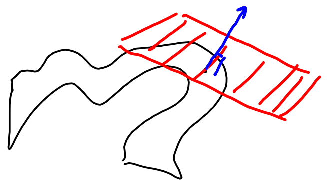
Nivåflater - normaler og tangenter



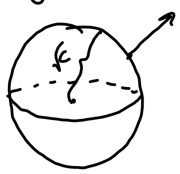
DEF: Anta at $f: A \to \mathbb{R}$ er en funksjon av n variabler, og velg $c \in \mathbb{R}$. Mengden

$$N_c := \{ \vec{x} \in A : f(x) = c \}$$

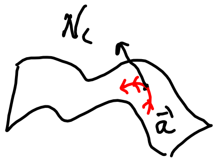
kalles en nivåflate for f.

Eks: $f(x,y,z) = x^2 + y^2 + z^2$.

N_c :



Normal: Fikser et punkt i N_c, kall det $\vec{a}$.



La $\vec{r}: (-\varepsilon, \varepsilon) \to N_c$ være en kurve med $\vec{r}(0) = \vec{a}$.

$\vec{r}\,'(0)$.

$$0 = (c)' = \big(f(r(t))\big)'(0) = \underline{\nabla f(\vec{a})} \cdot r'(0).$$

SETNING 3.7.2: La $f: A \subset \mathbb{R}^n \to \mathbb{R}$ være deriverbar og la $\vec{a} \in N_c$. Da står $\nabla f(\vec{a})$ vinkelrett på N_c i $\vec{a}$, i den forstand at dersom $\vec{r}: (-\varepsilon, \varepsilon) \to N_c$ er en deriverbar kurve med $\vec{r}(0) = \vec{a}$, da er $\nabla f(\vec{a}) \cdot \vec{r}\,'(0) = 0$.

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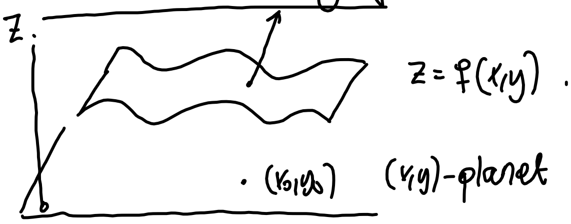
DEF: Tangentplanet til N_c i $\vec{a}$ er mengden av alle vektorer $\vec{x} \in \mathbb{R}^n$ s.a. $(\vec{x} - \vec{a}) \cdot \nabla f(\vec{a}) = 0$.

Normalen til en graf



Vi tenker på grafen som en nivåflate:

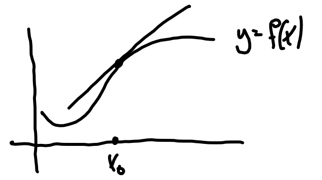
Definer $g(x,y,z) = z - f(x,y)$,

s.a. grafen $= N_0$

Normalen blir $\nabla g(x_0, y_0, f(x_0,y_0)) =$

$$\left(-\frac{\partial f}{\partial x}(x_0,y_0), -\frac{\partial f}{\partial y}(x_0,y_0), 1\right).$$

Tangentplanet: La $(x_0, y_0, z_0) \in N_0$.



$f(x_0) + f'(x_0)\cdot(x - x_0)$.

$$\left(-\frac{\partial f}{\partial x}(x_0,y_0), -\frac{\partial f}{\partial y}(x_0,y_0), 1\right) \cdot (x - x_0, y - y_0, z - z_0) = 0.$$

$$-\frac{\partial f}{\partial x}(x_0,y_0)\cdot(x - x_0) - \frac{\partial f}{\partial y}(x_0,y_0)\cdot(y - y_0) + z - z_0 = 0$$

$$z = \underset{\substack{\shortparallel \\ f(x_0,y_0)}}{z_0} + \frac{\partial f}{\partial x}(x_0,y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0,y_0)\cdot(y - y_0).$$

Lineariseringen til f i (x_0,y_0).

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EKS: $f(x,y) = x^2 - 2x + y^2 - 2y + 2 = (x-1)^2 + (y-1)^2$.

Finn normalen og tangentplanet til grafen til f i $(3/2, 3/2)$.

- $f(3/2, 3/2) = \frac{1}{2}$.
- $\frac{\partial f}{\partial x}(x,y) = 2(x-1)$ $\quad \frac{\partial f}{\partial x}(3/2, 3/2) = 1$.
- $\frac{\partial f}{\partial y}(x,y) = 2(y-1)$ $\quad \frac{\partial f}{\partial y}(3/2, 3/2) = 1$.

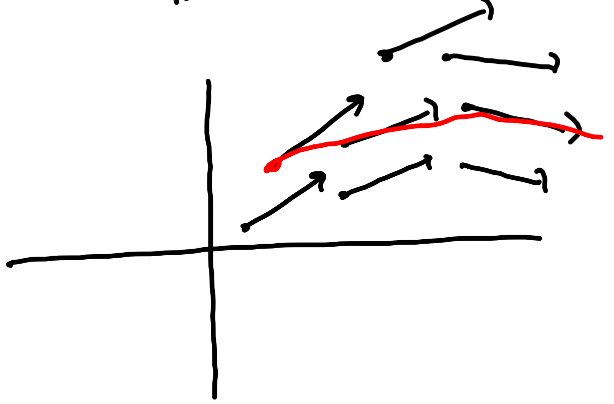
Normalen: $(-1, -1, 1)$.

Tangent:
$$z = f(3/2, 3/2) + \frac{\partial f}{\partial x}(3/2, 3/2) \cdot (x - 3/2) + \frac{\partial f}{\partial y}(3/2, 3/2) \cdot (y - 3/2)$$
$$= 1/2 + (x - 3/2) + (y - 3/2)$$
$$= -5/2 + x + y.$$

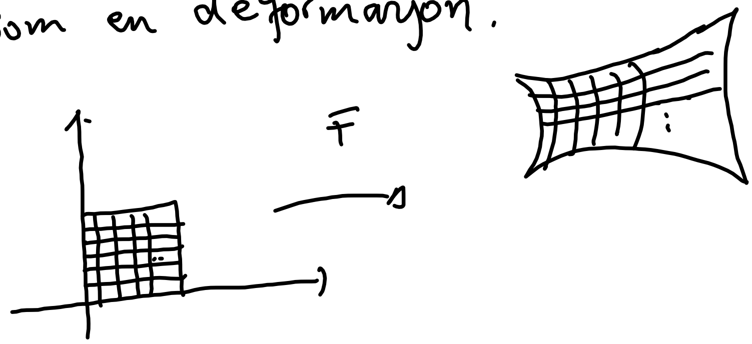
```
>> x=0:0.05:2;
>> y=x;
>> [x,y]=meshigrid(x,y);
Undefined function 'meshigrid' for input arguments of type 'double'.

Did you mean:
>> [x,y]=meshgrid(x,y);
>> mesh(x,y,(x-1).^2+ (y-1).^2)
>> hold on
>> mesh(x,y,-5/2+x+y)
>> quiver3(3/2,3/2,1/2,-1,-1,1)
>> axis equal
>>
```

Grafisk fremstilling av vektorfelter

Et vektorfelt på $\mathbb{R}^2$ e en avbildning $\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$.



Skal nå tenke på $F: \mathbb{R}^2 \to \mathbb{R}^2$ som en deformasjon.



Eks: $F(r,t) = (e^r \cdot \cos(t), e^r \cdot \sin(t))$.

$(z \mapsto e^z)$

```
>> r=linspace(0,1,30);
>> t=r;
>> f=inline('exp(r).*cos(t)')

f =

     Inline function:
     f(r,t) = exp(r).*cos(t)

>> g=inline('exp(r).*sin(t)')

g =

     Inline function:
     g(r,t) = exp(r).*sin(t)

>> for n=1:30
plot(f(r,t(n)),g(r,t(n)))
end
>> hold on
>> for n=1:30
plot(f(r,t(n)),g(r,t(n)))
end
>> for n=1:30
plot(f(r(n),t),g(r(n),t))
end
>>
```

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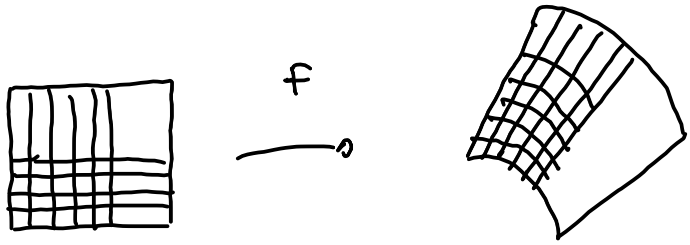


Merk: Jo mindre rutene er, jo mer kommer bildene av den til å se ut som rektangler.

$F(\vec{x}) = F(\vec{a}) + T_{\vec{a}}F(\vec{x}-\vec{a}) + \sigma(\vec{x}-\vec{a})$.

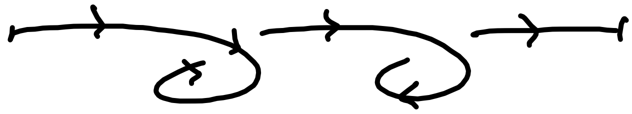
Nær $\vec{a}$ er det $T_{\vec{a}}F$ som "bestemmer".

- Husk at dersom $T: \mathbb{R}^2 \to \mathbb{R}^2$ er en lineæravbilding gitt ved $\vec{x} \mapsto A\vec{x}$, da skalerer T arealer med faktor $|\det A|$.

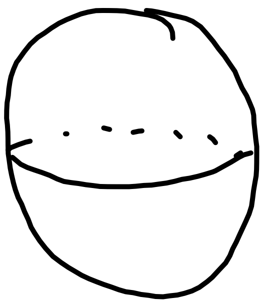
- Nær punktet $\vec{a}$ skaleres arealer nesten med en faktor lik determinanten til $T_{\vec{a}}F$.

$$T_{\vec{a}}F(\vec{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x}(\vec{a}) & \frac{\partial f_1}{\partial y}(\vec{a}) \\ \frac{\partial f_2}{\partial x}(\vec{a}) & \frac{\partial f_2}{\partial y}(\vec{a}) \end{bmatrix} \cdot \vec{x}.$$

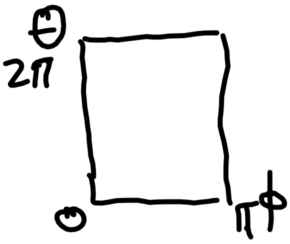
Parametriserte flater



$\vec{r} : [0,1] \longrightarrow \mathbb{R}^3$



$\longleftarrow (\sin\phi \cdot \cos\theta, \sin\phi \sin\theta, \cos\phi)$.



DEF: En parametrisert flate er en kontinuerlig avbildning $F : A \longrightarrow \mathbb{R}^n$ der A er et område i $\mathbb{R}^2$.