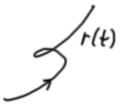
Parametrisert kurve

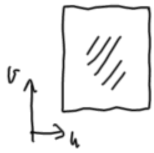
$r(t) = (x_1(t), \dots, x_n(t)) \in \mathbb{R}^n \quad t \in \mathbb{R}.$



Parmetrisert flate

$\bar{x}(u,v) = X(u,v)\,i + Y(u,v)\,j + Z(u,v)\,k = (X(u,v), Y(u,v), Z(u,v)) \in \mathbb{R}^3$

$(u,v) \in [a_1, a_2] \times [b_1, b_2]$

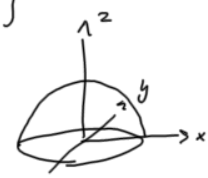


Enkleste eksempel:

$(\overset{X}{u}, \overset{Y}{v}, Z(u,v))$

Parametrisere enhetskula i $\mathbb{R}^3$ $\quad \{(x,y,z) \mid x^2+y^2+z^2 = 1\}$

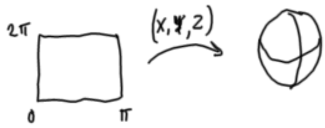
Øvre halvdel: $z = \sqrt{1-x^2-y^2}, \quad x^2+y^2 \le 1.$



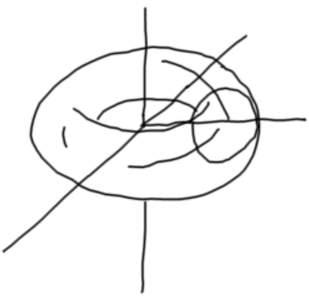
Kulekoordinater $\boxed{r=1}$

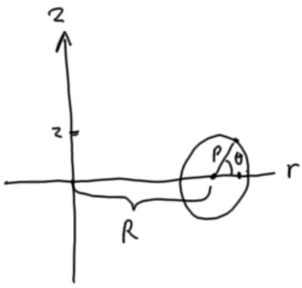
$x = r \sin\varphi \cos\theta$
$y = r \sin\varphi \sin\theta$
$z = r\cos\varphi$

$X(\varphi, \theta) = \sin\varphi\cos\theta \qquad \varphi \in [0, \pi]$
$Y(\varphi, \theta) = \sin\varphi \sin\theta \qquad \theta \in [0, 2\pi]$
$Z(\varphi, \theta) = \cos\varphi$



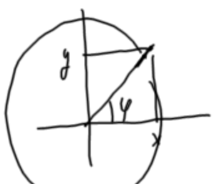
Parametrisering av Torus



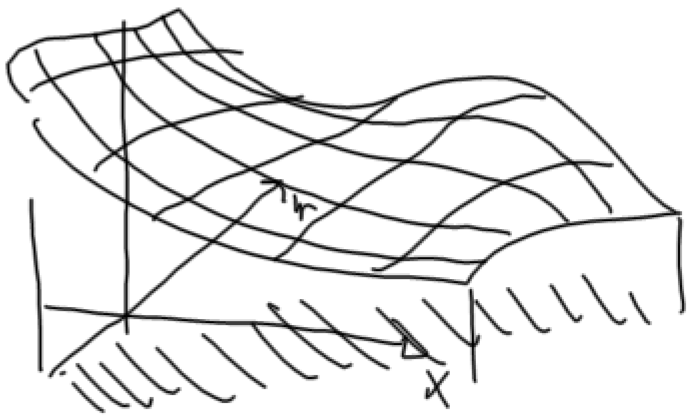


$\rightarrow r = R + \rho\cos\theta \qquad \theta \in [0, 2\pi]$
$z = \rho \sin\theta$

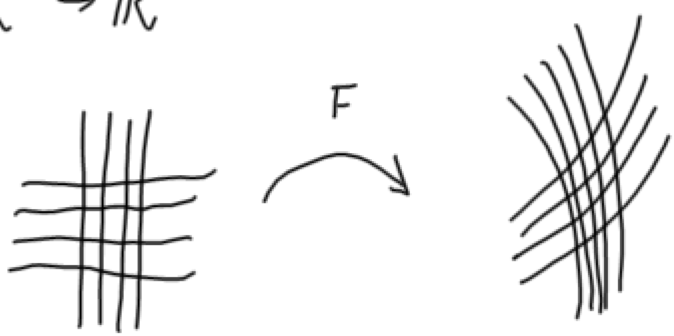
Polarkoordinater i (x,y) planet.



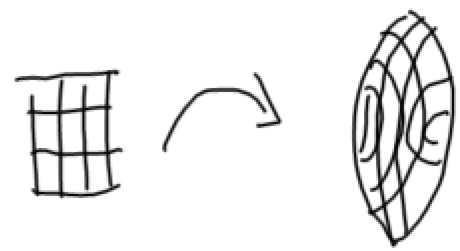
$x = r\cos(\varphi) = \cos(\varphi)(R + \rho\cos\theta)$
$y = r\sin(\varphi) = \sin(\varphi)(R+\rho\cos\theta)$
$z = \rho\sin\theta$
$(\theta, \varphi) \in [0, 2\pi]^2$



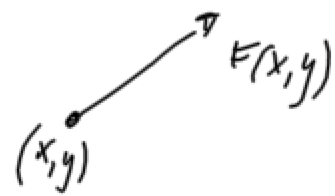
$$F : \mathbb{R}^2 \to \mathbb{R}^2$$

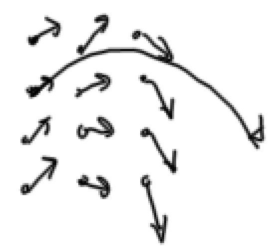


$$F(x,y) = \begin{pmatrix} x^2 - y^2 \\ 2xy \end{pmatrix}$$



Betrakte F som et vektorfelt.





Linearalgebra i $\mathbb{R}^n$

Løse systemer av lineære ligninger

Eks.

$$\begin{aligned} 2x - y &= 3 \\ x + 3y &= 4 \end{aligned} \quad \xrightarrow{I \to I - II} \quad \begin{aligned} x - 4y &= -1 \\ x + 3y &= 4 \end{aligned} \quad \xrightarrow{II \to II - I} \quad \begin{aligned} x - 4y &= -1 \\ 7y &= 5 \end{aligned} \quad \xrightarrow{II \to II/7} \quad \begin{aligned} x - 4y &= -1 \\ y &= 5/7 \end{aligned}$$

$$I: \; x - 4 \cdot \frac{5}{7} = -1 \rightarrow x = \frac{20-7}{7} = \frac{13}{7}$$

$$x = \frac{13}{7}$$
$$y = \frac{5}{7}$$

Gauss eliminasjon.

$$\boxed{\begin{aligned} 0x + 2y + z &= 1 \\ 3x + 5y + z &= 2 \\ x + 2y + z &= 1 \end{aligned}} \quad \xrightarrow{III \leftrightarrow I} \quad \begin{aligned} x + 2y + z &= 1 \\ 3x - 5y + z &= 2 \\ 2y + z &= 1 \end{aligned} \quad \xrightarrow{II \to II - 3I} \quad \begin{aligned} x + 2y + z &= 1 \\ +y + 2z &= +1 \\ 2y + z &= 1 \end{aligned} \quad \xrightarrow{III \to III - 2II} \quad \boxed{\begin{aligned} x + 2y + z &= 1 \\ y + 2z &= 1 \\ z &= +1/3 \end{aligned}}$$

Radoperasjoner

1) Bytte om på ligningene
2) Gange en ligning med et tall $\neq 0$.
3) Legge sammen/trekke fra rader.

} Forandrer vi IKKE løsningene.

Eks

$$\begin{aligned} x + 2y + z - u &= 3 \\ -x - y - 4z + 2u &= -1 \\ 2x + 5y + z &= 9 \\ x + 7z - 5u &= 0 \end{aligned} \quad \xrightarrow{\substack{II + I \\ III - 2I \\ IV - I}} \quad \begin{aligned} x + 2y + z - u &= 3 \\ y - 3z + u &= 2 \\ y - z + 2u &= 3 \\ -2y + 6z - 4u &= -3 \end{aligned} \quad \xrightarrow{\substack{III - II \\ IV + 2II}} \quad \begin{aligned} x + 2y + z - u &= 3 \\ y - 3z + u &= 2 \\ \cancel{2}z + u/2 &= 1/2 \\ \cancel{2}u &= 1/2 \end{aligned}$$

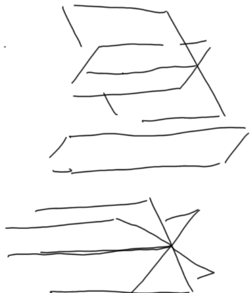
3 ligninger / 3 ukjente:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned} \qquad A\bar{x} = \bar{b}$$

Hver av ligningene sier at $\bar{x}$ ligger i et plan.

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$\bar{x}$ i et plan med normalvektor $\bar{n} = \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \end{pmatrix}$.



$A\bar{x} = \bar{b} \qquad (A \; \bar{b}). \leftarrow$

$$\begin{aligned} 2y + z &= -1 \\ 3x + 5y + z &= 2 \\ x + 2y + z &= 1 \end{aligned} \quad \begin{pmatrix} 0 & 2 & 1 & -1 \\ 3 & 5 & 1 & 2 \\ 1 & 2 & 1 & 1 \end{pmatrix} \overset{III \leftrightarrow I}{\sim} \begin{pmatrix} 1 & 2 & 1 & 1 \\ 3 & 5 & 1 & 2 \\ 0 & 2 & 1 & -1 \end{pmatrix} \overset{II - 3I}{\sim} \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & 1 & -1 \end{pmatrix}$$

$$\overset{III - 2II}{\sim} \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -3 & -3 \end{pmatrix} \overset{III/-3}{\sim} \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \leftarrow \quad \begin{aligned} x + 2y + z &= 1 \\ y + 2z &= 1 \\ z &= 1 \end{aligned}$$

En matrise er på trappeform hvis

A) Alle rader starter med 0 eller 1

B) Alle rader har sitt første 1 tall minst en søyle til høyre for raden over.

A er radekvivalent med B hvis vi kan komme fra A til B med radoperasjoner. $A \sim B$

Alle matriser er radekvivalent med en matrise på trappeform.

$$A = \begin{pmatrix} 2 & 4 & -2 \\ 1 & 2 & 0 \\ -3 & 1 & 3 \end{pmatrix}$$ Prøv å få A på trappeform.

$$A \overset{I \leftrightarrow II}{\sim} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & -2 \\ -3 & 1 & 3 \end{pmatrix} \overset{\substack{II - 2I \\ III + 3I}}{\sim} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & -2 \\ 0 & 7 & 3 \end{pmatrix} \overset{II/-2,\; II \leftrightarrow III}{\sim} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 7 & 3 \\ 0 & 0 & 1 \end{pmatrix} \overset{II/7}{\sim} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3/7 \\ 0 & 0 & 1 \end{pmatrix}$$