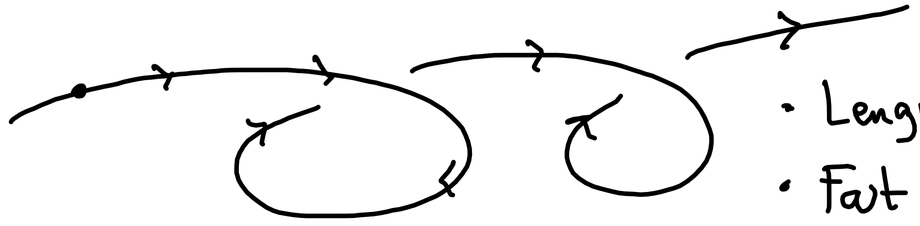
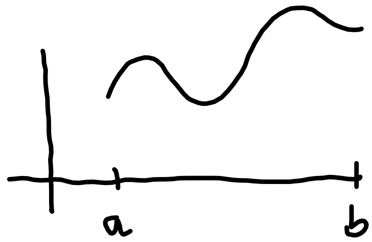
Parametriserte kurver



- Lengde
- Fart
- Hastighet
- Akselerasjon.

Eks 1.

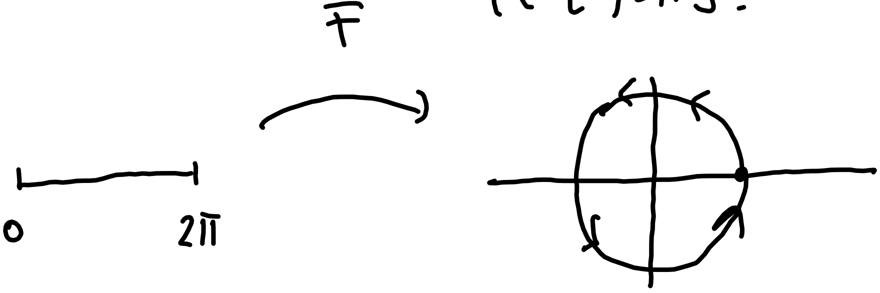


$y = f(x)$,

Eksempel på en parametrisert kurve over $I = [a,b]$.

Eks 2: Definere $F(t) = (\cos(t), \sin(t))$,

$t \in [0, 2\pi]$.



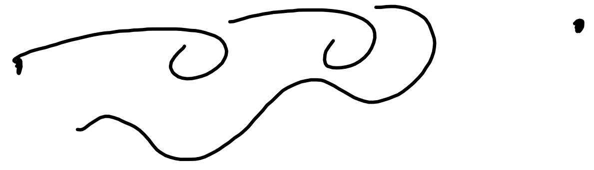
DEF 3.3.1: En parametrisert kurve e en kontinuerlig avbildning $\vec{r} : I \to \mathbb{R}^n$ (I e et intervall),

$r(t) = (x_1(t), x_2(t), \dots, x_n(t))$.

Eks: En bedrift fører n forskjellige varer. Lagerbeholdning

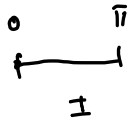
$$r(t) = (x_1(t), x_2(t), \dots, x_n(t)).$$

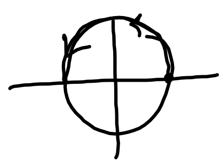
Konstant siden r e kont.



$(\cos(t), \sin(t))$

$t \in [0, \pi]$.

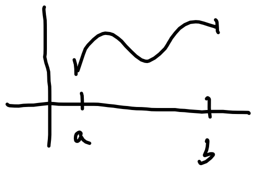




$y = \sqrt{1-x^2}$

$t \mapsto (t, \sqrt{1-t^2})$

Lengde



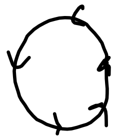
$y = g(x)$.

$$l = \int_a^b \sqrt{1 + g'(x)^2}\, dx.$$

DEF: Anta at $\vec{r}(t) = (x_1(t), \dots, x_n(t))$ er en parametrisert kurve, og anta at $x_1(t), \dots, x_n(t)$ er deriverbare. Da definerer vi lengden til kurven

$$L(a,b) = \int_a^b \sqrt{x_1'(t)^2 + x_2'(t)^2 + \dots + x_n'(t)^2}\, dt.$$

Eks: La $\vec{r}(t) = (\cos(t), \sin(t))$ $t \in [0, 2\pi]$



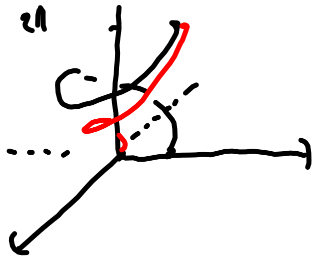
$$L(0, 2\pi) = \int_0^{2\pi} \sqrt{(-\sin(t))^2 + (\cos(t))^2}\, dt$$

$$= \int_0^{2\pi} dt = 2\pi.$$

Eks: La $\vec{r}(t)$ være den parametriserte kurven $(\cos(t), \sin(t), t)$

$$\vec{r}(t) = (t\cdot\cos t, t\sin t, t), \quad t\in[0,2\pi]$$

Finn et uttrykk for lengden.



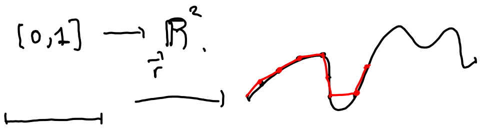
$$L(0,2\pi) = \int_0^{2\pi} \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2 + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{\cos^2 t - \cancel{2t\cos t\sin t} + t^2\sin^2 t + \sin^2 t + \cancel{2t\sin t\cos t} + t^2\cos^2 t + 1}\, dt$$

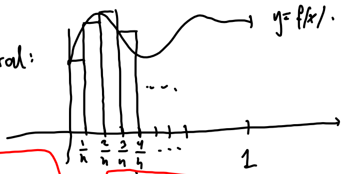
$$= \int_0^{2\pi} \sqrt{1 + t^2 + 1}\, dt = \underline{\int_0^{2\pi} \sqrt{2+t^2}\, dt}.$$

Hvorfor er definisjonen av lengde rimelig?

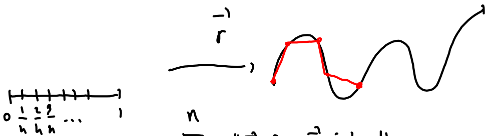
$\vec{r} : [0,1] \longrightarrow \mathbb{R}^2$.



Minne om integral:



$$\boxed{\int_0^1 f(x)\,dx = \lim_{n\to\infty} \sum_{j=0}^{n-1} \frac{1}{n} f\left(\frac{j}{n}\right)}$$



$$\lim_{n\to\infty} \sum_{j=1}^{n} \left\| \vec{r}\left(\tfrac{j}{n}\right) - \vec{r}\left(\tfrac{j-1}{n}\right) \right\|$$

$$= \lim_{n\to\infty} \sum_{j=1}^{n} \frac{1}{n} \cdot \left\| \frac{\vec{r}\left(\frac{j}{n}\right) - \vec{r}\left(\frac{j-1}{n}\right)}{1/n} \right\|$$

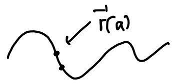
$$\overset{\text{for store } n}{\approx} \lim_{n\to\infty} \sum_{j=1}^{n} \frac{1}{n} \left\| \vec{r}\,'\left(\tfrac{j-1}{n}\right) \right\|$$

$$= \int_0^1 \|\vec{r}\,'(t)\|\,dt$$

$$= \int_0^1 \sqrt{x_1'(t)^2 + \dots + x_n'(t)^2}\,dt.$$

Dersom $\vec{x} \in \mathbb{R}^n$ så har vi $\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$

Fart : $\dfrac{\text{lengde}}{\text{tid}}$



$$\lim_{\varepsilon\to 0} \frac{1}{\varepsilon} \int_a^{a+\varepsilon} \|\vec{r}\,'(t)\|\,dt$$

$$= \|\vec{r}\,'(a)\|.$$

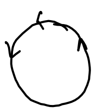
Definerer fart ved $v(t) = \|\vec{r}\,'(t)\|$.

Eks : $\vec{r}(t) = (\cos(t), \sin(t))$, $t \in [0, 2\pi]$.

Finn farten.

$\|\vec{r}\,'(t)\| = \|(-\sin(t), \cos(t))\|$

$= \sqrt{\sin^2 t + \cos^2 t} = 1.$



DEF : Hastighet er definert som $\vec{v}(t) = \vec{r}\,'(t)$.

DEF : Akselerasjon er definert som $\vec{a}(t) = \vec{v}\,'(t)$.