Mat 1110 — 8/2-17

| 3.1 : 7, 12 |
|---|
| 3.2 : 5 |
| 3.3 : 11, 12 |

(om vi rekker 2.8.2)

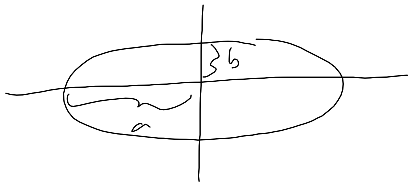
$\boxed{3.1.7}$ $\vec{r}(t) = (\underbrace{a\cos t}_{x}, \underbrace{b\sin t}_{y})$

$t \in [0, 2\pi]$

a) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$$\frac{(a\cdot\cos t)^2}{a^2} + \frac{(b\sin t)^2}{b^2} = \frac{\cancel{a^2}\cos^2 t}{\cancel{a^2}} + \frac{\cancel{b^2}\sin^2 t}{\cancel{b^2}}$$

$$= \cos^2 t + \sin^2 t = 1 \quad \text{OK}$$




b) $\vec{r}(t) = (a\cos t, b\sin t)$

$\vec{v}(t) = \vec{r}\,'(t) = (-a\sin t, b\cos t)$

$|\vec{v}(t)| = \sqrt{(-a\sin t)^2 + (b\cos t)^2} = \sqrt{b^2\cos^2 t + a^2\sin^2 t}$

$\vec{a}(t) = \vec{v}\,'(t) = (-a\cos t, -b\sin t)$

$\cancel{|\vec{a}(t)|} \neq v'(t) = a(t)$

c) $O = \displaystyle\int_0^{2\pi} \sqrt{a^2\sin^2 t + b^2\cos^2 t}\, dt$

$$s(2\pi) = \int_0^{2\pi} v(t)\, dt = \int_0^{2\pi} \underbrace{\sqrt{b^2\cos^2 t + a^2\sin^2 t}}\, dt$$

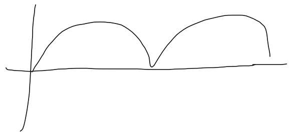
Set $a = 5$, $b = 3$

quad( @(t) sqrt(25.*sin(t).^2 + 9.*cos(t).^2), 0, 2*pi)

$= \underline{25.5270}$

$\boxed{3.1.12}$

$\vec{r}(t) = (rt - r\sin t,\ r - r\cos t)$



a) $\vec{v}(t) = \vec{r}\,'(t) = (r - r\cos t,\ r\sin t)$

$$v(t) = |\vec{v}(t)| = \sqrt{\underbrace{(r - r\cos t)^2}_{r^2(1-\cos t)^2} + (r\sin t)^2}$$

$$= r\sqrt{(1-\cos t)^2 + \sin^2 t}$$

$$= r\sqrt{1 - 2\cos t + \underbrace{\cos^2 t + \sin^2 t}_{1}}$$

$$= r\sqrt{2(1-\cos t)}$$

$$= \sqrt{2}\,r\sqrt{1-\cos t}$$

$\vec{a}(t) = \vec{v}\,'(t) = (r\sin t,\ r\cos t)$

b) $$S(2\pi) = \int_0^{2\pi} v(t)\,dt = r\sqrt{2}\int_0^{2\pi}\sqrt{1-\cos t}\,dt$$

c) Forklar: $\sqrt{1-\cos t} = \dfrac{|\sin t|}{\sqrt{1+\cos t}}$

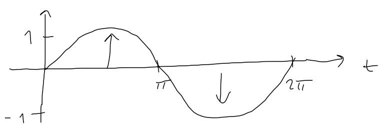
$$\sqrt{1-\cos t}\cdot\frac{\sqrt{1+\cos t}}{\sqrt{1+\cos t}} = \frac{\sqrt{1-\cos^2 t}}{\sqrt{1+\cos t}}$$

[Husk: $\cos^2 t + \sin^2 t = 1$
$\sin^2 t = \underbrace{1 - \cos^2 t}$ ]

$$= \frac{\sqrt{\sin^2 t}}{\sqrt{1+\cos t}} \leftarrow |\sin t|$$

$$= \frac{|\sin t|}{\sqrt{1+\cos t}}$$

d) $$r\sqrt{2}\int_0^{2\pi}\sqrt{1-\cos t}\,dt = r\sqrt{2}\int_0^{2\pi}\frac{|\sin t|}{\sqrt{1+\cos t}}\,dt$$




$$= r\sqrt{2}\left(\int_0^{\pi}\frac{\sin t}{\sqrt{1+\cos t}}\,dt - \int_{\pi}^{2\pi}\frac{\sin t}{\sqrt{1+\cos t}}\,dt\right)$$

$u = 1 + \cos t \rightarrow \dfrac{du}{dt} = -\sin t$

$du = -\sin t\,dt$

$$= r\sqrt{2}\left(-\int_{1+\cos 0}^{1+\cos\pi}\frac{1}{\sqrt{u}}\,du + \int_{1+\cos\pi}^{1+\cos 2\pi}\frac{1}{\sqrt{u}}\,du\right)$$

$\cos 0 = 1$
$\cos\pi = -1$
$\cos 2\pi = 1$

$$= r\sqrt{2}\left(\underbrace{-\int_2^0\frac{1}{\sqrt{u}}\,du}_{+\int_0^2\frac{1}{\sqrt{u}}\,du} + \int_0^2\frac{1}{\sqrt{u}}\,du\right)$$

$$= r\sqrt{2}\,2\int_0^2 u^{-\frac{1}{2}}\,du = r\sqrt{2}\,2\left[2\sqrt{u}\right]_0^2 = \underline{8r}$$

$\boxed{3.2.5}$

$$\vec{F}(x,y) = \begin{pmatrix} x^2 y \\ xy + x \end{pmatrix} \qquad \vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\vec{r}(t) = \sin t\,\vec{i} + \cos t\,\vec{j} \qquad \vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= (\underset{r_1}{\sin t}, \underset{r_2}{\cos t})$$

$$\vec{G}'(t) = \vec{F}'(\vec{r}(t)) = \vec{F}'(\vec{r}(t)) \cdot \vec{r}\,'(t)$$

$$\vec{G}'(t) = \frac{\partial \vec{F}}{\partial x} \cdot r_1' + \frac{\partial \vec{F}}{\partial y} \cdot r_2'$$

$$\left[ \frac{\partial \vec{F}}{\partial x} = \begin{pmatrix} 2xy \\ y+1 \end{pmatrix}, \quad \frac{\partial \vec{F}}{\partial y} = \begin{pmatrix} x^2 \\ x \end{pmatrix} \right.$$

$$\left. r_1' = \cos t, \quad r_2' = -\sin t \right]$$

$$\vec{G}'(t) = \begin{pmatrix} 2\sin t \cos t \\ \cos t + 1 \end{pmatrix} \cdot \cos t + \begin{pmatrix} \sin^2 t \\ \sin t \end{pmatrix} \cdot (-\sin t)$$

$$= \begin{pmatrix} 2\sin t \cos^2 t \\ \cos^2 t + \cos t \end{pmatrix} + \begin{pmatrix} -\sin^3 t \\ -\sin^2 t \end{pmatrix}$$

$$= \begin{pmatrix} \sin t(2\cos^2 t - \sin^2 t) \\ \cos^2 t + \cos t - \sin^2 t \end{pmatrix}$$

$\boxed{3.3.11}$ $\vec{r}(t) = \left( \frac{t^2}{2}, \frac{2\sqrt{2}}{9} t^{\frac{3}{2}}, \underbrace{\frac{t}{9}}_{z(t)} \right), 1 \le t \le 7$

$f(\vec{r}(t)) = \frac{1}{15} + \frac{1}{2} \frac{dz}{ds}$ (s buelengden)

$$\int_1^7 f(\vec{r}(t)) \cdot v(t)\, dt$$

$\vec{v}(t) = \left( t, \frac{\sqrt{2}}{3}\sqrt{t}, \frac{1}{9} \right)$

$\frac{ds}{dt} = v(t) = |\vec{v}(t)| = \sqrt{\underbrace{t^2 + \frac{2}{9}t + \frac{1}{81}}_{(t+\frac{1}{9})^2 = \left(\frac{9t+1}{9}\right)^2}}$

$= \frac{9t+1}{9}$

$\frac{dz}{dt} = \frac{dz}{ds} \cdot \frac{ds}{dt}$

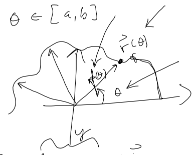
$\frac{1}{\cancel{9}} = \frac{dz}{ds} \cdot \frac{9t+1}{\cancel{9}} \rightarrow \frac{dz}{ds} = \frac{1}{9t+1}$

$$\int_1^7 f(\vec{r}(t)) \cdot v(t)\, dt = \int_1^7 \left( \frac{1}{15} + \frac{1}{2}\frac{1}{9t+1} \right)\left( \frac{9t+1}{9} \right) dt$$

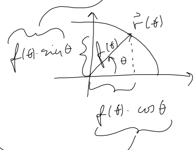
$$= \int_1^7 \frac{9t+1}{15 \cdot 9} + \frac{1}{18}\, dt = \int_1^7 \frac{1}{15}t + \left( \frac{1}{135} + \frac{1}{18} \right) dt$$

$$= \left[ \frac{1}{30}t^2 + \left( \frac{1}{135} + \frac{1}{18} \right) t \right]_1^7 = \underline{\underline{\frac{89}{45}}}$$

$\boxed{3.3.12}$ $r = f(\theta)$ , $\theta \in [a,b]$




a) $\vec{r}(\theta) = \overbrace{f(\theta)\cos\theta}^{x}\,\vec{i} + \overbrace{f(\theta)\sin(\theta)}^{y}\,\vec{j}$




b) VS: $v(\theta) = \sqrt{f(\theta)^2 + f'(\theta)^2}$

$$\vec{v}(\theta) = \vec{r}\,'(\theta) = \left(f'(\theta)\cdot\cos\theta - f(\theta)\cdot\sin\theta\right)\vec{i} + \left(f'(\theta)\cdot\sin\theta + f(\theta)\cdot\cos\theta\right)\vec{j}$$

$$v(\theta) = |\vec{v}(\theta)|$$

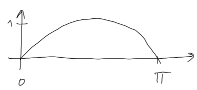
$$= \sqrt{\left(f'(\theta)\cos\theta - f(\theta)\sin\theta\right)^2 + \left(f'(\theta)\sin\theta + f(\theta)\cos\theta\right)^2}$$

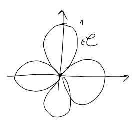
$$= \left(\left(f'(\theta)\cos\theta\right)^2 + \left(f(\theta)\sin\theta\right)^2 + \left(f'(\theta)\sin\theta\right)^2 + \left(f(\theta)\cos\theta\right)^2\right)^{\frac{1}{2}}$$

$$= \left(f'(\theta)^2\underbrace{\left(\cos^2\theta + \sin^2\theta\right)}_{1} + f(\theta)^2\underbrace{\left(\cos^2\theta + \sin^2\theta\right)}_{1}\right)^{\frac{1}{2}}$$

$$= \sqrt{f'(\theta)^2 + f(\theta)^2}$$

c) $f(\theta) = \sin\theta$ , $\theta \in [0,\pi]$



$$s(\pi) = \int_0^\pi v(\theta)\,d\theta$$

$$= \int_0^\pi \sqrt{\underbrace{\cos^2\theta + \sin^2\theta}_{1}}\,d\theta = \int_0^\pi d\theta = \underline{\underline{\pi}}$$

d) $\int_C g\,ds$ , $C$ fra (c)

$g(x,y) = x\cdot y$

$$\vec{r}(\theta) = (\cos\theta\sin\theta,\ \sin^2\theta)$$

$$\int_C g\,ds = \int_0^\pi g(\vec{r}(\theta))\cdot v(\theta)\,d\theta$$

$$= \int_0^\pi (\cos\theta\sin\theta)\cdot(\sin^2\theta)\,d\theta$$

$$= \int_0^\pi \cos\theta\cdot\sin^3\theta\,d\theta$$

$$\begin{bmatrix} u = \sin\theta \\ du = \cos\theta\,d\theta \end{bmatrix} = \int_0^0 u^3\,du = \underline{\underline{0}}$$