3.1.2

$\vec{r}(t) = (\cos t, t\sin t)$

$\vec{v}(t) = \vec{r}\,'(t) = (-\sin t, \sin t + t\cos t)$

$v(t) = (\sin^2 t + \sin^2 t + 2t\sin t\cos t + t^2\cos^2 t)^{1/2}$

$= (2\sin^2 t + t\sin 2t + t^2\cos^2 t)^{1/2}$

$\vec{a}(t) = \vec{v}\,'(t) = (-\cos t, 2\cos t - t\sin t)$

side 122

$\vec{a}(t) = a(t)\vec{T}(t) + v(t)\vec{T}\,'(t)$

$T(t) = \dfrac{\vec{v}(t)}{v(t)}$ (enhetstangent)

$T'(t) \perp T(t)$, $(T(t) \cdot T(t))' = (1)' = 0$

$$\underset{\displaystyle 2T(t)\cdot T'(t)}{\shortparallel}$$

$a(t) = \vec{a}(t) \cdot T(t)$

Her $T(t) = \dfrac{(-\sin t, \sin t + \cos t)}{\sqrt{2\sin^2 t + t\sin 2t + t^2\cos^2 t}}$

Regn ut $a(t) = \vec{a}(t) \cdot T(t)$

(og dere skal få fasitsvaret)

$r'(t) = v(t)T(t)$

$\vec{a}(t) = r''(t) = v'(t)T(t) + v(t)T'(t)$

Så alternativt kan vi regne ut

$a(t) = v'(t)$

3.1.7

$\vec{r}(t) = (a\cos t, b\sin t), t \in [0, 2\pi]$

$(a > 0, b > 0)$

a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{(a\cos t)^2}{a^2} + \frac{(b\sin t)^2}{b^2} =$

$= \frac{\cancel{a^2}\cos^2 t}{\cancel{a^2}} + \frac{\cancel{b^2}\sin^2 t}{\cancel{b^2}} = \cos^2 t + \sin^2 t = 1$

(Kurven blir en ellipse)

b) $\vec{v}(t) = \vec{r}\,'(t) =$

$= (-a\sin t,\ b\cos t)$

$v(t) = \sqrt{a^2\sin^2 t + b^2\cos^2 t}$

$\vec{a}(t) = \vec{v}\,'(t) = (-a\cos t, -b\sin t) = -\vec{r}(t)$.

c) $\ell(E) = \int_0^{2\pi} v(t)\,dt = \int_0^{2\pi} \sqrt{a^2\sin^2 t + b^2\cos^2 t}\,dt$

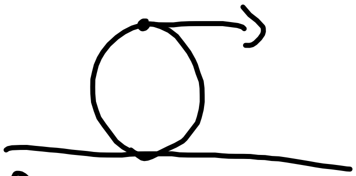
$a = 5,\ b = 3$

$\ell(E) = \int_0^{2\pi} \sqrt{25\sin^2 t + 9\cos^2 t}\,dt =$

$\overset{\text{Maple}}{=}$ 25,527

3.1.12

$$\vec{r}(t) = (rt - r\sin t, r - r\cos t)$$



Sykloiden (eksempel 4)

a)

$$\vec{v}(t) = \vec{r}\,'(t) = (r - r\cos t, r\sin t)$$

$$v(t) = \sqrt{(r - r\cos t)^2 + r^2\sin^2 t} =$$

$$= \sqrt{r^2 - 2r^2\cos t + \underbrace{r^2\cos^2 t + r^2\sin^2 t}_{r^2}}$$

$$= \sqrt{2r^2(1-\cos t)} = r\sqrt{2}\sqrt{1-\cos t}$$

$$\vec{a}(t) = (r\sin t, r\cos t)$$

b) Buelengde av sykloide

$$S = \int_0^{2\pi} v(t)\,dt = r\sqrt{2}\int_0^{2\pi} \sqrt{1-\cos t}\,dt$$

c) $$\sqrt{1-\cos t} = \sqrt{\frac{(1-\cos t)(1+\cos t)}{1+\cos t}} =$$

$$= \sqrt{\frac{1-\cos^2 t}{1+\cos t}} = \sqrt{\frac{\sin^2 t}{1+\cos t}} = \frac{|\sin t|}{\sqrt{1+\cos t}}$$

d) Vi får da

$$S = \sqrt{2}\,r \int_0^{2\pi} \frac{|\sin t|}{\sqrt{1+\cos t}}\,dt =$$

$$= \sqrt{2}\,r\left(\int_0^{\pi} \frac{\sin t}{\sqrt{1+\cos t}}\,dt - \int_{\pi}^{2\pi} \frac{\sin t}{\sqrt{1+\cos t}}\,dt\right)$$

$$\int \frac{\sin t}{\sqrt{1+\cos t}}\,dt = -\int \frac{du}{\sqrt{u}} = -2\sqrt{u} + C$$

$$u = 1+\cos t,\ du = -\sin t\,dt \quad = -2\sqrt{1+\cos t} + C$$

$$S = \sqrt{2}\,r\left(\Big|_0^{\pi}\left(-2\sqrt{1+\cos t}\right) - \Big|_{\pi}^{2\pi}\left(-2\sqrt{1+\cos t}\right)\right) =$$

$$= \sqrt{2}\,r\left(2\sqrt{2} + 2\sqrt{2}\right) = \underline{\underline{8r}}$$

<u>3.1.5</u> c)

$$x(t) = \sin(2t)\cos t$$
$$y(t) = \sin(2t)\sin t$$

Tegne denne med MATLAB