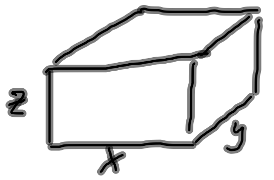
5.10.4



overflate: $f(x,y,z) = xy + 2xz + 2yz$

bibetingelse:
volum: $g(x,y,z) = xyz = V$

$$\nabla f = \begin{pmatrix} y+2z \\ x+2z \\ 2x+2y \end{pmatrix} \qquad \nabla g = \begin{pmatrix} yz \\ xz \\ xy \end{pmatrix}$$

$\nabla g = 0$: da må noen av x,y,z være 0, som er umulig hvis volum skal være 0

$$\nabla f = \lambda \nabla g : \quad \begin{pmatrix} y+2z \\ x+2z \\ 2x+2y \end{pmatrix} = \lambda \begin{pmatrix} yz \\ xz \\ xy \end{pmatrix} \begin{array}{l} \cdot x \\ \cdot y \\ \cdot z \end{array}$$

$$\Rightarrow \begin{pmatrix} xy+2xz \\ xy+2yz \\ 2xz+2yz \end{pmatrix} = \lambda \begin{pmatrix} xyz \\ xyz \\ xyz \end{pmatrix}$$

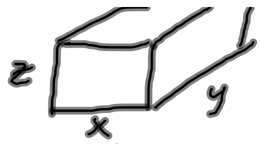
Sammenlign 1. og 2. $\underline{xy} + \underline{2}x\underline{z} = \underline{xy} + \underline{2}y\underline{z} \Rightarrow x = y$

2. og 3. $xy + \underline{2yz} = 2xz + \underline{2yz} \Rightarrow y = 2z$

$\Rightarrow x = y = 2z$

$g(x,y,z) = V \Rightarrow xyz = V \Rightarrow 4z^3 = V \Rightarrow \underline{\underline{z = \left(\frac{V}{4}\right)^{\frac{1}{3}}}}.$ $\quad x = y = 2z = 2\left(\frac{V}{4}\right)^{\frac{1}{3}} = \underline{\underline{(2V)^{\frac{1}{3}}}}$

5.10.6 $(\rightarrow 5.9.13)$



$$A(x,y,z) = xy + 2xz + 2yz$$

$$g(x,y,z) = 4x + 4y + 4z = 56 \qquad \nabla g = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} \neq 0$$

$$\nabla A = \lambda \nabla g : \quad \begin{pmatrix} y + 2z \\ x + 2z \\ 2x + 2y \end{pmatrix} = \lambda \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$$

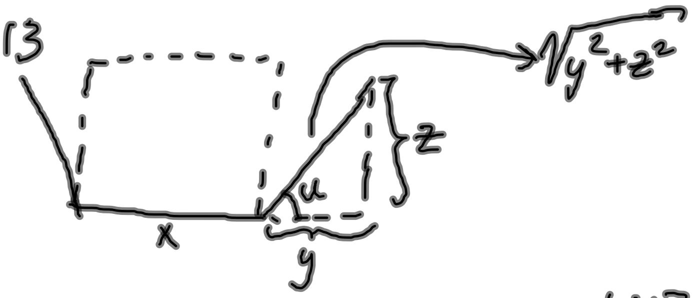
1. og 2. : $\quad y + 2z = x + 2z \Rightarrow x = y$

2. og 3. : $\quad x + 2z = 2x + 2y = 4x \Leftrightarrow 2z = 3x \Leftrightarrow z = \frac{3}{2}x$

$$4x + 4y + 4z = 56 \Leftrightarrow 4x + 4x + 4\tfrac{3}{2}x = 56 \Leftrightarrow 14x = 56 \Leftrightarrow \underline{\underline{x = 4}}$$

$\underline{\underline{y = 4}} \qquad z = \frac{3}{2} \cdot 4 = \underline{\underline{6}}$

5.10.13



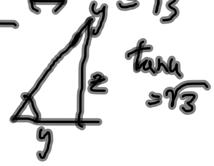
$$A(x,y,z) = xz + 2\cdot\frac{1}{2}yz = xz + yz$$

bibetingelse: $g(x,y,z) = x + 2\sqrt{y^2+z^2} = b$

$$\nabla A = \lambda \nabla g: \quad \begin{pmatrix} z \\ z \\ x+y \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \frac{2y}{\sqrt{y^2+z^2}} \\ \frac{2z}{\sqrt{y^2+z^2}} \end{pmatrix} \to \neq 0$$

1. og 2.: $1 = \frac{2y}{\sqrt{y^2+z^2}} \Leftrightarrow y^2+z^2 = 4y^2 \Leftrightarrow \underline{3y^2 = z^2} \Leftrightarrow \frac{z}{y} = \sqrt{3}$

$\Rightarrow u = \frac{\pi}{3}$



likning 1: $\lambda = z$

likning 3: $x+y = \frac{2z^2}{\sqrt{y^2+z^2}} = \frac{6y^2}{\sqrt{4y^2}} = \frac{6y^2}{2y} = 3y \Leftrightarrow x = 2y$

lengden på siderenna $= \sqrt{y^2+z^2} = 2y$

$\Rightarrow$ alle sidene i renna er like lange, så de må være $\frac{b}{3}$ alle sammen.

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2. $f(x,y) = 2x + 4y$

$g(x,y) = x^2 + y^2 - 4 = 0$

max/min på $S = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 4\}$

S er lukket og begrenset, siden f er kont. så har den både max. og min. på S.

$\nabla f = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ $\nabla g = \begin{pmatrix} 2x \\ 2y \end{pmatrix}$

$\nabla g = 0 \Leftrightarrow x = y = 0$, som ikke passer inn: $g(x,y) = 0$

$\nabla f = \lambda \nabla g$: $\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \lambda \begin{pmatrix} 2x \\ 2y \end{pmatrix} \Rightarrow \begin{matrix} 2 = 2\lambda x \\ 4 = 2\lambda y \end{matrix}$ $\lambda, x, y \neq 0$

del 2. på 1. : $\frac{2\lambda y}{2\lambda x} = 2 \Rightarrow \frac{y}{x} = 2 \Leftrightarrow y = 2x$

bibetingelse: $x^2 + y^2 = 4$: $x^2 + 4x^2 = 4 \Leftrightarrow x = \pm \frac{2}{\sqrt{5}}$

kandidater: $\left(\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$, $\left(-\frac{2}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right)$

$2x + 4y = \frac{4}{\sqrt{5}} + \frac{16}{\sqrt{5}} = \frac{20}{\sqrt{5}} = 4\sqrt{5}$

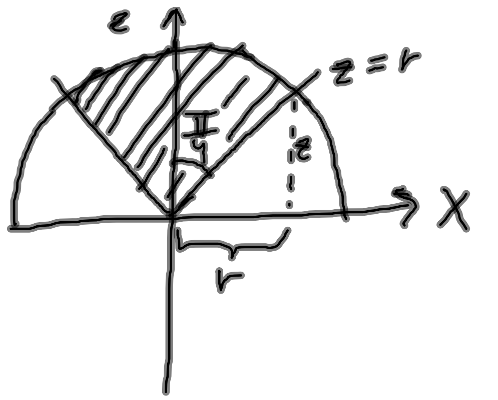
maksimum ($f > 0$) minimum ($f < 0$)

V 2008

3.

$$\begin{pmatrix} 2 & 1 & 3000 \\ 1 & 1 & 1000 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1000 \\ 2 & 1 & 3000 \end{pmatrix} \overset{II-2I}{\sim} \begin{pmatrix} 1 & 1 & 1000 \\ 0 & -1 & 1000 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2000 \\ 0 & 1 & -1000 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3000 \\ 1000 \end{pmatrix} = 2000\,\vec{v}_1 - 1000\,\vec{v}_2$$

5. 

$0 \le \rho \le 2$

$0 \le \phi \le \frac{\pi}{4}$

$0 \le \theta \le 2\pi$

$$x^2 + y^2 = \rho^2 \cos^2\theta \sin^2\phi + \rho^2 \sin^2\theta \sin^2\phi$$
$$= \rho^2 \sin^2\phi$$

6. $$A = \iint \Big(\underbrace{\frac{\partial Q}{\partial x}}_{Q\, = x} - \underbrace{\frac{\partial P}{\partial y}}_{P = 0} \Big)\, dx\, dy$$