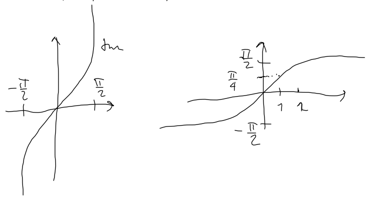
Plenum 15/2

3.4: 5, 14
3.5: 1, 11
3.6: 1, 8, 10

$\boxed{3.4.5}$ $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$

$$\vec{F}(x,y,z) = (yz, x, xy) = yz\underbrace{\vec{i}}_{(1,0,0)} + x\underbrace{\vec{j}}_{(0,1,0)} + xy\underbrace{\vec{k}}_{(0,0,1)}$$

$$\vec{r}(t) = (t, \arctan t, t) \quad , \ t \in [0,1]$$



$$\int_{\mathcal{C}} \vec{F} \cdot d\vec{r} = \int_0^1 \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

$$= \int_0^1 (\tan\arctan t, t, t \cdot \arctan t) \cdot (1, \frac{1}{1+t^2}, 1)\, dt$$

$$= \int_0^1 2t \arctan t + \frac{t}{1+t^2}\, dt$$

$$= \underbrace{2\int_0^1 t \arctan t \, dt}_{\substack{\text{delvis int.} \\ (1)}} + \underbrace{\int_0^1 \frac{t}{1+t^2}\, dt}_{\substack{\text{substit.} \\ (2)}} \qquad \begin{aligned} u &= 1+t^2 \\ du &= 2t\, dt \end{aligned}$$

$$(1): \int_0^1 \overset{u'}{t} \cdot \overset{v}{\arctan t}\, dt = \Big[\overset{u}{\tfrac{1}{2}t^2} \overset{v}{\arctan t}\Big]_0^1 - \frac{1}{2}\int_0^1 \overset{u \cdot v'}{\frac{t^2}{1+t^2}}\, dt$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - 0\right) - \frac{1}{2}\int_0^1 \frac{(t^2+1) - 1}{t^2+1}\, dt$$

$$= \frac{\pi}{8} - \frac{1}{2}\int_0^1 1 - \frac{1}{t^2+1}\, dt$$

$$= \frac{\pi}{8} - \frac{1}{2}\Big[t - \arctan t\Big]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2}\Big[1 - \frac{\pi}{4}\Big] = \frac{\pi}{4} - \frac{1}{2}$$

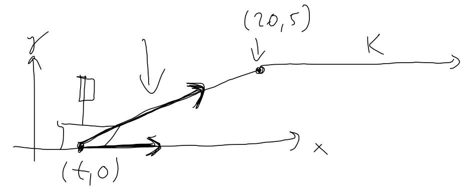
$$(2): \int_0^1 \frac{t}{1+t^2}\, dt = \frac{1}{2}\int_1^2 \frac{1}{u}\, du = \frac{1}{2}\Big[\ln u\Big]_1^2$$

$$= \frac{1}{2}\ln 2$$

$$\int_{\mathcal{C}} \vec{F} \cdot d\vec{r} = 2 \cdot \left(\frac{\pi}{4} - \frac{1}{2}\right) + \frac{1}{2}\ln 2$$

$$= \underline{\underline{\frac{\pi}{2} - 1 + \frac{1}{2}\ln 2}}$$

$\boxed{3.4.14}$



a) $$W = K\int_0^{20} \frac{20-t}{\sqrt{25+(20-t)^2}}\,dt$$

$$W = \int_C \vec{F}\cdot d\vec{r}$$

retning kraft $= (20-t, 5)$

$\vec{r}(t) = (t, 0)$, $\vec{r}\,'(t) = (1, 0)$

$$\vec{F}(\vec{r}(t)) = \left(\frac{(20-t)K}{\sqrt{25+(20-t)^2}}, \frac{5K}{\sqrt{25+(20-t)^2}}\right)$$

$$W = \int_C \vec{F}\cdot d\vec{r} = \int_0^{20} \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$$

$$= \int_0^{20} \frac{(20-t)K}{\sqrt{25+(20-t)^2}}\,dt$$

b) $$W = K\int_0^{20} \frac{(20-t)}{\sqrt{25+(20-t)^2}}\,dt$$

$$\left[\begin{array}{l} u = 25+(20-t)^2, \quad \int_{425}^{25} \\ du = -2(20-t)\,dt \end{array}\right]$$

$$= -\frac{K}{2}\int_{425}^{25} \frac{1}{\sqrt{u}}\,du \qquad -\int_a^b = \int_b^a$$

$$= \frac{K}{2}\int_{25}^{425} u^{-\frac{1}{2}}\,du \qquad \sqrt{u} = u^{\frac{1}{2}}, \quad \frac{1}{\sqrt{u}} = u^{-\frac{1}{2}}$$

$$= \frac{K}{2}\cdot\frac{1}{-\frac{1}{2}+1}\left[u^{-\frac{1}{2}+1}\right]_{25}^{425}$$

$$= K\left[u^{\frac{1}{2}}\right]_{25}^{425} = \underline{5K(\sqrt{17}-1)}$$

Husk!

Skalarfelt : $f : \mathbb{R}^n \to \mathbb{R}$

Vektorfelt : $F : \mathbb{R}^n \to \mathbb{R}^n$

$\nabla f : \mathbb{R}^n \to \mathbb{R}^n$

$$\int_C \nabla f \cdot d\vec{r} = f(\vec{b}) - f(\vec{a})$$

$\boxed{3.5.1}$ $\vec{F}(x,y) = (\underbrace{[2xy + 2x]}_{F_1}, \underbrace{x^2}_{F_2})$

$\overset{?}{=} \nabla f$

$$F(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

$$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$$ Må holde for at f skal finnes

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f\right) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} f\right)$$

$$\left.\begin{aligned} \frac{\partial}{\partial y} F_1 &= 2x \\ \frac{\partial}{\partial x} F_2 &= 2x \end{aligned}\right\} \text{ likhet!}$$

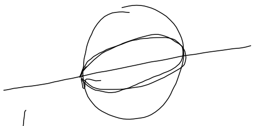
$$f(x,y) = \int 2xy + 2x \, dx$$
$$= x^2 y + x^2 + C(y)$$

$$\frac{\partial}{\partial y} f(x,y) = x^2 + C'(y)$$

$$\Rightarrow C'(y) = 0 \rightarrow C(y) = C$$

$$\underline{\underline{f(x,y) = x^2 y + x^2 + C}}$$

$\boxed{3.5.11}$ $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$



$$\vec{F}(x,y,z) = \left(z e^{xz+y},\ e^{xz+y} + 2z,\ x e^{xz+y} + 2y \right)$$

$$\overset{?}{=} \nabla f \qquad \frac{\partial f}{\partial x} \qquad \frac{\partial f}{\partial y} \qquad \frac{\partial f}{\partial z}$$

$$\frac{\partial}{\partial y}\left(z e^{xz+y}\right) = z e^{xz+y}$$

$$\frac{\partial}{\partial x}\left(e^{xz+y} + 2z\right) = z e^{xz+y}$$

$$\frac{\partial}{\partial z}\left(z e^{xz+y}\right) = e^{xz+y} + xz e^{xz+y}$$

$$\frac{\partial}{\partial x}\left(x e^{xz+y} + 2y\right) = e^{xz+y} + xz e^{xz+y}$$

$$\frac{\partial}{\partial z}\left(e^{xz+y} + 2z\right) = x e^{xz+y} + 2$$

$$\frac{\partial}{\partial y}\left(x e^{xz+y} + 2y\right) = x e^{xz+y} + 2$$

startpkt.

$$\int_{\mathcal{C}} \vec{F} \cdot d\vec{r} = \int_{\mathcal{C}} \nabla f \cdot d\vec{r} = f(\vec{b}) - f(\vec{a})$$

sluttpkt.

$$= f(\vec{a}) - f(\vec{a}) = 0$$

$$f(x,y,z) = \int e^{xz+y} + 2z \, dy$$

$$= e^{xz+y} + 2zy + C_1(x,z)$$

$$f(x,y,z) = \int x e^{xz+y} + 2y \, dz$$

$$= \frac{x}{x} e^{xz+y} + 2zy + C_2(x,y)$$

$$\left[\int e^{kx} dx = \frac{1}{k} e^{kx} \right] \qquad = e^{xz+y} + 2zy + C_2(x,y)$$

$$f(x,y,z) = \int z e^{xz+y} dx$$

$$= e^{xz+y} + C_3(y,z)$$

$$f(x,y,z) = e^{xz+y} + 2zy + C$$

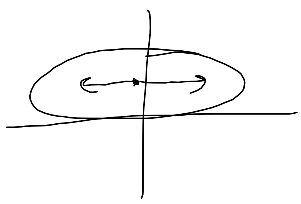
$\boxed{3.6.1}$ $4x^2 + 9y^2 + 32x - 18y + 37 = 0$

$$4(x^2 + 2\cdot 4x + 4^2) - 64$$

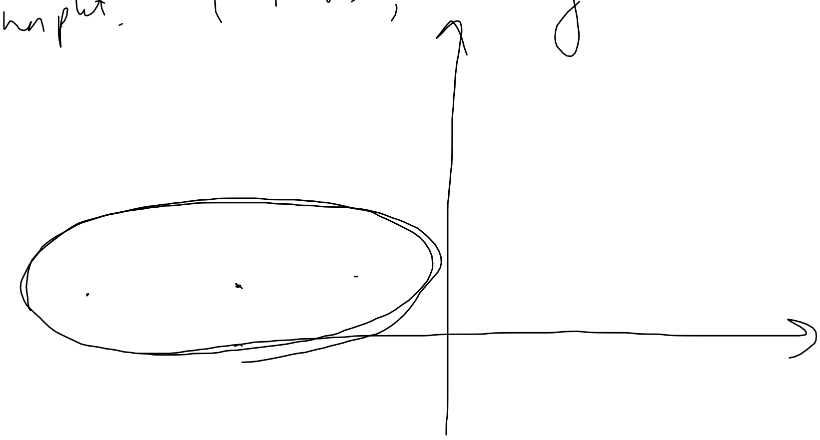
$$+ 9(y^2 - 2y + 1^2) - 9 + 37 = 0$$

$$4(x+4)^2 + 9(y-1)^2 = 36 \quad \Big/ \cdot \frac{1}{36}$$

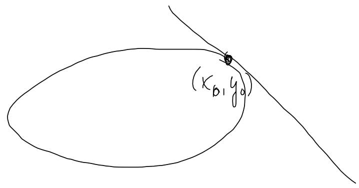
$$\underbrace{\frac{(x+4)^2}{3^2} + \frac{(y-1)^2}{2^2} = 1}_{\text{ellipse}}$$



- Sentrum $(-4, 1)$
- Halvakser $a = 3, \; b = 2$
- Brennvidde $c = \sqrt{a^2 - b^2} = \sqrt{9-4} = \sqrt{5}$
- Brennpkt. $(-4-\sqrt{5}, 1)$ og $(-4+\sqrt{5}, 1)$



$\boxed{3.6.8}$



$$\frac{x x_0}{a^2} + \frac{y y_0}{b^2} = 1$$

$$\text{tangent:} \quad y(x) = y'(x_0)(x - x_0) + \underbrace{y(x_0)}_{y_0}$$

$$y(x) = y'(x_0)(x - x_0) + y_0$$

$$\frac{x^2}{a^2} + \frac{y(x)^2}{b^2} = 1$$

deriver begge sider mhp x.

$$\frac{2x}{a^2} + \frac{2y(x) \cdot y'(x)}{b^2} = 0$$

$$\frac{2x_0}{a^2} + \frac{2y(x_0) \cdot y'(x_0)}{b^2} = 0$$

$$\frac{2x_0}{a^2} + \frac{2y_0 y'(x_0)}{b^2} = 0$$

$$y'(x_0) = -\frac{b^2 x_0}{a^2 y_0}$$

$\downarrow$

$$y(x) = -\frac{b^2 x_0}{a^2 y_0}(x - x_0) + y_0$$

$$= -\frac{b^2 x_0 x}{a^2 y_0} + \frac{b^2 x_0^2}{a^2 y_0} + y_0$$

$$= -\frac{b^2 x_0 x}{a^2 y_0} + \frac{b^2}{y_0}\underbrace{\left(\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2}\right)}_{=1}$$

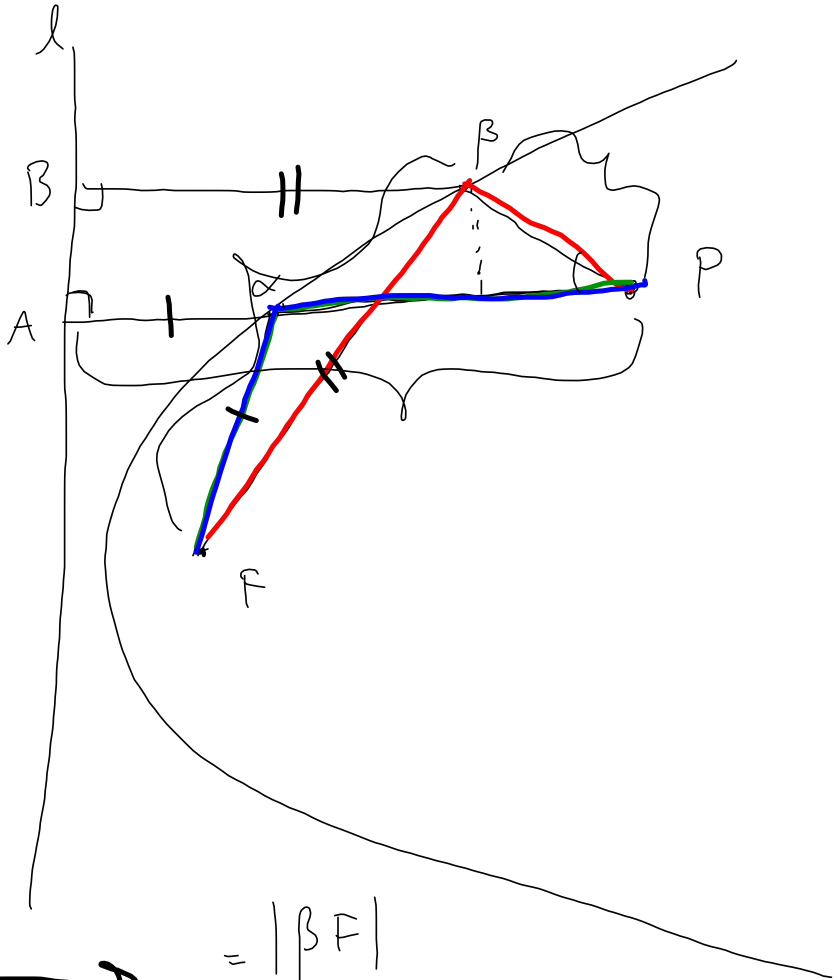
$$= -\frac{b^2 x_0 x}{a^2 y_0} + \frac{b^2}{y_0} \quad \Big/ \cdot \frac{y_0}{b^2}$$

$$\frac{y_0 \cdot y}{b^2} = -\frac{x_0 x}{a^2} + 1$$

$$\frac{y_0 \cdot y}{b^2} + \frac{x_0 x}{a^2} = 1$$

$\square$

$\boxed{3.6.10}$



$$|PA| \le |P\beta| + \overbrace{|\beta B|}^{=|\beta F|} = |P\beta| + |\beta F| =$$

$$\parallel$$

$$|P\alpha| + |\alpha A| = |P\alpha| + |\alpha F| =$$