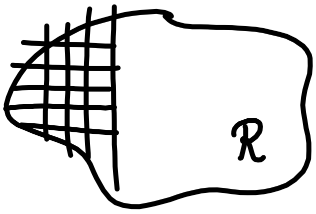
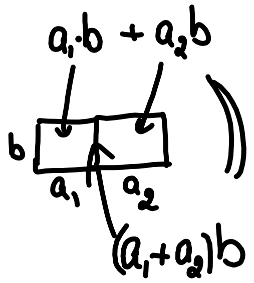
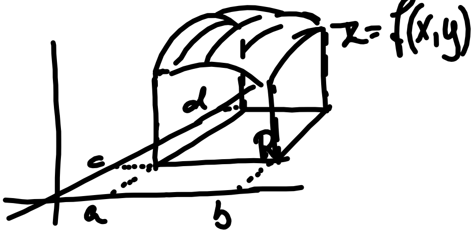
Dobbeltintegraler (Trippel-, ...)

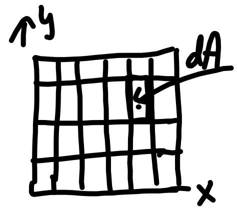
Hva er et areal?



Eneste vi kan regne ut areal av er $\square$ b $\leftrightarrow$ ab
a







Funksjonen er tilnærmet konstant

$dA = dx\,dy$

$f(x,y) \cdot dx\,dy$

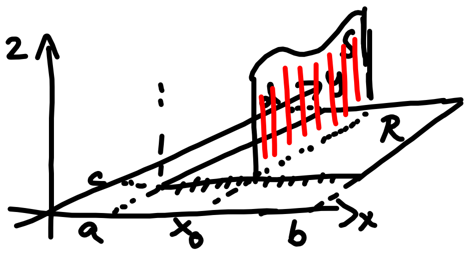
<u>f integrerbar</u>: $\underline{\iint_R} f(x,y)\,dx\,dy = \overline{\iint_R} f(x,y)\,dx\,dy$

$$= \iint_R f(x,y)\,dx\,dy$$

<u>Teorem</u> f kontinuerlig $\Rightarrow$ f integrerbar

$\square$

Hvordan regne ut $\iint_R f(x,y)\,dx\,dy$?

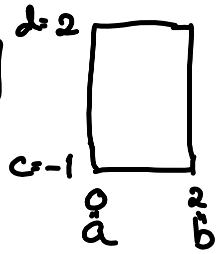


areal av S: $\int_c^d f(x_0,y)\,dy = g(x_0)$

$\int_a^b g(x)\,dx$

$$\iint_R f(x,y)\,dx\,dy = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

Eks. $f(x,y) = 4xy + 3y^2 \quad R = [0,2] \times [-1,2]$



$$\iint_R f(x,y)\,dx\,dy = \int_0^2 \left(\int_{-1}^2 4xy + 3y^2\,dy \right) dx$$

$$= \int_0^2 \left(\left[2xy^2 + y^3 \right]_{-1}^2 \right) dx$$

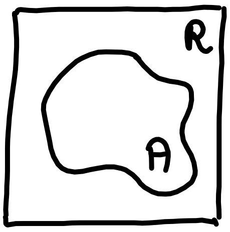
$$= \int_0^2 6x + 9 \; dx$$

$$= \left[3x^2 + 9x \right]_0^2 = \underline{\underline{30}}$$

$$\int_{-1}^2 \left(\int_0^2 4xy + 3y^2\,dx \right) dy = \int_{-1}^2 8y + 6y^2\,dy = \underline{\underline{30}}$$

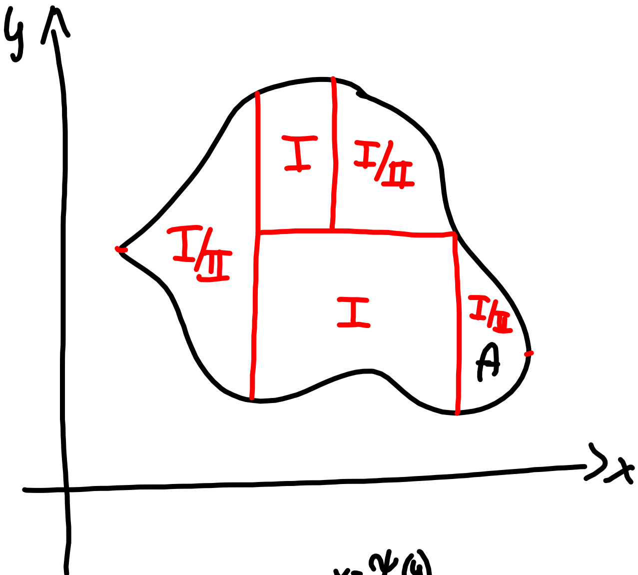
$R = [a,b] \times [c,d]$
f kontinuelig i R

$$\iint_R f\,dx\,dy = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx = \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy$$

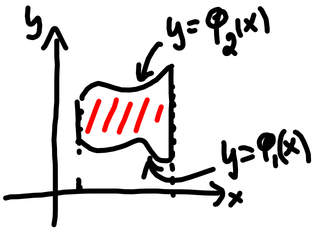


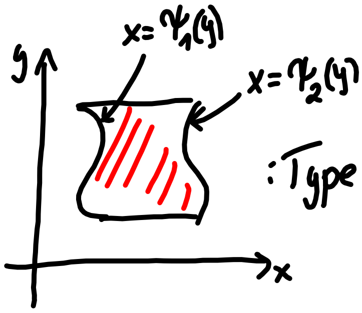
$$f_A(x,y) = \begin{cases} f(x,y) & (x,y)\in A \\ 0 & \text{ellers} \end{cases}$$

f er integrerbar over A
hvis f_A er integrerbar over R
(uavhengig av R)

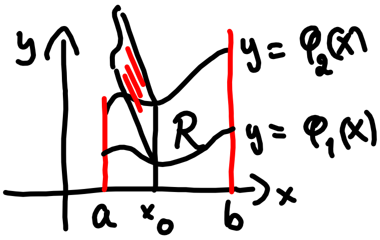


Type I:





: Type II



$$\iint_R f(x,y)\,dx\,dy = \int_a^b \left(\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)\,dy \right) dx$$

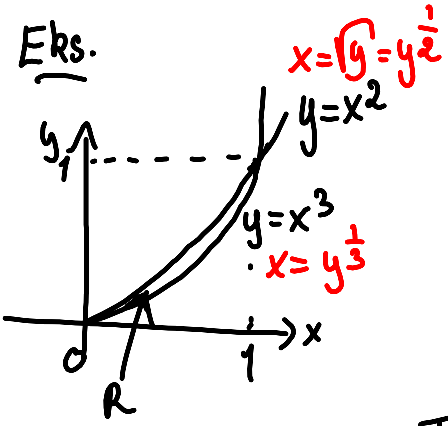
Type I.

Regner ut:

$$\int_{\varphi_1(x_0)}^{\varphi_2(x_0)} f(x_0,y)\,dy = g(x_0)$$

Type II: $\displaystyle\int_c^d \left(\int_{\psi_1(y)}^{\psi_2(y)} f(x,y)\,dx \right) dy$

Eks.



Type I: $\displaystyle\int_0^1 \left(\int_{x^3}^{x^2} 1\,dy \right) dx$

$$= \int_0^1 [y]_{x^3}^{x^2}\,dx$$

$$= \int_0^1 x^2 - x^3\,dx = \underline{\underline{\frac{1}{12}}}$$

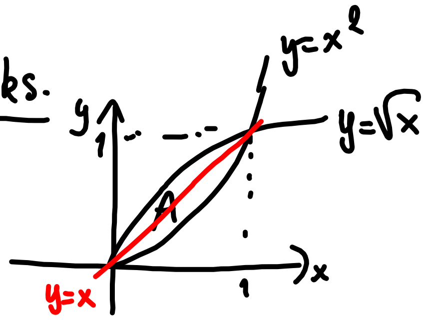
$$\text{area}(R) = \iint_R 1\,dx\,dy$$

Type II:

$$\int_0^1 \left(\int_{y^{\frac{1}{2}}}^{y^{\frac{1}{3}}} 1\,dx \right) dy = \int_0^1 [x]_{y^{\frac{1}{2}}}^{y^{\frac{1}{3}}}\,dy$$

$$= \int_0^1 y^{\frac{1}{3}} - y^{\frac{1}{2}}\,dy = \underline{\underline{\frac{1}{12}}}$$

Eks.



$f(x,y) = x - y$ over A

Type I:

$$\int_0^1 \left(\int_{x^2}^{\sqrt{x}} x - y \, dy \right) dx$$

$$= \int_0^1 \left[xy - \frac{1}{2}y^2 \right]_{x^2}^{\sqrt{x}} dx$$

$$= \int_0^1 x\sqrt{x} - \frac{1}{2}x - xx^2 + \frac{1}{2}(x^2)^2 \, dx$$

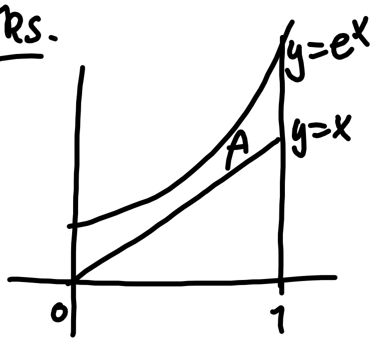
$$= \int_0^1 x^{\frac{3}{2}} - \frac{1}{2}x - x^3 + \frac{1}{2}x^4 \, dx$$

$$= \left[\frac{2}{5}x^{\frac{5}{2}} - \frac{1}{4}x^2 - \frac{1}{4}x^4 + \frac{1}{10}x^5 \right]_0^1$$

$$= \frac{2}{5} - \frac{1}{4} - \frac{1}{4} + \frac{1}{10} = \frac{8}{20} - \frac{5}{20} - \frac{5}{20} + \frac{2}{20}$$

$$= \underline{\underline{0}}$$

Eks.



$f(x,y) = x + y^2$ over A

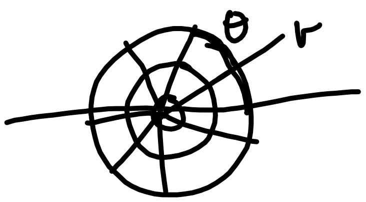
Type I:

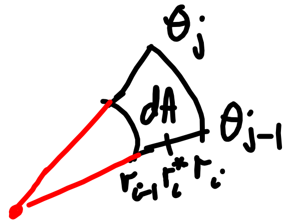
$$\int_0^1 \left(\int_{y=x}^{y=e^x} x + y^2 \, dy \right) dx$$

$$= \int_0^1 \left[xy + \frac{1}{3}y^3 \right]_{y=x}^{y=e^x} dx$$

$$= \int_0^1 \left(xe^x + \frac{1}{3}e^{3x} - x^2 - \frac{1}{3}x^3 \right) dx = \underline{\underline{1}}$$

Dobbeltintegraler i polarkoordinater





$$dA = \frac{1}{2} r_i (\theta_j - \theta_{j-1}) r_i - \frac{1}{2} r_{i-1} (\theta_j - \theta_{j-1}) r_{i-1}$$

$$= \frac{1}{2}(r_i^2 - r_{i-1}^2)(\theta_j - \theta_{j-1})$$

$$= \frac{1}{2}(r_i + r_{i-1})(r_i - r_{i-1})(\theta_j - \theta_{j-1})$$

$$= \underbrace{\frac{r_i + r_{i-1}}{2}}_{r_i^*}(r_i - r_{i-1})(\theta_j - \theta_{j-1})$$

$$= r_i^* \cdot (r_i - r_{i-1})(\theta_j - \theta_{j-1})$$

Partisjon:

A: $0 < r_1 < r_2 < \dots < r_n = R$

$0 \leq \theta_1 < \theta_2 < \dots < \theta_m = \Theta \ (= 2\pi)$

på tegningen

Funksjon

$z = f(x, y)$ over A

$$\sum_{i=1}^{n} \sum_{j=1}^{m} f(x_{ij}^*, y_{ij}^*)(r_i - r_{i-1})(\theta_j - \theta_{j-1}) r_i^*$$

$$\longrightarrow \iint_A f(r\cos\theta, r\sin\theta)\, r \, dr \, d\theta$$