PARTIELLE DERIVERTE $\frac{\partial f}{\partial x_i}$

GRADIENTEN TIL EN FUNKSJON ∇f

DERIVERBARE FUNKSJONER

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SAMME FOR VEKTORVALUERTE FUNKSJONER

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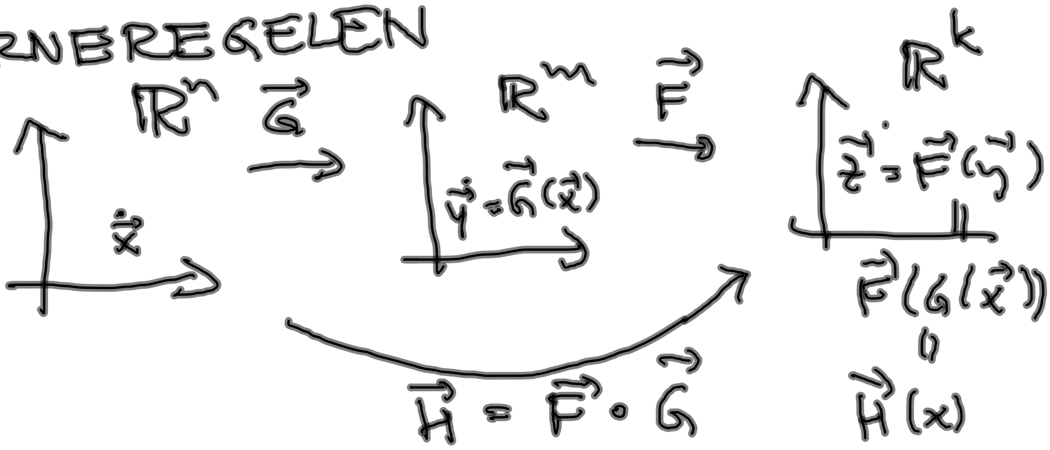
KJERNEREGELEN



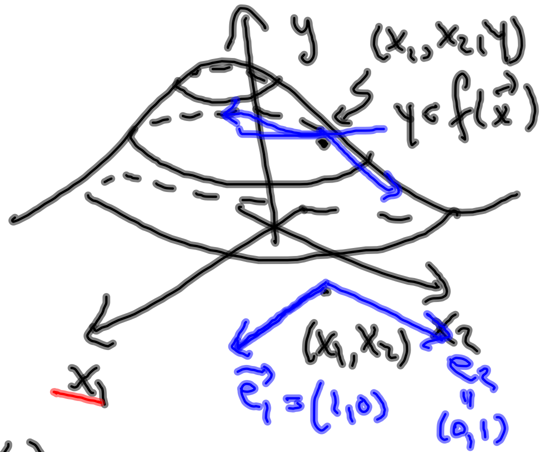
$$\vec{H}'(\vec{x}) = \vec{F}'(G(\vec{x})) \cdot \vec{G}'(\vec{x})$$

La $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ være en funksjon

$\vec{x} = (x_1, \dots, x_n) \longmapsto f(\vec{x}) = f(x_1, \dots, x_n)$

Hvis

$$\lim_{h \to 0} \frac{f(x_1 + h, x_2) - f(x_1, x_2)}{h}$$



eksisterer kalles grensen

$$\frac{\partial f}{\partial x_1}(\vec{x}) = \frac{\partial f}{\partial x_1}(x_1, x_2)$$

For $1 \leq j \leq n$ er den j'te partielle deriverte til f lik

$$\frac{\partial f}{\partial x_j}(\vec{x}) = \lim_{h \to 0} \frac{f(x_1, \dots, x_j + h, \dots, x_n) - f(x_1, \dots, x_n)}{h}$$

$$= \lim_{h \to 0} \frac{f(\vec{x} + h\vec{e}_j) - f(\vec{x})}{h}$$

GRADIENT

$$\nabla f(\vec{x}) = \left(\frac{\partial f}{\partial x_1}(\vec{x}), \dots, \frac{\partial f}{\partial x_n}(\vec{x})\right) \in \mathbb{R}^n$$

nabla

f vokser raskest i retningen $\nabla f(\vec{x})$

ANNET NAVN:

derivert

$$\nabla f(\vec{x}) = f'(\vec{x})$$

$n=1$: $\nabla f(x) = \left(\frac{\partial f}{\partial x}(x)\right) = \left(\frac{df}{dx}(x)\right) = (f'(x))$

Eksempel!

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$$

$$\vec{u} = (u_1, u_2) \longmapsto f(\vec{u}) = u_1^2 + u_2^2$$

$$\frac{\partial f}{\partial u_1}(\vec{u}) = 2u_1 \qquad \frac{\partial f}{\partial u_2}(\vec{u}) = 2u_2$$

$$\nabla f(\vec{u}) = (2u_1, 2u_2)$$

$$\nabla f(u_1, u_2) = 2(u_1, u_2)$$

VEKTORVALUERTE FUNKSJONER

$$\vec{F} : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$$(x_1, \dots, x_n) = \vec{x} \longmapsto \vec{y} = \vec{F}(\vec{x}) = (y_1, \dots, y_m)$$

$$y_i = F_i(\vec{x})$$

$$\vec{F}(\vec{x}) = (F_1(\vec{x}), \dots, F_m(\vec{x}))$$

<u>DEF</u>: Jacobi-matrisen til $\vec{F}$ i $\vec{x}$ er

$$\vec{F}'(\vec{x}) = \begin{bmatrix} \nabla F_1(\vec{x}) \\ \vdots \\ \nabla F_m(\vec{x}) \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1}(\vec{x}) & \cdots & \frac{\partial F_1}{\partial x_n}(\vec{x}) \\ \vdots & \ddots & \vdots \\ \frac{\partial F_m}{\partial x_1}(\vec{x}) & \cdots & \frac{\partial F_m}{\partial x_n}(\vec{x}) \end{bmatrix}$$

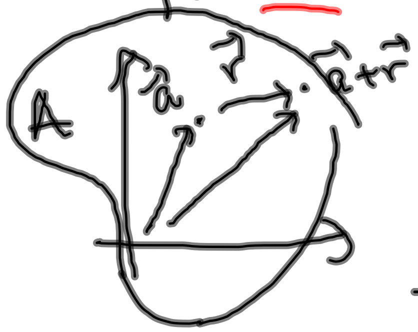
$m \times n$ matrise

Deriverbare funksjoner

La $A \subset \mathbb{R}^n$, $\vec{a} \in A$ et indre punkt, $f : A \longrightarrow \mathbb{R}$. Vi sier at f er **deriverbar** i $\vec{a}$ hvis

sigma

$$f(\vec{a}+\vec{r}) = f(\vec{a}) + \nabla f(\vec{a}) \cdot \vec{r} + \sigma(\vec{r})$$



der $\sigma(\vec{r}) \longrightarrow 0$ raskere enn $\vec{r}$, dvs.

$$\lim_{\vec{r} \longrightarrow \vec{0}} \frac{|\sigma(\vec{r})|}{|\vec{r}|} = 0.$$

La $A \subset \mathbb{R}^n$, $\vec{a} \in A$ et indre punkt

$\vec{F} : A \to \mathbb{R}^m$

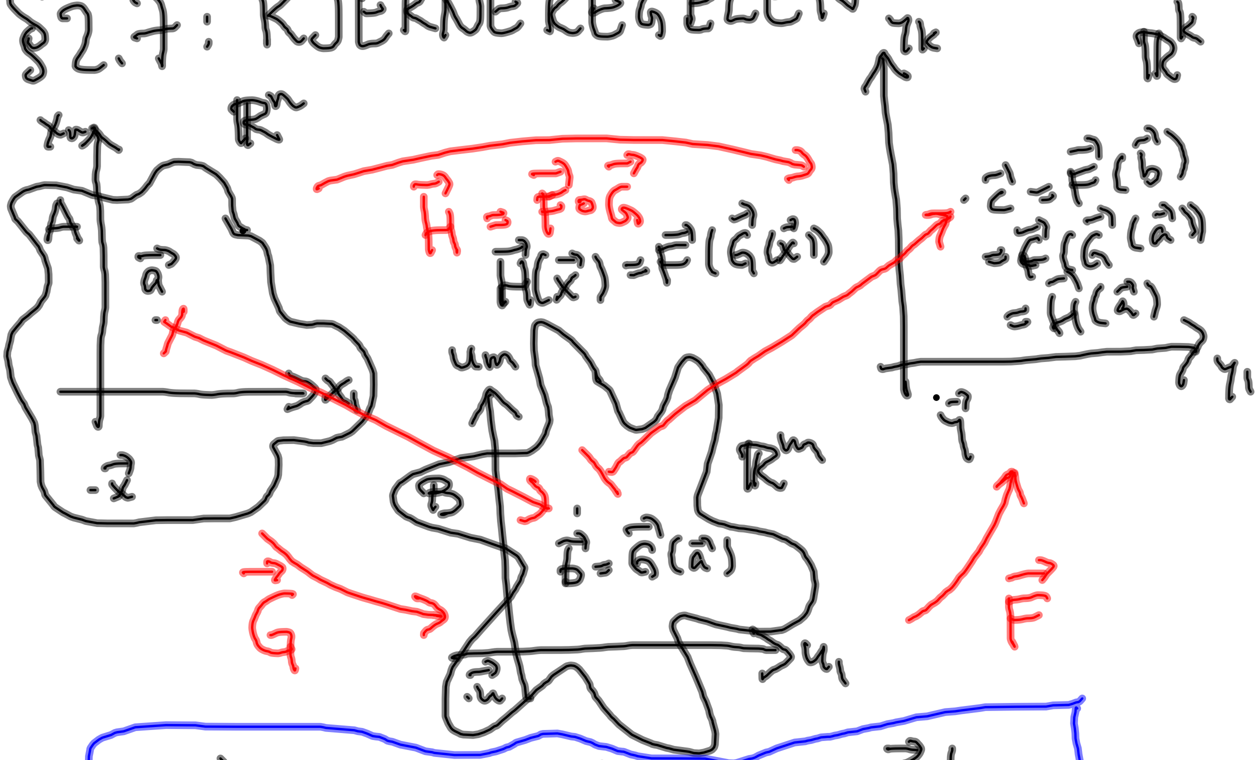
DEF: $\vec{F}$ ER DERIVERBAR I $\vec{a}$ HVIS

$$\vec{F}(\vec{a}+\vec{r}) = \vec{F}(\vec{a}) + \vec{F}'(\vec{a})\,\vec{r} + \vec{\sigma}(\vec{r})$$

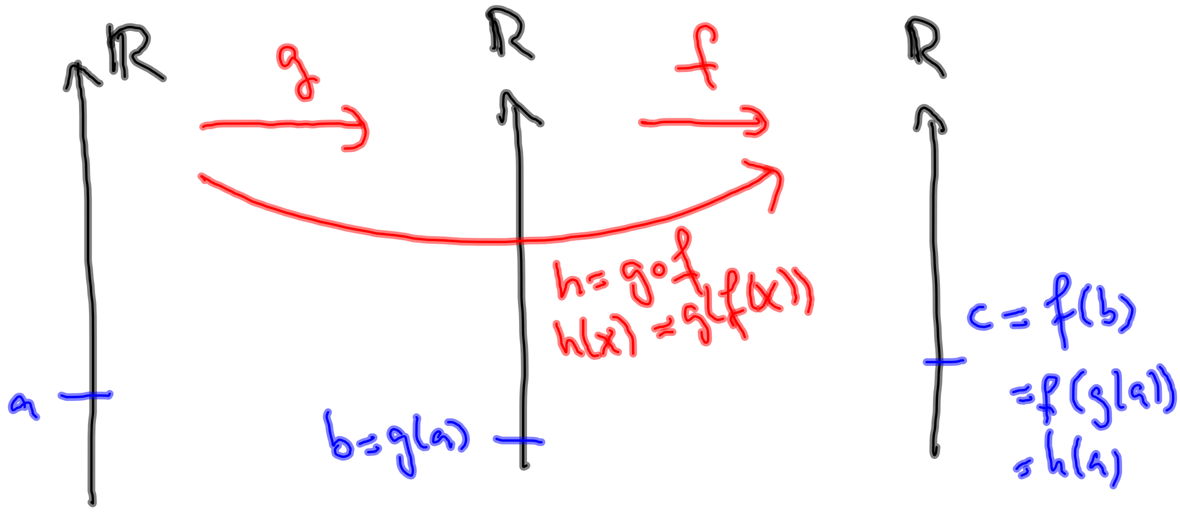
DER $\vec{\sigma}(\vec{r}) \to \vec{0}$ RASKERE ENN $\vec{r} \to \vec{0}$ (NÅR $\vec{r} \to \vec{0}$)

DVS.
$$\lim_{\vec{r}\to\vec{0}} \frac{|\vec{\sigma}(\vec{r})|}{|\vec{r}|} = 0.$$

§2.7: KJERNEREGELEN



$$\boxed{\vec{H}'(\vec{a}) = \vec{F}'(\vec{G}(\vec{a})) \cdot \vec{G}'(\vec{a})}$$



$$\left.\begin{array}{l} g \text{ deriverbar i } a \\ f \text{ deriverbar i } b \end{array}\right\} \Longrightarrow h \text{ er deriverbar i } a$$

$$h'(a) = g'(f(a)) \cdot f'(a)$$

Eks

$$\begin{tikzcd} \mathbb{R}^2 \arrow[rr, "h = f\circ\vec{G}"] \arrow[dr, "\vec{G}"'] & & \mathbb{R} \\ & \mathbb{R}^2 \arrow[ur, "f"'] & \end{tikzcd}$$

$f(u_1,u_2) = u_1^2 + u_2^2$

$\vec{x} = (x_1,x_2) \longmapsto \vec{u} = \vec{G}(\vec{x})$

$= (G_1(\vec{x}), G_2(\vec{x}))$

$G_1(x_1,x_2) = x_1^2 - x_2^2$

$G_2(x_1,x_2) = 2x_1x_2$

$$\vec{G}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1^2 - x_2^2 \\ 2x_1x_2 \end{pmatrix}$$

$$\begin{aligned} h(x_1,x_2) &= f(\vec{G}(x_1,x_2)) \\ &= (x_1^2 - x_2^2)^2 + (2x_1x_2)^2 \\ &= x_1^4 - 2x_1^2x_2^2 + x_2^4 + 4x_1^2x_2^2 \\ &= x_1^4 + 2x_1^2x_2^2 + x_2^4 \\ &= (x_1^2 + x_2^2)^2 \end{aligned}$$

Vil sjekke at

$$h'(\vec{x}) = f'(\vec{u}) \cdot \vec{G}'(\vec{x})$$

$$f(u_1,u_2) = u_1^2 + u_2^2$$

$$f'(\vec{u}) = \nabla f(\vec{u}) = (2u_1, 2u_2)$$

$$\vec{G}(x_1,x_2) = (x_1^2 - x_2^2,\ 2x_1x_2) = \begin{bmatrix} x_1^2 - x_2^2 \\ 2x_1x_2 \end{bmatrix}$$

$$\vec{G}'(\vec{x}) = \begin{bmatrix} \nabla G_1(\vec{x}) \\ \nabla G_2(\vec{x}) \end{bmatrix} = \begin{bmatrix} \frac{\partial G_1}{\partial x_1}(\vec{x}) & \frac{\partial G_1}{\partial x_2}(\vec{x}) \\ \frac{\partial G_2}{\partial x_1}(\vec{x}) & \frac{\partial G_2}{\partial x_2}(\vec{x}) \end{bmatrix}$$

$$= \begin{bmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{bmatrix}$$

$$f'(\vec{u}) \cdot \vec{G}'(\vec{x}) = \vec{f}'(\vec{G}(\vec{x})) \cdot \vec{G}'(\vec{x})$$

$$= \begin{bmatrix} 2u_1 & 2u_2 \end{bmatrix} \begin{bmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{bmatrix}$$

$u_1 = G_1(\vec{x}) = x_1^2 - x_2^2$ $\qquad$ $u_2 = G_2(\vec{x}) = 2x_1x_2$

$$= \begin{bmatrix} 2(x_1^2 - x_2^2) & 2 \cdot 2x_1x_2 \end{bmatrix} \begin{bmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{bmatrix}$$

$$= \begin{bmatrix} 2(x_1^2 - x_2^2) \cdot 2x_1 + 2 \cdot 2x_1x_2 \cdot 2x_2 & 2(x_1^2 - x_2^2)(-2x_2) + 2 \cdot 2x_1x_2 \cdot 2x_1 \end{bmatrix}$$

$$= \begin{bmatrix} 4x_1^3 - 4x_1x_2^2 + 8x_1x_2^2 & -4x_1^2x_2 + 4x_2^3 + 8x_1^2x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 4x_1^3 + 4x_1x_2^2 & 4x_1^2x_2 + 4x_2^3 \end{bmatrix}$$

Sammenliken med

$$h(x_1, x_2) = x_1^4 + 2x_1^2 x_2^2 + x_2^4$$

der

$$\nabla h(\vec{x}) = h'(\vec{x})$$

$$= \underline{\underline{\left[4x_1^3 + 4x_1 x_2^2 \;\vdots\; 4x_1^2 x_2 + 4x_2^3 \right]}}$$