4.5.

2a)

$$\left(\begin{array}{ccc}\multicolumn{3}{c}{\overbrace{\qquad\qquad}^{A}}\\ 1 & 1 & 2\\ -1 & 1 & 1\\ 2 & 3 & 3\end{array}\right.\left|\begin{array}{ccc}\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{array}\right) \quad -2$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0\\ 0 & 2 & 3 & 1 & 1 & 0\\ 0 & 1 & -1 & -2 & 0 & 1\end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0\\ 0 & 1 & -1 & -2 & 0 & 1\\ 0 & 2 & 3 & 1 & 1 & 0\end{array}\right) -2$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0\\ 0 & 1 & -1 & -2 & 0 & 1\\ 0 & 0 & 5 & 5 & 1 & -2\end{array}\right) \frac{1}{5}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0\\ 0 & 1 & -1 & -2 & 0 & 1\\ 0 & 0 & 1 & 1 & \frac{1}{5} & \frac{-2}{5}\end{array}\right) -2$$

$$\sim \left(\begin{array}{cccccc} 1 & 1 & 0 & -1 & -\frac{2}{5} & \frac{4}{5}\\ 0 & 1 & 0 & -1 & \frac{1}{5} & 3/5\\ 0 & 0 & 1 & 1 & 1/5 & -2/5\end{array}\right) -1$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3/5 & 1/5\\ 0 & 1 & 0 & -1 & 1/5 & 3/5\\ 0 & 0 & 1 & 1 & 1/5 & -2/5\end{array}\right) \quad A^{-1} = \begin{pmatrix} 0 & -3/5 & 1/5\\ -1 & 1/5 & 3/5\\ 1 & 1/5 & -2/5\end{pmatrix}$$

4.5.2 b)

$$\underbrace{\left(\begin{array}{ccc:ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 0 & 0 & 1 & 0 \\ 4 & 16 & -6 & 0 & 0 & 1 \end{array}\right)}_{B} \begin{array}{l} -2\ -4 \\ \\ \\ \end{array}$$

$$\sim \left(\begin{array}{ccc:ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 8 & -6 & -2 & 1 & 0 \\ 0 & 24 & -18 & -4 & 0 & 1 \end{array}\right) \begin{array}{l} \\ -3 \\ \\ \end{array}$$

$$\sim \left(\begin{array}{ccc:ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 8 & -6 & -2 & 1 & 0 \\ 0 & 0 & 0 & 2 & -3 & 1 \end{array}\right)$$

Vi ser av dette (siden vi får rad med 0-er) at B ikke er radekvivalent med I_3 dvs B er ikke inverterbar.

4.5.6

a) Skal invertere B

$$\left(\begin{array}{ccc:ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right)\begin{array}{c} \\ 2 \\ \\ \end{array}$$

$$\underbrace{\begin{matrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 1 \end{matrix}}_{= B}$$

$$\left(\begin{array}{ccc:ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 2 & 1 \end{array}\right)\begin{array}{c} \\ \\ \frac{1}{3} \end{array} \sim \left(\begin{array}{ccc:ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2/3 & 1/3 \end{array}\right)\begin{array}{c} \\ \\ -1 \end{array}$$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/3 & -1/3 \\ 0 & 0 & 1 & 0 & 2/3 & 1/3 \end{array}\right)\begin{array}{c} \\ -2 \\ \\ \end{array} \sim \left(\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -2/3 & 2/3 \\ 0 & 1 & 0 & 0 & 1/3 & -1/3 \\ 0 & 0 & 1 & 0 & 2/3 & 1/3 \end{array}\right)$$

$$B^{-1} = \begin{pmatrix} 1 & -2/3 & 2/3 \\ 0 & 1/3 & -1/3 \\ 0 & 2/3 & 1/3 \end{pmatrix}$$

4.5.6 b)

Skal løse
$$\begin{aligned} x + 2y &= 5 \\ y + z &= 3 \\ -2y + z &= 3 \end{aligned} \qquad \underbrace{\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 1 \end{pmatrix}}_{= B \text{ (fra a))}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = B^{-1} \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{2}{3} & \frac{2}{3} \\ 0 & \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix}}}$$

c)
$$\begin{aligned} x + 2y \quad &= 5 \\ y + z \quad &= 3 \\ -2y + (a+1)z &= b^2 - 10 \end{aligned}$$

Skal drøfte dette systemet

$$\begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & -2 & 1+a & b^2-10 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 3+a & b^2-4 \end{pmatrix}$$

Ser at koeffisientmatrisen er invertibel når $a \neq -3$. Får da én løsning.

Når $a = -3$ og $b = 2$ eller $b = -2$ blir siste likning $0 = 0$ og vi får ∞-mange løsninger. Når $a = -3$ og $b \neq 2$ og $b \neq -2$ blir siste likning $0 = b^2 - 4 \neq 0$. Så får ingen løsninger.

4.6

2 Gitt $\vec{a}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \vec{a}_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \vec{a}_3 = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

$\vec{b} = \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$. Skal skrive $\vec{b}$ som lin. komb. av $\vec{a}_i$'ene.

$$x\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + y\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + z\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 & -2 \\ 0 & 2 & -1 & 5 \\ -1 & 1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 3 & -2 \\ 0 & 2 & -1 & 5 \\ 0 & 2 & 5 & -1 \end{bmatrix} \begin{matrix} \\ -1 \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 1 & 3 & -2 \\ 0 & 2 & -1 & 5 \\ 0 & 0 & 6 & -6 \end{bmatrix} \begin{matrix} \\ \\ \frac{1}{6} \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 3 & -2 \\ 0 & 2 & -1 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$x + y + 3z = -2$

$2y - z = 5$

$z = -1$

$y = \frac{5+z}{2} = 2$

$x = -2 - y - 3z$

$= -2 - 2 + 3 = -1$

$(-1)\vec{a}_1 + 2\vec{a}_2 + (-1)\vec{a}_3 = \vec{b}$

4.6 Kan enhver vektor i $\mathbb{R}^n$ skrives som en lin. komb. av $\vec{a}_1, \dots, \vec{a}_n$?

a) $\vec{a}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, $\vec{a}_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$$\begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix} \tfrac{1}{2} \sim \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix} \sim I_2$$

svar ja!

b) $\vec{a}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{a}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $\vec{a}_3 = \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 4 \\ 3 & -1 & 7 \end{bmatrix} \begin{matrix} -2 \;\; -3 \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & -4 & 4 \end{bmatrix} \begin{matrix} \\ (-2) \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$ ikke radekvivalent med I_3

(siden vi har en rad med 0'er)

Svar: Nei!

4.6.41

$$\begin{pmatrix} 7 \\ 4 \\ -3 \\ 1 \end{pmatrix} = \vec{b} \text{ som en lin. komb. av.}$$

$$\vec{a}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \quad \vec{a}_2 = \begin{pmatrix} 2 \\ 1 \\ 3 \\ -4 \end{pmatrix}, \quad \vec{a}_3 = \begin{pmatrix} 5 \\ -7 \\ 6 \\ 3 \end{pmatrix} \quad a_4 = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 3 \end{pmatrix}$$

Definer $A = \begin{pmatrix} 1 & 2 & 5 & 2 \\ 0 & 1 & -7 & -1 \\ -1 & 3 & 6 & 0 \\ 2 & -4 & 3 & 3 \end{pmatrix}$ $\vec{b}$ som over

Bruk Matlab. Definer A og $\vec{b}$

$X = A \backslash b$ Matlab svarer da med

$$X = \begin{pmatrix} 11.1111 \\ 1.8889 \\ 0.4074 \\ -4.9630 \end{pmatrix}$$

For hånd

$[A : \vec{b}]$ radreduseres $\sim \left[I_4 \;\vdots\; \begin{matrix} 100/9 \\ 17/9 \\ 11/27 \\ -134/27 \end{matrix} \right]$

(4x5matrise)

4.6.6

Kan enhver vektor i $\mathbb{R}^4$ skrives som en lineær komb. av de 5 oppgitte vektorene?

Bruk Matlab.

$$\begin{bmatrix} 1 & 0 & 2 & -1 & 0 \\ -2 & -3 & -1 & 3 & 2 \\ 3 & 4 & 10 & -1 & 1 \\ 2 & 1 & 5 & 2 & 0 \end{bmatrix} = A$$

Bruker Matlab

$$\mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Hver linje har pivotelement

Svaret er ja!

4.8

a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

b) $\begin{array}{c} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{array}\begin{array}{c} \\ (-3) \end{array} \sim \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{array}{c} \\ \frac{1}{2} \\ \\ \end{array} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{2} & 1 \end{pmatrix}$

e) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{array}{c} 4 \\ \\ \\ \end{array} \sim \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2)

$$\begin{pmatrix}1&2\\-1&1\end{pmatrix}\begin{matrix}1\\ \hookleftarrow\end{matrix} \sim \begin{pmatrix}1&2\\0&3\end{pmatrix}\begin{matrix}\\ \frac{1}{3}\end{matrix} \sim \begin{pmatrix}1&2\\0&1\end{pmatrix}\begin{matrix}\hookleftarrow\\ -2\end{matrix} \sim \begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$\begin{pmatrix}1&0\\0&1\end{pmatrix} = \begin{pmatrix}1&-2\\0&1\end{pmatrix}\begin{pmatrix}1&0\\0&\frac{1}{3}\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}1&2\\-1&1\end{pmatrix}$$

$$\begin{pmatrix}1&2\\-1&1\end{pmatrix} = \begin{pmatrix}1&0\\1&1\end{pmatrix}^{-1}\begin{pmatrix}1&0\\0&\frac{1}{3}\end{pmatrix}^{-1}\begin{pmatrix}1&-2\\0&1\end{pmatrix}^{-1}\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$= \begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}1&0\\0&3\end{pmatrix}\begin{pmatrix}1&2\\0&1\end{pmatrix}$$

produkt av elementære matriser.

3) Skal skrive som produkt av elementære

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & -2 & 3 \\ 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{matrix} \\ \\ \frac{1}{4} \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} \\ \\ -1 \end{matrix} \sim \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} \\ -2 \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ -1 & -2 & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & -2 & 3 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} \end{pmatrix}^{-1}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

produkt av elementære matriser.