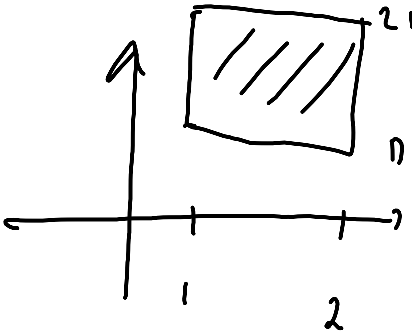
<u>6.1.1</u> d) e) og g)

d) $R = [1,2] \times [\pi, 2\pi]$



$$\iint_R x\cos(xy)\,dx\,dy = \int_1^2 \left(\int_\pi^{2\pi} x\cos(xy)\,dy \right) dx$$

$$= \int_1^2 \left(\Big[\sin(xy) \Big]_\pi^{2\pi} \right) dx$$

$$= \int_1^2 \big(\sin(2\pi x) - \sin(\pi x) \big)\, dx$$

$$= \left[\frac{-\cos(2\pi x)}{2\pi} + \frac{\cos(\pi x)}{\pi} \right]_1^2$$

$$= \left(\cancel{\frac{-1}{2\pi}} + \frac{1}{\pi} \right) - \left(\cancel{\frac{-1}{2\pi}} - \frac{1}{\pi} \right) = \underline{\underline{\frac{2}{\pi}}}$$

6.1.1. e)

$$R = [0,2] \times [1,2]$$

$$\iint_R xy e^{x^2 y}\,dx\,dy = \int_1^2 \left(\int_0^2 xy e^{x^2 y}\,dx \right) dy$$

$$= \int_1^2 \left[\frac{1}{2} e^{x^2 y} \right]_0^2 dy$$

$$= \int_1^2 \frac{1}{2}\left(e^{4y} - 1\right) dy = \frac{1}{2}\left[\frac{1}{4} e^{4y} - y\right]_1^2$$

$$= \frac{1}{2}\left[\left(\frac{1}{4} e^{8} - 2\right) - \left(\frac{1}{4} e^{4} - 1\right)\right]$$

$$= \frac{1}{8}\left(e^{8} - e^{4}\right) - \frac{1}{2}$$

6.1.1.g

$R = [1, \sqrt{3}] \times [0,1]$

$$\iint_R \frac{1}{1+x^2y}\,dxdy =$$

$$= \int_1^{\sqrt{3}} \left(\int_0^1 \frac{1}{1+x^2y}\,dy \right) dx$$

$$= \int_1^{\sqrt{3}} \left[\frac{1}{x^2} \ln(1+x^2y) \right]_0^1 dx$$

$$= \int_1^{\sqrt{3}} \frac{1}{x^2} \ln(1+x^2)\,dx$$

$u' = \frac{1}{x^2}, \; u = -\frac{1}{x}, \; v = \ln(1+x^2)$

$$= \left[-\frac{1}{x} \ln(1+x^2) \right]_1^{\sqrt{3}} + \int_1^{\sqrt{3}} \frac{1}{\cancel{x}} \frac{2\cancel{x}}{1+x^2}\,dx$$

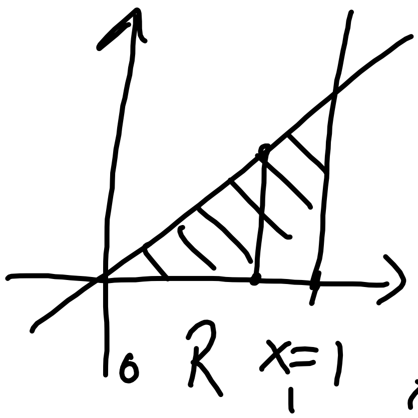
$$= \left(-\frac{1}{\sqrt{3}} \ln 4 + \ln 2 \right) + 2 \left[\arctan x \right]_1^{\sqrt{3}}$$

$$= -\frac{1}{\sqrt{3}} \ln 4 + \ln 2 + 2 \left(\frac{\pi}{3} - \frac{\pi}{4} \right)$$

$$= \frac{(\sqrt{3}-2)}{\sqrt{3}} \ln 2 + \frac{\pi}{6} = \underline{\underline{\frac{3-2\sqrt{3}}{3} \ln 2 + \frac{\pi}{6}}}$$

6.2.1. e) f) g)

6.2.1 e)



$x \in [0,1]$

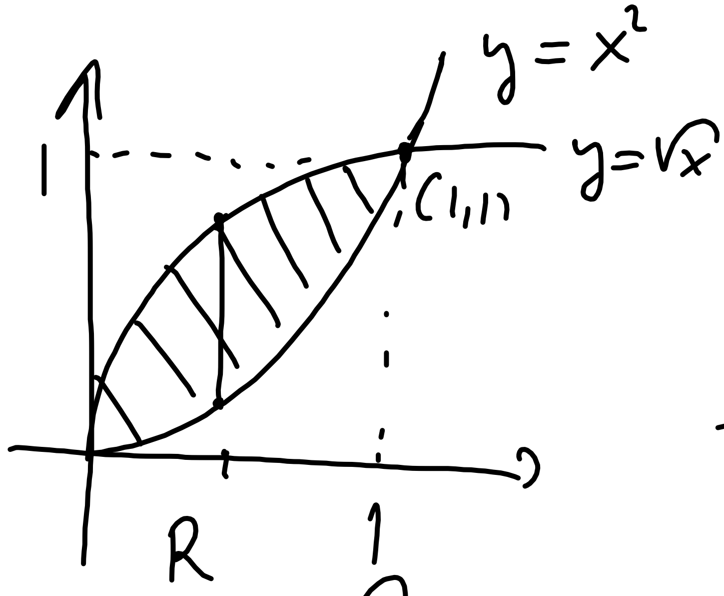
$0 \leq y \leq x$

$$\iint_R e^{x^2}\,dx\,dy = \int_0^1 \left(\int_0^x e^{x^2}\,dy \right) dx$$

$$= \int_0^1 \left(\left[e^{x^2} y \right]_{y=0}^{y=x} \right) dx = \int_0^1 x e^{x^2}\,dx$$

$$= \left[\frac{1}{2} e^{x^2} \right]_0^1 = \frac{1}{2}(e-1)$$

6.2.1. f)



$$\iint_R x^2 y \, dx\, dy$$

$$\int_0^1 \int_{x^2}^{\sqrt{x}} x^2 y \, dy\, dx = \int_0^1 \left[\frac{1}{2} x^2 y^2\right]_{y=x^2}^{y=\sqrt{x}} dx$$

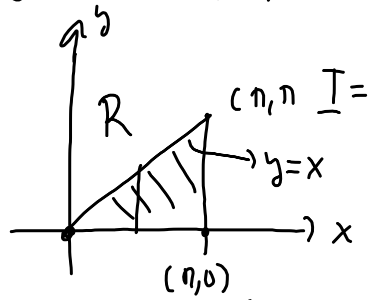
$$= \int_0^1 \frac{1}{2}\left(x^3 - x^6\right) dx = \frac{1}{2}\left[\frac{x^4}{4} - \frac{x^7}{7}\right]_0^1$$

$$= \frac{1}{2}\left[\frac{1}{4} - \frac{1}{7}\right] = \underline{\underline{\frac{3}{56}}}$$

6.2.1. g)

Trekant med hjørner

$(0,0),\ (\pi,0),\ (\pi,\pi)$



$$I=\iint_R x\cos(x+y)\,dx\,dy$$

$$I=\int_0^{\pi}\left(\int_0^{x} x\cos(x+y)\,dy\right)dx$$

$$=\int_0^{\pi}\Big[x\sin(x+y)\Big]_0^{x}\,dx$$

$$=\int_0^{\pi}(x\underset{u'}{\sin 2x}-x\sin x)\,dx=I_1+I_2$$

$$=\left(\left[x\left(\frac{-\cos 2x}{2}\right)\right]_0^{\pi}+\int_0^{\pi}\frac{\cos 2x}{2}\,dx\right)$$

$$-\left(\Big[-x\cos x\Big]_0^{\pi}+\int_0^{\pi}\cos x\,dx\right)$$

$$\left(\left(-\frac{\pi}{2}\right)+\left[\frac{\sin 2x}{4}\right]_0^{\pi}\right)$$

$$-\left(\pi+\Big[\sin x\Big]_0^{\pi}\right)=-\frac{\pi}{2}-\pi$$

$$=\underline{-\frac{3\pi}{2}}$$