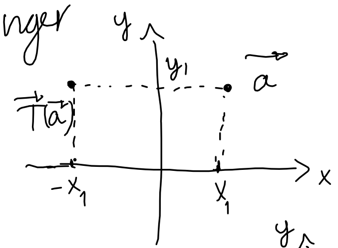
Plenum 2/2-16

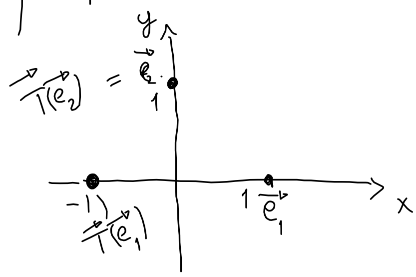
1.9: 4, 8, 11 2.8: 2
1.10: 2, 5
2.7: 2, 8

1.9: Lineæravbildninger



4) $\vec{T}: \mathbb{R}^2 \to \mathbb{R}^2$

$\vec{T}(\underbrace{\vec{e}_1}_{\begin{bmatrix}1\\0\end{bmatrix}}) = \begin{bmatrix}-1\\0\end{bmatrix}$



$\vec{T}(\underbrace{\vec{e}_2}_{\begin{bmatrix}0\\1\end{bmatrix}}) = \begin{bmatrix}0\\1\end{bmatrix}$

Matrisen til $\vec{T}$ er $\underline{\underline{A = \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}}}$.

8) $\vec{T}: \mathbb{R}^2 \to \mathbb{R}^2$, speiler om x-akse, θ pos. ret.

MERK: $\vec{T}(\vec{x}) = \vec{G}(\vec{H}(\vec{x}))$ der $\vec{H}: \mathbb{R}^2 \to \mathbb{R}^2$ er lin. avb. som speiler om x-aksen, og $\vec{G}: \mathbb{R}^2 \to \mathbb{R}^2$ er lin. avb. som roterer med θ i pos. ret.

$$\underbrace{\text{Matrisen til } \vec{T}}_{A} = \underbrace{\text{matrisen til } \vec{G}}_{C} \cdot \underbrace{\text{matrisen til } \vec{H}}_{B}$$

Hvorfor? $A\vec{x} = \vec{T}(\vec{x}) = \vec{G}(\vec{H}(\vec{x})) = \vec{G}(B\vec{x}) = C(B\vec{x})$

$$= (CB)\vec{x}$$

$$\Downarrow$$

$$A = CB$$

Sett f. eks. $\vec{x} = \vec{e}_1$, $\vec{x} = \vec{e}_2$

Matrisen til $\vec{G}$ er $C = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ → Eks. 1.9.7

— " — $\vec{H}$ er $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ → Eks. 1.9.6

Så matrisen til $\vec{T}$ er

$$A = CB = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$$

11.) $\vec{a} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

a) Finn x, y, z, u s.a. $\vec{e}_1 = x\vec{a} + y\vec{b}$

$$\vec{e}_2 = z\vec{a} + u\vec{b}$$

$$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = x\begin{bmatrix} -2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2x + y \\ x + 3y \end{bmatrix} \Rightarrow \begin{aligned} 1 &= -2x + y \\ 0 &= x + 3y \end{aligned}$$

$x = -3y$

$6y + y = 1 \Rightarrow y = \frac{1}{7}$, $x = -\frac{3}{7}$

$$\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = z\begin{bmatrix} -2 \\ 1 \end{bmatrix} + u\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2z+u \\ z+3u \end{bmatrix}$$

$$\Downarrow$$

$$0 = -2z + u \rightsquigarrow u = 2z \rightsquigarrow u = \underline{\underline{\tfrac{2}{7}}}$$

$$1 = z + 3u \rightsquigarrow 7z = 1 \Rightarrow z = \underline{\underline{\tfrac{1}{7}}}$$

b) $\vec{T}: \mathbb{R}^2 \to \mathbb{R}^2$, $\vec{T}(\vec{a}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{T}(\vec{b}) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\vec{T}(\vec{e}_1) \underset{a)}{=} \vec{T}(x\vec{a} + y\vec{b}) \underset{\vec{T}\text{ lineær}}{=} x\vec{T}(\vec{a}) + y\vec{T}(\vec{b})$$

$$\underset{a)}{=} -\frac{3}{7}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{7}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -\frac{3}{7} + \frac{1}{7} \\ -\frac{3}{7} - \frac{1}{7} \end{bmatrix} = \underline{\underline{\begin{bmatrix} -\frac{2}{7} \\ -\frac{4}{7} \end{bmatrix}}}$$

$$\vec{T}(\vec{e}_2) \underset{a)}{=} \vec{T}(z\vec{a} + u\vec{b}) \underset{\text{lineær}}{=} z\vec{T}(\vec{a}) + u\vec{T}(\vec{b}) \underset{a)}{=} \frac{1}{7}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{2}{7}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} \frac{3}{7} \\ -\frac{1}{7} \end{bmatrix}}}$$

c) Matrisen til $\vec{T}$ er:

$$A = \underline{\underline{\begin{bmatrix} -\frac{2}{7} & \frac{3}{7} \\ -\frac{4}{7} & -\frac{1}{7} \end{bmatrix}}}$$

1.10: Affinavbildninger

2) $\vec{r}(t) = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$; L

$$\vec{F}(x,y,z) = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\vec{r}(t) = \begin{bmatrix} 2+t \\ -1 \\ 3+2t \end{bmatrix}$$

$$\vec{F}(\vec{r}(t)) = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{bmatrix} \begin{bmatrix} 2+t \\ -1 \\ 3+2t \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+t+1+6+4t \\ 0-3-6-4t \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 \\ -9 \end{bmatrix} + \begin{bmatrix} 5t \\ -4t \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

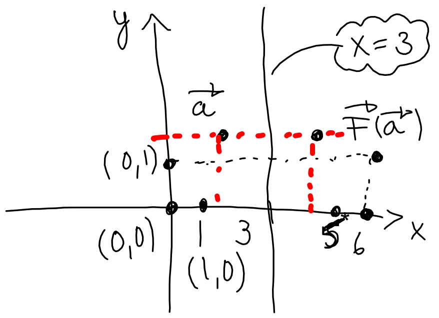
$$= \underline{\begin{bmatrix} 11 \\ -10 \end{bmatrix} + \begin{bmatrix} 5 \\ -4 \end{bmatrix}} t = \underline{\begin{bmatrix} 5t+11 \\ -4t-10 \end{bmatrix}}$$

5) a) $\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$

$$\vec{F}(0,0) = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$\vec{F}(1,0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$\vec{F}(0,1) = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$



$\vec{F}$ affin $\Rightarrow \vec{F}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = A \begin{bmatrix} x \\ y \end{bmatrix} + \vec{b}$

$$\underline{\vec{F}(0,0)} = A \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \vec{b} = \underline{\vec{b}}$$

Konstantleddet $\vec{b} = \underline{\begin{bmatrix} 6 \\ 0 \end{bmatrix}}$.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\vec{F}(1,0) \overset{!}{=} A \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix} = \underline{\begin{bmatrix} a_{11} + 6 \\ a_{21} \end{bmatrix}} = \underline{\begin{bmatrix} 5 \\ 0 \end{bmatrix}} \Rightarrow \begin{matrix} a_{11} + 6 = 5 \\ a_{21} = 0 \end{matrix}$$

$$= \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$\Downarrow$$

$$a_{11} = -1$$

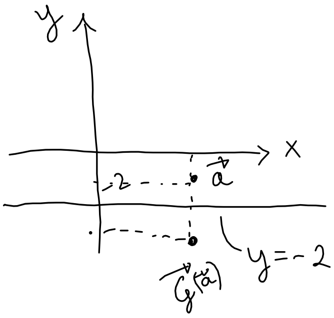
$$a_{21} = 0$$

$$\underbrace{\begin{bmatrix}6\\1\end{bmatrix}}_{\text{Figur}} = \underbrace{\vec{F}(0,1)}_{\text{affin}} = A\begin{bmatrix}0\\1\end{bmatrix} + \begin{bmatrix}6\\0\end{bmatrix} = \begin{bmatrix}a_{12}+6\\a_{22}+0\end{bmatrix}$$

$$\begin{aligned}a_{12}+6&=6\\ a_{22}&=1\end{aligned} \quad\Rightarrow\quad \begin{aligned}a_{12}&=0\\ a_{22}&=1\end{aligned}$$

$$\Rightarrow\ A = \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} \qquad \left(\vec{b} = \begin{bmatrix}6\\0\end{bmatrix}\right)$$

b)



2.7: Kjerneregelen

2) $f(u,v) = ue^{-v}$, $g(x,y,z) = 2xy+z$, $h(x,y,z) = 2y(z+x)$

$$k(x,y,z) = f\big(\underbrace{g(x,y,z)}_{\text{"spiller rollen som u"}},\ \underbrace{h(x,y,z)}_{\text{"spiller rollen som v"}}\big)$$

$$\frac{\partial k}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial g}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial h}{\partial x} = e^{-2y(z+x)}\,2y - (2xy+z)e^{-2y(z+x)}\,2y$$

Kjerneregel på komponentform

$$= 2ye^{-2y(z+x)}\big(1-(2xy+z)\big)$$