4.2

2

d) $\begin{pmatrix} 1 & 2 & 1 & 1 \\ -1 & 3 & -1 & 2 \\ 1 & 12 & 1 & 7 \end{pmatrix}$ (-1)

$\sim \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 5 & 0 & 3 \\ 0 & 10 & 0 & 6 \end{pmatrix}$ (-2) $\sim \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 5 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\frac{1}{5}$

$\sim \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & \frac{3}{5} \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (på trappeform)

4.2.9

a)

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 & 5 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & -2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 7 \end{pmatrix} \begin{array}{c} -1 \\ \\ \\ \\ \end{array}$$

$$\sim \begin{pmatrix} 1 & 2 & 0 & 2 & 5 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix} \begin{array}{c} \\ 2 \\ \\ \\ \end{array} \sim \begin{pmatrix} 1 & 2 & 0 & 2 & 5 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 3 & 3 & 6 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 0 & 2 & 5 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 3 & 3 & 6 \end{pmatrix} \begin{array}{c} \\ \\ -3 \\ \\ \end{array} \sim \begin{pmatrix} 1 & 2 & 0 & 2 & 5 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ (trappeform)

b) Skal løse likningssystem som har A som utvidet matrise

Likningssystemet er ekvivalent med

$$\begin{aligned} x + 2y \quad + 2u &= 5 \\ y + z + u &= 3 \\ z + u &= 2 \end{aligned}$$

u kan velges fritt

$$\begin{aligned} z &= 2 - u \\ y &= 3 - z - u = 1 \\ x &= 5 - 2y - 2u = 3 - 2u \end{aligned}$$

4.2.10

A, B, C tre byer

Firma leier ut 120 biler

har kontor i A, B og C

$$A \xrightarrow{60\%} A \qquad A \xrightarrow{30\%} B \qquad A \xrightarrow{10\%} C$$

$$B \xrightarrow{30\%} A \qquad B \xrightarrow{50\%} B \qquad B \xrightarrow{20\%} C$$

$$C \xrightarrow{60\%} A \qquad C \xrightarrow{10\%} B \qquad C \xrightarrow{30\%} C$$

x_0 antall biler i A ved et tidspunkt

y_0 —"— i B $t = 0$

z_0 —"— i C

x_1 antall biler i A ved tiden $t = 1$

y_1 —"— i B

z_1 —"— i C

Ønsker at $\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \cdots$

(fordelingen skal være stabil)

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 0{,}6 & 0{,}3 & 0{,}6 \\ 0{,}3 & 0{,}5 & 0{,}1 \\ 0{,}1 & 0{,}2 & 0{,}3 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{Må løse}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0{,}6 & 0{,}3 & 0{,}6 \\ 0{,}3 & 0{,}5 & 0{,}1 \\ 0{,}1 & 0{,}2 & 0{,}3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Samme som å løse:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 0{,}6 & 0{,}3 & 0{,}6 \\ 0{,}3 & 0{,}5 & 0{,}1 \\ 0{,}1 & 0{,}2 & 0{,}3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0{,}4 & -0{,}3 & -0{,}6 \\ -0{,}3 & 0{,}5 & -0{,}1 \\ -0{,}1 & -0{,}2 & 0{,}7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -3 & -6 \\ -3 & 5 & -1 \\ -1 & -2 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

4.2.10 forts.

$$\begin{pmatrix} 4 & -3 & -6 \\ -3 & 5 & -1 \\ -1 & -2 & 7 \end{pmatrix} \sim \begin{pmatrix} -1 & -2 & 7 \\ -3 & 5 & -1 \\ 4 & -3 & -6 \end{pmatrix} (-1)$$

$$\begin{pmatrix} 1 & 2 & -7 \\ -3 & 5 & -1 \\ 4 & -3 & -6 \end{pmatrix} \begin{matrix} 3 & -4 \\ \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 2 & -7 \\ 0 & 11 & -22 \\ 0 & -11 & 22 \end{pmatrix} \begin{matrix} \\ (+1) \\ \\ \end{matrix}$$

$$\begin{pmatrix} 1 & 2 & -7 \\ 0 & 11 & -22 \\ 0 & 0 & 0 \end{pmatrix} \frac{1}{11} \sim \begin{pmatrix} 1 & 2 & -7 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

Dvs jeg får $\quad x + 2y - 7z = 0$

$$y - 2z = 0$$

z kan velges fritt $\quad y = 2z$,

$$x = 7z - 2y = 3z$$

Må ha $\quad x + y + z = 3z + 2z + z = 6z = 120$

$z = 20, \; y = 40, \; x = 60$

fordeling i C, B og A.

4.3

2c)

$$C = \begin{array}{cc} \begin{pmatrix} 1 & 1 & 1 & 2 \\ -1 & 2 & -1 & 3 \\ 2 & 2 & 1 & 1 \end{pmatrix} & \begin{array}{c} -2 \\ \\ \\ \end{array} \end{array} \sim \begin{array}{cc} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 3 & 0 & 5 \\ 0 & 0 & -1 & -3 \end{pmatrix} & \begin{array}{c} \\ \frac{1}{3} \\ (-1) \end{array} \end{array}$$

$$\sim \begin{array}{cc} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & \frac{5}{3} \\ 0 & 0 & 1 & 3 \end{pmatrix} & \begin{array}{c} \\ \\ -1 \end{array} \end{array} \sim \begin{array}{cc} \begin{pmatrix} 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & 3 \end{pmatrix} & \begin{array}{c} \\ -1 \\ \\ \end{array} \end{array}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & -8/3 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & 3 \end{pmatrix} \quad \text{(Redusert trappeform)}$$

4.3.4

6) $$\begin{aligned} 2x - y + z + 3u &= -4 \\ -x + 2y + 4z + 3u &= 2 \\ -2x + y + 3z - 4u &= -1 \end{aligned}$$

$$A = \begin{bmatrix} 2 & -1 & 1 & 3 & -4 \\ -1 & 2 & 4 & 3 & 2 \\ -2 & 1 & 3 & -4 & -1 \end{bmatrix}$$

$$B = \mathrm{rref}(A) \overset{\text{Matlab}}{=}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3.5 & 0.5 \\ 0 & 1 & 0 & 3.75 & 3.75 \\ 0 & 0 & 1 & -0.25 & -1.25 \end{bmatrix}$$

$$\begin{aligned} x \quad + \tfrac{7}{2}u &= \tfrac{1}{2} \\ y \quad + \tfrac{15}{4}u &= \tfrac{15}{4} \\ z \quad - \tfrac{1}{4}u &= -\tfrac{5}{4} \end{aligned}$$

u kan velges fritt

$z = -\frac{5}{4} + \frac{1}{4}u, \; y = \frac{15}{4} - \frac{15}{4}u, \; x = \frac{1}{2} - \frac{7}{2}u$

4.4

1 d) $A\vec{x} = \vec{b}$

Här $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$

$$\left(\begin{array}{cccc} 1 & -2 & 3 & 1 \\ 2 & -1 & 3 & 0 \\ 1 & 0 & 1 & -2 \end{array}\right) \begin{array}{c} -2 \quad -1 \\ \\ \\ \end{array} \sim \left(\begin{array}{cccc} 1 & -2 & 3 & 1 \\ 0 & 3 & -3 & -2 \\ 0 & 2 & -2 & -3 \end{array}\right) \begin{array}{c} \\ -\frac{2}{3} \\ \\ \end{array}$$

$$\left(\begin{array}{cccc} 1 & -2 & 3 & 1 \\ 0 & 3 & -3 & -2 \\ 0 & 0 & 0 & -\frac{5}{3} \end{array}\right)$$

Sista likning blir då

$0 \cdot z = -\frac{5}{3}$

dvs $0 = -\frac{5}{3}$

Likningssystemet har ingen lösning.

4.4.4

$$A = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & -1 & 1 \\ 6 & 0 & -6 & 7 \\ 2 & 1 & 0 & 0 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 0 \\ 1 \\ h \\ 0 \end{pmatrix}$$

a) Skal bringe A på trappeform

b) Skal avgjøre når $A x = \vec{b}$ har løsning og finne løsning

$$C = \begin{pmatrix} 0 & 1 & 2 & 0 & 0 \\ 1 & 0 & -1 & 1 & 1 \\ 6 & 0 & -6 & 7 & h \\ 2 & 1 & 0 & 0 & 0 \end{pmatrix} \sim \begin{array}{c} \begin{pmatrix} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 6 & 0 & -6 & 7 & h \\ 2 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{l} -6 \; -2 \\ \\ \\ \\ \end{array}$$

$$\sim \begin{pmatrix} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & h-6 \\ 0 & 1 & 2 & -2 & -2 \end{pmatrix} \begin{array}{l} \\ -1 \\ \\ \\ \end{array} \sim \begin{pmatrix} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & h-6 \\ 0 & 0 & 0 & -2 & -2 \end{pmatrix} \begin{array}{l} \\ \\ 2 \\ \\ \end{array}$$

$$\begin{pmatrix} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & h-6 \\ 0 & 0 & 0 & 0 & 2h-14 \end{pmatrix}$$

En trappeform for A

blir da
$$\begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
(ikke redusert)

4.4.4 forts.

Likningsystemet er ekvivalent med

$$\begin{aligned} x \qquad - z + u &= 1 \\ y + 2z \qquad &= 0 \\ u &= h-6 \\ 0 &= 2h-14 \end{aligned}$$

Må ha $2h-14=0$, dvs $h=7$

får da $u=1$, $y=-2z$, $x=z-u+1=z$

z kan velges fritt.

4.4.5

$$C = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & a^2-a & 3 \\ -1 & 1 & -3 & a \end{pmatrix}\begin{matrix} -2 \quad 1 \\ \\ \\ \end{matrix}$$

Skal få denne på trappeform

$$\sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 1 & -2 & a+1 \end{pmatrix}\begin{matrix} \\ -1 \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 0 & a-a^2 & a \end{pmatrix}\begin{matrix} \\ \\ \frac{1}{a-a^2} \end{matrix}$$

Hvis $a-a^2 \neq 0$ dvs $a \neq 0$ og $a \neq 1$

$$C \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 0 & 1 & \frac{1}{1-a} \end{pmatrix} \quad (\text{trappeform})$$

Hvis $a = 0$ $\quad C \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Hvis $a = 1$ $\quad C \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

4.4.5 forts.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & a^2-a \\ -1 & 1 & -3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ 3 \\ a \end{pmatrix}$$

Når har $A\vec{x} = \vec{b}$

én, ∞, eller ingen løsninger

Vi ser at $A\vec{x} = \vec{b}$ har utvidet matrise

C fra pkt. a)

Hadde $C \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 0 & a^2-a & a \end{pmatrix}$

Dvs. likningssystemet har denne matrisen som utvidet matrise

Hvis $a \neq 0$ og $a \neq 1$

$A \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & a^2-a-2 \\ 0 & 0 & 1 \end{pmatrix}$ Har pivot i hver rad og kolonne. Entydig løsning.

$(A \sim I_3)$

Hvis $a=0$ $C \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Gir systemet
$$\begin{aligned} x \quad + z &= 1 \\ y - 2z &= 1 \end{aligned}$$

Ser at z kan velges fritt, ∞-mange løsn.

Hvis $a=1$

$C \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ Siste likning $0=1$ ingen løsning.