix} -0{,}1 & -0{,}1 \\ -0{,}1 & -0{,}1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$$\Rightarrow \vec{v}_1 = \underline{\underline{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}}$$