Plenum 5/4-16

4.1: 1, 6 4.3: 5

4.2: 2c, 10 4.4: 4, 5

4.1:

1) (I) $x + 2y - z = 3$

(II) $2x + 3y - 3z = -1$

(III) $-x + 2y + 3z = 1$

Gang (III) m/ 2 og legg til (II):

$$x + 2y - z = 3$$
$$7y + 3z = 1$$
$$-x + 2y + 3z = 1$$

(I) og legg til (III):

$$x + 2y - z = 3$$
$$7y + 3z = 1 \qquad \left(y + \frac{3}{7}z = \frac{1}{7}\right)$$
$$4y + 2z = 4$$

-4(II) + (III):

$$x + 2y - z = 3$$
$$y + \frac{3}{7}z = \frac{1}{7}$$
$$\frac{2}{\cancel{7}}z = \frac{24}{\cancel{7}}$$
$$\downarrow$$
$$z = 12$$

$$2z - \frac{12}{7}z = \frac{14-12}{7}z = \frac{2}{7}z$$

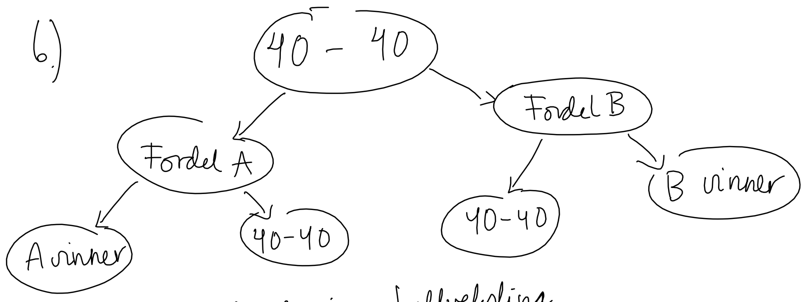
$$4 - \frac{4}{7} = \frac{28-4}{7} = \frac{24}{7}$$

$$y + \frac{3}{7}12 = \frac{1}{7}$$
$$y = \frac{1}{7} - \frac{3 \cdot 12}{7} = \frac{-35}{7} = -5$$

$$x + 2(-5) - 12 = 3$$
$$x = 25$$

Svar: $(25, -5, 12)$

6)



A; 60% for å vinne ballveksling

x = spiller As sanns. for å vinne game fra 40-40

y = —— " —————————— fordel A

z = —— " —————————— fordel B

a) $\underbrace{x}_{\text{sanns. vinne fra 40-40}} = \underbrace{0{,}6}_{\text{sanns. A vinner neste ball}}\ \underbrace{y}_{\text{sann. vinne fra fordel A}} + \underbrace{0{,}4}_{\text{sanns. for at blir fordel B fra 40-40}}\ z$ sanns. A vinner når fordel B

sanns. for at blir fordel A og A vinner fra fordel A → Tilsvarende.

$\underbrace{y}_{\text{sanns. vinner fra fordel A}} = \underbrace{0{,}4}_{\text{sanns. B vinner neste ball → tilbake til 40-40}}\ \underbrace{x}_{\text{sanns. A vinner fra 40-40}} + \underbrace{0{,}6}_{\text{sanns. A vinner neste ball og dermed gamet}}$

sanns. går fra fordel A til 40-40 og deretter vinner A

$\underbrace{z}_{\text{sanns. A vinner fra fordel B}} = \underbrace{0{,}6}_{\text{sanns. A vinner neste ball}}\ \underbrace{x}_{\text{sanns. A vinner fra 40-40}}$

sanns. tilbake til 40-40 og deretter vinner A

b)
$$\begin{cases} x = 0{,}6y + 0{,}4z \\ y = 0{,}4x + 0{,}6 \\ z = 0{,}6x \end{cases} \iff \begin{cases} 0{,}76x = 0{,}6y \\ y = 0{,}4x + 0{,}6 \\ z = 0{,}6x \end{cases}$$

$$\iff \begin{cases} 0{,}52x = 0{,}36 \\ y = 0{,}4x + 0{,}6 \\ z = 0{,}6x \end{cases} \Rightarrow x = \frac{36}{52} = \underline{\frac{9}{13}}$$

$$y = \frac{4}{10}\,\frac{9}{13} + \frac{6}{10} = \dots = \underline{\frac{57}{65}}$$

$$z = \frac{6}{10}\,\frac{9}{13} = \dots = \frac{27}{65}$$

Svar: $\left(\frac{9}{13}, \frac{57}{65}, \frac{27}{65}\right)$

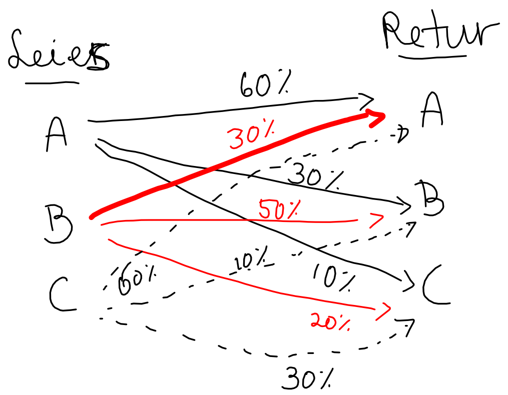
4.2:

2) c)
$$\begin{bmatrix} 1 & 1 & -2 & 3 \\ 2 & 1 & 3 & 0 \\ -1 & 0 & -5 & 2 \end{bmatrix} \overset{\substack{-2\mathrm{I}+\mathrm{II} \\ \mathrm{I}+\mathrm{III}}}{\sim} \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -1 & 7 & -6 \\ 0 & 1 & -7 & 5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & -7 & 5 \\ 0 & -1 & 7 & -6 \end{bmatrix} \overset{\mathrm{II}+\mathrm{III}}{\sim} \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & -7 & 5 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & -7 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

10.)



120 bil

$a, b, c = \#$ biler i A, B og C

$$\begin{cases} a + b + c = 120 \\ a = 0{,}6a + 0{,}3b + 0{,}6c \\ b = 0{,}3a + 0{,}5b + 0{,}1c \\ c = 0{,}1a + 0{,}2b + 0{,}3c \end{cases}$$

$$\begin{cases} a + b + c = 120 \\ -0{,}4a + 0{,}3b + 0{,}6c = 0 \\ 0{,}3a - 0{,}5b + 0{,}1c = 0 \\ 0{,}1a + 0{,}2b - 0{,}7c = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 120 \\ -0{,}4 & 0{,}3 & 0{,}6 & 0 \\ 0{,}3 & -0{,}5 & 0{,}1 & 0 \\ 0{,}1 & 0{,}2 & -0{,}7 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ -4 & 3 & 6 & 0 \\ 3 & -5 & 1 & 0 \\ 1 & 2 & -7 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ 0 & 7 & 10 & 480 \\ 0 & -8 & -2 & -360 \\ 0 & 1 & -8 & -120 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & -8 & -2 & -360 \\ 0 & 7 & 10 & 480 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & 0 & -66 & -1320 \\ 0 & 0 & 66 & 1320 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & 0 & -66 & -1320 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$c = \underline{\underline{20}}$

$b - 8c = -120 \Rightarrow b = 160 - 120 = \underline{\underline{40}}$

$a + b + c = 120$

$a = 120 - 40 - 20 = \underline{\underline{60}}$

4.3:

5) $2x - y + z = b_1$
$-x + 3y + 2z = b_2$
$3x - 4y - z = b_3$

$$\text{rref}\left(\begin{bmatrix} 2 & -1 & 1 \\ -1 & 3 & 2 \\ 3 & -4 & -1 \end{bmatrix}\right) \underset{\text{MATLAB}}{=} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$; ikke identitetsmatrisen.

Så ligningssystemet har ikke entydige løsninger for alle b_1, b_2, b_3.

4.4: $A\vec{x}=\vec{b}$

4.) $$\underbrace{\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & -1 & 1 \\ 6 & 0 & -6 & 7 \\ 2 & 1 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{matrix} 0 \\ 1 \\ h \\ 0 \end{matrix}}_{\vec{b}} \sim \begin{bmatrix} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 6 & 0 & -6 & 7 & h \\ 2 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & h-6 \\ 0 & 1 & 2 & -2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & h-6 \\ 0 & 0 & 0 & -2 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & h-6 \\ 0 & 0 & 0 & 0 & 2h-14 \end{bmatrix}$$

A på trappeform $\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Hvis $h \neq 7$ (dvs. $2h-14 \neq 0$); Siste ligning sier $0 =$ noe som ikke er 0;

INGEN LØSNING!

Hvis $h = 7$: ∞ mange løsninger (søyle 3 er ikke pivotsøyle, så x_3 er en fri variabel).

$$x_4 = h-6 = 7-6 = 1$$

$$x_2 = 0 - 2x_3 = -2x_3$$

$$x_1 = 1 - x_4 + x_3 = 1 - 1 + x_3 = x_3 \text{ og } x_3 \text{ fri.}$$

Dvs: $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

5) $A\vec{x}=\vec{b}$:

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & a^2-a \\ -1 & 1 & -3 \end{bmatrix}}_{A}\ \underbrace{\begin{bmatrix} 1 \\ 3 \\ a \end{bmatrix}}_{\vec{b}} \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 1 & -2 & a+1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 0 & -a^2+a & a \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 0 & -a(a-1) & a \end{bmatrix}$$

3 tilfeller: $a \notin \{0,1\}$: $\left(\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 0 & 1 & -\frac{1}{a-1} \end{bmatrix}\right)$

$a=0$: $\left(\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right)$

$a=1$: $\left(\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)$

Hvis $a \notin \{0,1\}$: Entydig løsning (alle søyler, utenom siste er pivot).

Hvis $a=0$: ∞ mange løsninger (søyle 3 er ikke pivot $\Rightarrow x_3$ fri)

Hvis $a=1$: Siste ligning; $0=1 \Rightarrow$ INGEN LØSNINGER.