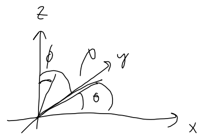
Plenum 22/2

$\boxed{3.7.4}$ kule:

$$x = \rho \cos\theta \sin\phi$$
$$y = \rho \sin\theta \sin\phi$$
$$z = \rho \cos\phi$$

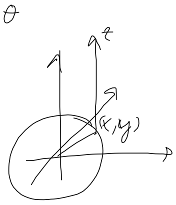
$$x^2 + y^2 + z^2 = \rho^2$$



Sylinder:

$$x = r \cos\theta$$
$$y = r \sin\theta$$
$$z = z$$

$$x^2 + y^2 = r^2$$



a) $f(x,y,z) = (x^2+y^2)e^{-z^2}$

sylinder: $f(r,\theta,z) = r^2 e^{-z^2} = f(r,z)$

kule: $f(\rho,\phi,\theta) = (\rho^2\cos^2\theta\sin^2\phi + \rho^2\sin^2\theta\sin^2\phi)\cdot e^{-(\rho\cos\phi)^2}$

$$= \rho^2\sin^2\phi \cdot e^{-\rho^2\cos^2\phi} = f(\rho,\phi)$$

b) $f(x,y,z) = \dfrac{1}{x^2+y^2+z^2}$

kule: $f(\rho,\theta,\phi) = \dfrac{1}{\rho^2} = f(\rho)$

sylinder: $f(r,\theta,z) = \dfrac{1}{r^2+z^2} = f(r,z)$

c) $f(x,y,z) = z \arctan\left(\dfrac{y}{x}\right)$

kule:

$$f(\rho,\theta,\phi) = \rho\cos\phi \cdot \arctan\left(\frac{\cancel{\rho}\sin\theta\cancel{\sin\phi}}{\cancel{\rho}\cos\theta\cancel{\sin\phi}}\right)$$
$$= \rho\cos\phi \arctan(\tan\theta)$$
$$= \rho\cos\phi\cdot\theta = f(\rho,\theta,\phi)$$

sylinder:

$$f(r,\theta,z) = z\arctan\left(\frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}\right)$$
$$= z\cdot\theta = f(\theta,z)$$

$\vec{x} = (x,y)$

$\boxed{3.7.5}$ c)

$$z = \underbrace{f(\vec{a})} + \underbrace{\nabla f(\vec{a})} (\vec{x} - \vec{a})$$

$$f(x,y) = x^2 y - x y^2 \quad , \quad \vec{a} = (2,-2)$$

$$f(2,-2) = 2^2 \cdot (-2) - 2 \cdot (-2)^2 = -8 - 8 = \underline{-16}$$

$$\nabla f(x,y) = \left(\frac{\partial}{\partial x} f , \frac{\partial}{\partial y} f \right)$$

$$\frac{\partial}{\partial x} f = 2xy - y^2 , \quad \frac{\partial}{\partial y} f = x^2 - 2xy$$

$$\frac{\partial}{\partial x} f(2,-2) = 2 \cdot 2 \cdot (-2) - (-2)^2 = -12$$

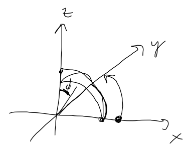
$$\frac{\partial}{\partial y} f(2,-2) = 2^2 - 2 \cdot 2 \cdot (-2) = 12$$

$$z = -16 + (-12 , 12) \cdot \begin{pmatrix} x - 2 \\ y + 2 \end{pmatrix}$$

$$= -16 + \left[(x-2) \cdot (-12) + (y+2) \cdot 12 \right]$$

$$= -12x + 12y + 32$$

$\boxed{3.9.2}$ $\rho = 2$



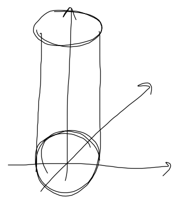
$x = \rho \cos\theta \sin\phi$

$y = \rho \sin\theta \sin\phi$

$z = \rho \cos\phi$

$\vec{r}(\theta,\phi) = 2\cos\theta\sin\phi\,\vec{i} + 2\sin\theta\sin\phi\,\vec{j} + 2\cos\phi\,\vec{k}$, $\theta \in [0, \pi/2]$, $\phi \in [0, \frac{\pi}{2}]$

$\boxed{3.9.4}$ $y^2 + z^2 = 9 = 3^2$

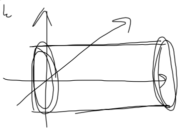


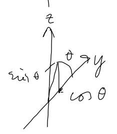
$x = x$

$y = r\cos\theta$

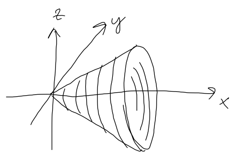
$z = r\sin\theta$

$\vec{r}(x,\theta) = x\vec{i} + 3\cos\theta\vec{j} + 3\sin\theta\vec{k}$





$\boxed{3.9.5}$ $x = \sqrt{y^2+z^2}$



$\vec{r}(y,z) = \sqrt{y^2+z^2}\,\vec{i} + y\vec{j} + z\vec{k}$

$\vec{r}(x,\theta) = x\vec{i} + x\cos\theta\vec{j} + x\sin\theta\vec{k}$

$\boxed{3.9.6}$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$X = \frac{x}{a}$, $Y = \frac{y}{b}$, $Z = \frac{z}{c}$

$X^2 + Y^2 + Z^2 = 1$

$X = \cos\theta\sin\phi$

$Y = \sin\theta\sin\phi$

$Z = \cos\phi$

$x = a\cos\theta\sin\phi$

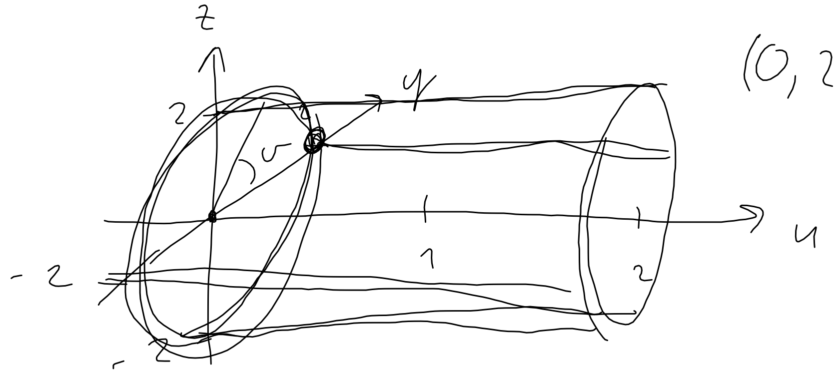
$y = b\sin\theta\sin\phi$

$z = c\cos\phi$

$\vec{r}(\theta,\phi) = a\cos\theta\sin\phi\,\vec{i} + b\sin\theta\sin\phi\,\vec{j} + c\cos\phi\,\vec{k}$

$\boxed{3.9.7}$ $\vec{r}(u,v) = u\vec{i} + 2\cos v\,\vec{j} + 2\sin v\,\vec{k}$

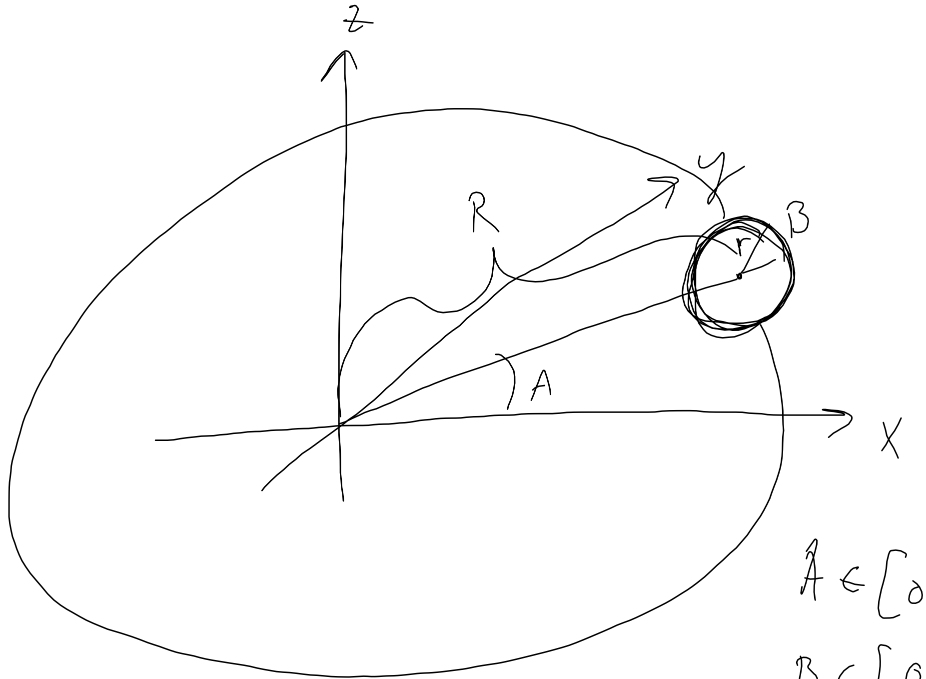
$0 \le u \le 2$, $0 \le v < 2\pi$



(0, 2, 0)

$\boxed{3.9.10}$ Matlab

$\boxed{3.9.14}$ Matlab



$A \in [0, 2\pi)$

$B \in [0, 2\pi)$

$R = 5$

$r = 3$