

6.1:

1) g)  $\iint_R \frac{1}{1+x^2 y} dx dy ; R = [1, \sqrt{3}] \times [0, 1]$

$$\iint_R \frac{1}{1+x^2 y} dx dy = \int_1^{\sqrt{3}} \int_0^1 \frac{1}{1+x^2 y} dy dx$$

bitte  
int. reihenfolge

$$= \int_1^{\sqrt{3}} \left[ \frac{\ln(1+x^2 y)}{x^2} \right]_{y=0}^1 dx$$

$$= \int_1^{\sqrt{3}} \left( \frac{\ln(1+x^2)}{x^2} - \frac{\ln(1)}{x^2} \right) dx = \int_1^{\sqrt{3}} \frac{\ln(1+x^2)}{x^2} dx$$

$$= \left[ -\frac{\ln(1+x^2)}{x} \right]_{x=1}^{\sqrt{3}} + \int_1^{\sqrt{3}} \frac{2x}{(1+x^2)^2} dx$$

Devis int:  $\int u v' = [uv] - \int v u'$

$$u = \ln(1+x^2)$$

$$u' = \frac{1}{x^2}$$

$$= -\frac{\ln(4)}{\sqrt{3}} + \ln(2) + 2 [\arctan(x)]_{x=1}^{\sqrt{3}}$$

$$v = -\frac{1}{x} = -x^{-1}$$

$$u' = \frac{1}{1+x^2} \cdot 2x$$

$$= -\frac{2 \ln(2)}{\sqrt{3}} + \ln(2) + 2 \arctan(\sqrt{3}) - 2 \arctan(1)$$

$$= \left(1 - \frac{2\sqrt{3}}{3}\right) \ln(2) + \frac{2\pi}{3} - \frac{\pi}{2}$$

$$= \left(1 - \frac{2\sqrt{3}}{3}\right) \ln(2) + \frac{\pi}{6}$$

$$\tan\left(\frac{\pi}{4}\right) = 1$$

$$\arctan(1) = \frac{\pi}{4}$$

$$2 \arctan(1) = \frac{\pi}{2}$$

$$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\arctan(\sqrt{3}) = \frac{\pi}{3}$$

$$2 \arctan(\sqrt{3}) = \frac{2\pi}{3}$$

Roten um 0 halbe  
Roten um 1 halbe  
Roten um 2 halbe  
...

beachte!

|                            | 0 | $\frac{\pi}{6}$      | $\frac{\pi}{4}$      | $\frac{\pi}{3}$      | $\frac{\pi}{2}$ |
|----------------------------|---|----------------------|----------------------|----------------------|-----------------|
| sin                        | 0 | $\frac{1}{2}$        | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1               |
| cos                        | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$        | 0               |
| $\frac{\sin}{\cos} = \tan$ | 0 | $\frac{1}{\sqrt{3}}$ | 1                    | $\sqrt{3}$           | siehe def.      |