$\boxed{4.2.7}$

$$\begin{aligned} 2x + 4y - 4z &= b_1 \\ 2x - y + 3z &= b_2 \\ x - y + 2z &= b_3 \end{aligned}$$

$$\begin{pmatrix} 2 & 4 & -4 \\ 2 & -1 & 3 \\ 1 & -1 & 2 \end{pmatrix} \overset{?}{\sim} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underset{2\text{III}-\text{I}}{\overset{\text{II}-\text{I}}{\sim}} \begin{pmatrix} 2 & 4 & -4 \\ \boxed{0} & \boxed{-5} & \boxed{7} \\ \boxed{0} & \boxed{-6} & \boxed{0} \end{pmatrix} \underset{\text{II} \leftrightarrow \text{III}}{\overset{\text{III}\cdot\left(-\frac{1}{6}\right),\ \text{I}\cdot\frac{1}{2}}{\sim}} \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & \boxed{-5} & 7 \end{pmatrix}$$

$$\overset{\text{III}+5\text{II}}{\sim} \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 7 \end{pmatrix} \overset{\text{III}\cdot\frac{1}{7}}{\sim} \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\uparrow \quad \uparrow \quad \uparrow$$

$$\sim I_3$$

$\boxed{4.2.10}$ a $\overset{\text{antall biler}}{\vee}$ i A , b antall biler i B,

c – " – C.

$$a + b + c = 120$$

$$\begin{cases} a = 0.6a + 0.3b + 0.6c \\ b = 0.3a + 0.5b + 0.1c \\ c = 0.1a + 0.2b + 0.3c \end{cases}$$

$$\downarrow$$

$$-0.4a + 0.3b + 0.6c = 0$$

$$0.3a - 0.5b + 0.1c = 0$$

$$0.1a + 0.2b - 0.7c = 0$$

$$\begin{pmatrix} 1 & 1 & 1 & 120 \\ -0.4 & 0.3 & 0.6 & 0 \\ 0.3 & -0.5 & 0.1 & 0 \\ 0.1 & 0.2 & -0.7 & 0 \end{pmatrix}$$

$$\begin{matrix} 10\cdot\mathrm{II} \\ 10\cdot\mathrm{III} \\ \sim \\ 10\cdot\mathrm{IV} \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 120 \\ -4 & 3 & 6 & 0 \\ 3 & -5 & 1 & 0 \\ 1 & 2 & -7 & 0 \end{pmatrix}$$

$$\begin{matrix} \mathrm{II}+4\mathrm{I} \\ \mathrm{III}-3\mathrm{I} \\ \sim \\ \mathrm{IV}-\mathrm{I} \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 120 \\ 0 & 7 & 10 & 480 \\ 0 & -8 & -2 & -360 \\ 0 & \boxed{1} & -8 & -120 \end{pmatrix}$$

$$\begin{matrix} \mathrm{IV}\leftrightarrow\mathrm{II} \\ \sim \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & -8 & -2 & -360 \\ 0 & 7 & 10 & 480 \end{pmatrix}$$

$$\begin{matrix} \mathrm{III}+8\mathrm{II} \\ \sim \\ \mathrm{IV}-7\mathrm{II} \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 120 \\ 6 & 1 & -8 & -120 \\ 0 & 0 & -66 & -1320 \\ 0 & 0 & 66 & 1320 \end{pmatrix}$$

$$\begin{matrix} \mathrm{IV}+\mathrm{III} \\ \sim \\ \mathrm{III}\cdot\left(-\frac{1}{66}\right) \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{matrix} \mathrm{II}+8\mathrm{III} \\ \sim \\ \mathrm{I}-\mathrm{III} \end{matrix} \begin{pmatrix} 1 & 1 & 0 & 100 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{matrix} \mathrm{I}-\mathrm{II} \\ \sim \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 60 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$a = 60$

$b = 40$

$c = 20$

$\boxed{4.3.2}$ a) $A = \begin{pmatrix} 1 & 2 \\ \boxed{3} & 5 \end{pmatrix} \overset{\text{II}-3\text{I}}{\sim} \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$

$$\overset{\text{II}\cdot(-1)}{\sim} \underbrace{\begin{pmatrix} 1 & \boxed{2} \\ 0 & 1 \end{pmatrix}}_{\text{trappeform}} \overset{\text{I}-2\text{II}}{\sim} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

b) $B = \begin{pmatrix} 2 & 1 & 3 & 4 \\ \boxed{1} & 1 & 0 & 3 \end{pmatrix} \overset{\text{I}\leftrightarrow\text{II}}{\sim} \begin{pmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & 3 & 4 \end{pmatrix}$

$$\overset{\text{II}-2\text{I}}{\sim} \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & -1 & 3 & -2 \end{pmatrix}$$

$$\overset{\text{II}(-1)}{\sim} \begin{pmatrix} 1 & \boxed{1} & 0 & 3 \\ 0 & 1 & -3 & 2 \end{pmatrix}$$

$\uparrow$

$$\overset{\text{I}-\text{II}}{\sim} \begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -3 & 2 \end{pmatrix}$$

$\uparrow \quad \uparrow$

c) $C = \begin{pmatrix} \boxed{1} & 1 & 1 & 2 \\ -1 & 2 & -1 & 3 \\ 2 & 2 & 1 & 1 \end{pmatrix}$

$$\underset{\text{III}-2\text{I}}{\overset{\text{II}+\text{I}}{\sim}} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 3 & 0 & 5 \\ 0 & 0 & -1 & -3 \end{pmatrix}$$

$$\underset{(-1)\,\text{III}}{\overset{\frac{1}{3}\cdot\text{II}}{\sim}} \begin{pmatrix} 1 & \boxed{1 \quad 1} & 2 \\ 0 & 1 \quad 0 & 5/3 \\ 0 & 0 \quad 1 & 3 \end{pmatrix}$$

$$\overset{\text{I}-\text{II}-\text{III}}{\sim} \begin{pmatrix} 1 & 0 & 0 & 2-\frac{5}{3}-3 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & -8/3 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

d) $D = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -2 & -4 & 5 \\ -1 & -2 & 8 \end{pmatrix} \underset{\substack{\text{III}-2\text{IV} \\ \text{IV}+\text{I}}}{\overset{\text{II}-2\text{I}}{\sim}} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & -11 \\ 0 & 0 & 11 \end{pmatrix}$

$$\underset{\text{IV}\cdot\left(\frac{1}{11}\right)}{\overset{\text{III}+\text{IV}}{\sim}} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \underset{\text{II}\leftrightarrows\text{IV}}{\overset{\text{I}-3\text{IV}}{\sim}} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\boxed{4.4.4}$ a)

$$\left(\begin{array}{cccc|c} 0 & 1 & 2 & 0 & 0 \\ 1 & 0 & -1 & 1 & 1 \\ 6 & 0 & -6 & 7 & h \\ 2 & 1 & 0 & 0 & 0 \end{array}\right)$$

$$\underset{\sim}{\text{I} \leftrightarrow \text{II}} \left(\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ \boxed{6} & 0 & -6 & 7 & h \\ \boxed{2} & 1 & 0 & 0 & 0 \end{array}\right)$$

$$\underset{\text{IV}-2\text{I}}{\overset{\text{III}-6\text{I}}{\sim}} \left(\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & h-6 \\ 0 & 1 & 2 & -2 & -2 \end{array}\right)$$

$$\underset{\sim}{\text{IV}-\text{II}} \begin{pmatrix} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & h-6 \\ 0 & 0 & 0 & -2 & -2 \end{pmatrix}$$

$$\underset{\text{IV}-\text{III}}{\overset{\text{IV}\cdot\left(-\frac{1}{2}\right)}{\sim}} \begin{pmatrix} 1 & 0 & -1 & \textcircled{1} & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & h-6 \\ 0 & 0 & 0 & \underset{\uparrow}{0} & 7-h \end{pmatrix}$$

b) $A\vec{x} = \vec{b}$?

$\underline{\underline{h=7}}$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\underset{\sim}{\text{I}-\text{III}} \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = x_3$

$x_1 - x_3 = 0$

$x_2 + 2x_3 = 0$

$x_4 = 1$

$x_2 = -2x_3$

$$\underbrace{\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} x_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_2 + \left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix}\right) x_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} x_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} x_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} x_4} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix}\right) x_3$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_4 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 0 \\ 1 \end{pmatrix} x_3$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} x_3$$

$\boxed{4.4.5}$

$$C = \left(\begin{array}{ccc:c} \boxed{1} & 0 & 1 & 1 \\ 2 & 1 & a^2-a & 3 \\ -1 & 1 & -3 & a \end{array}\right) \qquad \overbrace{A\vec{x}}\; = \vec{b}$$

a)

$$C \underset{III+I}{\overset{II-2I}{\sim}} \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & \boxed{1} & -2 & a+1 \end{pmatrix}$$

$$\overset{III-II}{\sim} \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 0 & \underbrace{-a^2+a} & a \end{pmatrix} = D$$

$$\frac{1}{-a^2+a} \leftarrow = 0 \,? \qquad -a^2+a = -a(a-1) = 0$$

$$a = 0 ,\ a = 1$$

b) $A\vec{x} = \vec{b}$

$$a \neq 0,1 : \quad D \overset{III \cdot \frac{1}{-a^2+a}}{\sim} \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 0 & 1 & \frac{1}{-a+1} \end{pmatrix}$$

$$\sim \left(I_3 \,\vdots\, \tilde{b} \right)$$

$$a = 0 : \quad D = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

uendelig mange løsninger.

$$\underline{\underline{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} z}}$$

$$a = 1 : \quad D = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x + z = 1$$
$$y - 2z = 1$$
$$0 = 1$$

$\Rightarrow$ ingen løsning!

$\boxed{4.5.6}$ $B = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 1 \end{pmatrix}$

a) $\left(B \;\vdots\; I_3 \right) \sim \left(I_3 \;\vdots\; B^{-1} \right)$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & \boxed{-2} & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\overset{\mathrm{III}+2\mathrm{II}}{\sim} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 2 & 1 \end{array}\right)$$

$$\overset{\mathrm{III}\cdot\frac{1}{3}}{\sim} \left(\begin{array}{ccc|ccc} 1 & \boxed{2} & 0 & 1 & 0 & 0 \\ 0 & 1 & \boxed{1} & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2/3 & 1/3 \end{array}\right)$$

$$\overset{\mathrm{II}-\mathrm{III}}{\sim} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/3 & -1/3 \\ 0 & 0 & 1 & 0 & 2/3 & 1/3 \end{array}\right)$$

$$\overset{\mathrm{I}-2\mathrm{II}}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2/3 & 2/3 \\ 0 & 1 & 0 & 0 & 1/3 & -1/3 \\ 0 & 0 & \text{[illegible]} & 0 & 2/3 & 1/3 \end{array}\right)$$

$$\underbrace{\qquad\qquad}_{B^{-1}}$$

b)

$$\begin{aligned} x + 2y &= 5 \\ y + z &= 3 \\ -2y + z &= 3 \end{aligned} \qquad \vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$B\vec{x} = \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix}$$

$$\underbrace{B^{-1}B}\vec{x} = B^{-1}\begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix}$$

$$\underbrace{I_3\vec{x}} = B^{-1}\begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} 1 & -2/3 & 2/3 \\ 0 & 1/3 & -1/3 \\ 0 & 2/3 & 1/3 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 5-2+2 \\ 1-1 \\ 2+1 \end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix}}}$$

c)

$$\begin{aligned} x + 2y &= 5 \\ y + z &= 3 \\ -2y + (a+1)z &= b^2 - 10 \end{aligned}$$

$$\begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & \boxed{-2} & a+1 & b^2-10 \end{pmatrix}$$

$$\overset{\mathrm{III}+2\mathrm{II}}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & \underline{a+3} & b^2-4 \end{array}\right)$$

<u>én løsning</u>: $a + 3 \neq 0$

$a \neq -3$

<u>uendelig mange</u>: $a + 3 = 0$ & $b^2 - 4 = 0$

$a = -3$ & $b = \pm 2$

<u>ingen løsning</u>: $a + 3 = 0$ & $b^2 - 4 \neq 0$