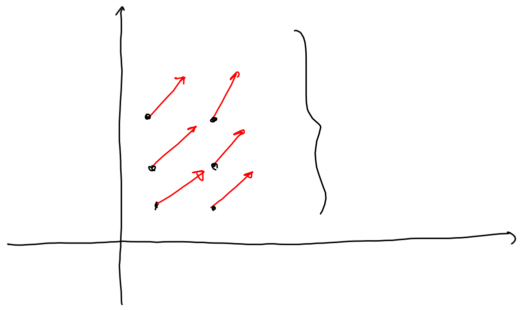
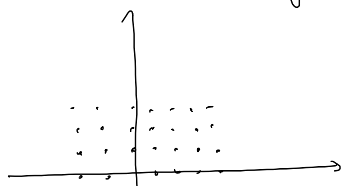
Grafiske fremstillinger av vektorfelt (3.8)

Vektorfelt: $\vec{F}: \mathbb{R}^n \to \mathbb{R}^n$, $\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$, $\vec{F}: \mathbb{R}^3 \to \mathbb{R}^3$

$\vec{F}(x,y)$ = vind i (x,y)



MATLAB: $\vec{F}(x,y) = xy\,\vec{i} + e^{x+y}\vec{j}$



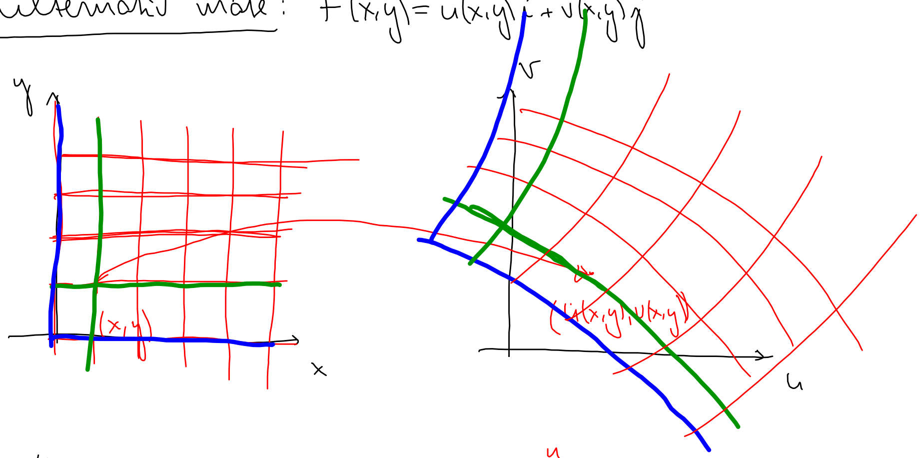
x = -5:0.5:5   u = ∼∼   (første komponenten til F)

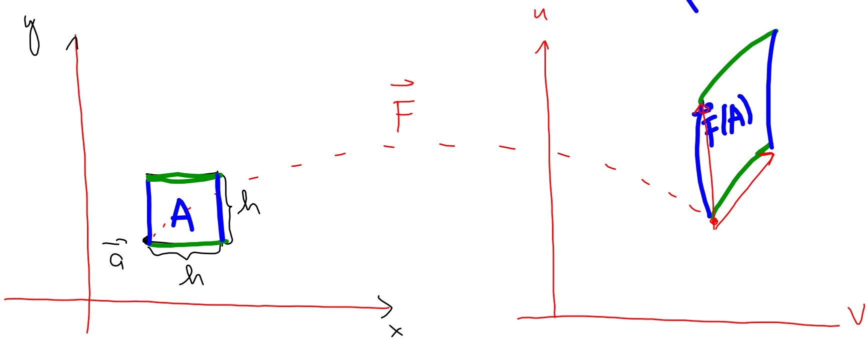
y = -5:0.5:5   v = ∼∼   (annen — " —)

[x,y] = meshgrid(x,y)   quiver(x,y,u,v)   u = x.*y

v = exp(x+y)

Alternativ måte: $\vec{F}(x,y) = u(x,y)\vec{i} + v(x,y)\vec{j}$





Hva skjer med arealet: Hvordan er areal($\vec{F}(A)$) sammenlignet med areal(A)?

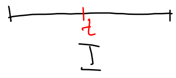
Svar: Hvis h er liten, vil

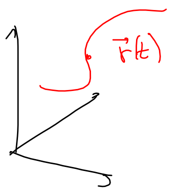
$$\text{areal}(\vec{F}(A)) \approx |\det(\vec{F}'(a))|\ \text{areal}(A)$$

Moral: $|\det \vec{F}'(a)|$ er arealforstørrelsesfaktoren i området rundt $\vec{a}$

Parametriserte flater (3.9)

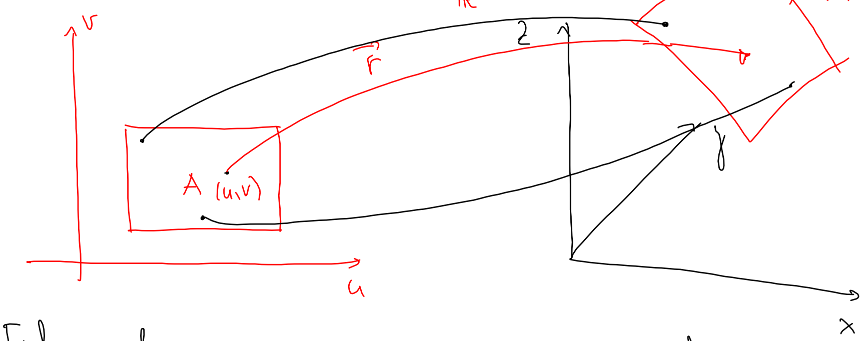
Parametrisert kurve: $\vec{r} : I \to \mathbb{R}^3$, $I \subset \mathbb{R}$



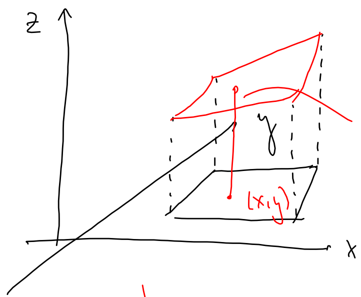


Parametrisert flate: $\vec{r} : A \to \mathbb{R}^3$, $A \subset \mathbb{R}^2$

$\vec{r}(u,v) = X(u,v)\vec{i} + Y(u,v)\vec{j} + Z(u,v)\vec{k}$



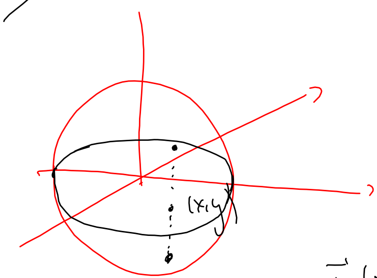
Eksempel:



$z = f(x,y)$

$\vec{r}(x,y) = x\vec{i} + y\vec{j} + f(x,y)\vec{k}$

Trøttel.



$x^2 + y^2 + z^2 = R^2$

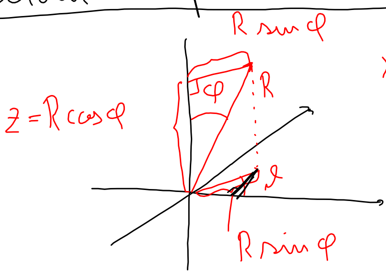
$z^2 = R^2 - x^2 - y^2$

$z = \pm\sqrt{R^2 - x^2 - y^2}$

$\vec{r}(x,y) = x\vec{i} + y\vec{j} + \sqrt{R^2 - x^2 - y^2}\,\vec{k}$

(øvre halvkule)

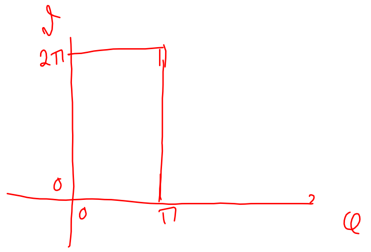
Alternativ parametrisering

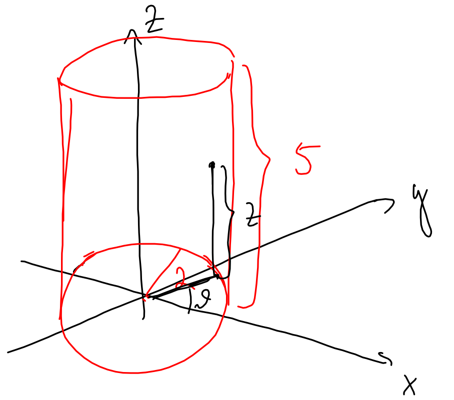


$x = R\sin\varphi\cos\theta \qquad y = R\sin\varphi\sin\theta$

$\vec{r}(\varphi,\theta) = R\sin\varphi\cos\theta\,\vec{i} + R\sin\varphi\sin\theta\,\vec{j} + R\cos\varphi\,\vec{k}$

$0 \le \varphi \le \pi, \quad 0 \le \theta \le 2\pi$



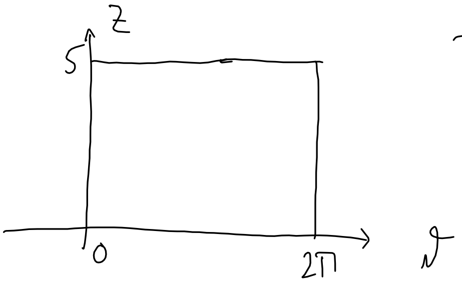


Parametrisering:

$x = 2\cos\vartheta \qquad 0 \le \vartheta \le 2\pi$

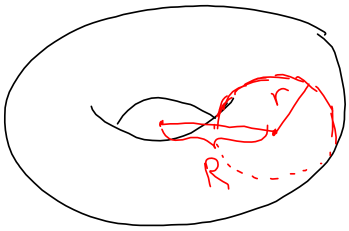
$y = 2\sin\vartheta \qquad 0 \le z \le 5$

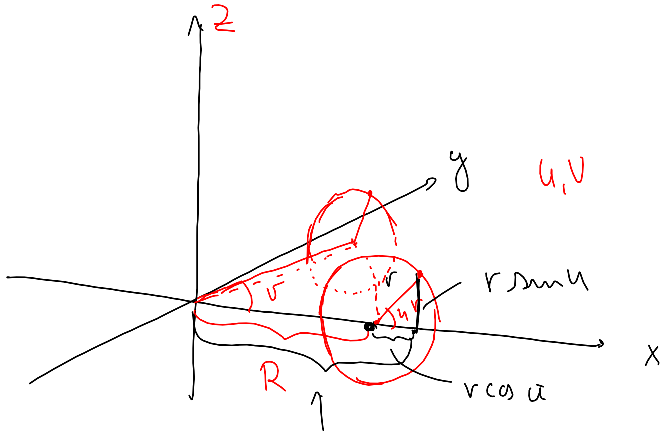
$z = z$



$$\vec{r}(\vartheta, z) = 2\cos\vartheta\,\vec{i} + 2\sin\vartheta\,\vec{j} + z\,\vec{k}$$

Torus (overflaten til en smultring)



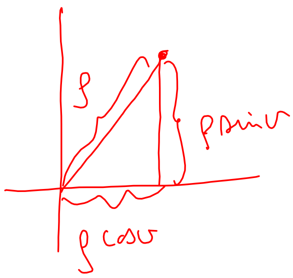


$$\underbrace{R + r\cos u}_{\rho}$$

$z(u,v) = r\sin u$

$x(u,v) = (R + r\cos u)\cos v$

$y(u,v) = (R + r\cos u)\sin v$



$$\vec{r}(u,v) = (R + r\cos u)\cos v\,\vec{i} + (R + r\cos u)\sin v\,\vec{j} + r\sin u\,\vec{k}$$

$0 \le u \le 2\pi$

$0 \le v \le 2\pi$