1.9.8

$$\begin{pmatrix}0\\1\end{pmatrix} \longrightarrow \begin{pmatrix}0\\-1\end{pmatrix} \longrightarrow \begin{pmatrix}\sin\theta\\-\cos\theta\end{pmatrix}$$

$$\begin{pmatrix}1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}$$

matrisen til T: $\underline{\underline{\begin{pmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{pmatrix}}}$ $\begin{pmatrix}T(\vec{e}_1) & T(\vec{e}_2)\end{pmatrix}$

1.9.9

$$A_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$A_{-\theta}$: roterer $-\theta$ grader. Roterer vi først med θ grader, og deretter $-\theta$, kommer vi tilbake til startpunktet, slik at $A_{-\theta}A_\theta\vec{x} = \vec{x}$, for alle $\vec{x}$, slik at $A_{-\theta}A_\theta = I_2$. Ved regning:

$$A_{-\theta}A_\theta = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos^2\theta + \sin^2\theta & -\cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

1.9.14

$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

a) $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad A\vec{v}_1 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1+2 \\ 2+1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= 3\vec{v}_1$

Derfor er $\vec{v}_1$ egenvektor med $\lambda_1 = 3$ som tilhørende egenverdi.

b) $\vec{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\quad A\vec{v}_2 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1-2 \\ 2-1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} = -1\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$= -\vec{v}_2$

Derfor er $\vec{v}_2$ egenvektor med $\lambda_2 = -1$ som tilhørende egenverdi.

c) $\vec{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\quad$ Vi skal finne x, y slik at $\vec{a} = x\vec{v}_1 + y\vec{v}_2$

c) $\vec{a} = x\vec{v_1} + y\vec{v_2}$

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} = x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$3 = x + y$ (legg sammen)

$-1 = x - y$ $\Rightarrow 2x = 2 \Rightarrow x = 1$

(trekk fra)

$\Rightarrow 3 + 1 = 2y \Rightarrow y = 2$

$\Rightarrow x = 1,\ y = 2$

$= \vec{a} = \vec{v_1} + 2\vec{v_2}$

$$A^{10}\vec{a} = A^{10}(\vec{v_1} + 2\vec{v_2}) = A^{10}\vec{v_1} + 2A^{10}\vec{v_2}$$

$$= 3^{10}\vec{v_1} + 2(-1)^{10}\vec{v_2} = 3^{10}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 2(-1)^{10}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

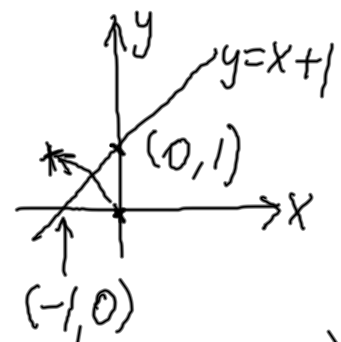
$$= \underline{\underline{\begin{pmatrix} 3^{10} + 2 \\ 3^{10} - 2 \end{pmatrix}}}$$

1.10.6

$F(0,0) = (-1,1)$

$F(0,1) = (0,1)$

$F(-1,0) = (-1,0)$



F kan skrives

$$F(x,y) = A\begin{pmatrix} x \\ y \end{pmatrix} + \vec{c} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$(0,0) \to (-1,1)$

$$F(0,0) = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \Rightarrow F(x,y) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$F(0,1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Longrightarrow \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

1.10.8

Et generelt plan: $ax + by + cz = d$

$$z = -\frac{a}{c}x - \frac{b}{c}y + \frac{d}{c} = \underbrace{\begin{pmatrix} -\frac{a}{c} & -\frac{b}{c} \end{pmatrix}}_{\substack{1\times 2 \\ A}} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{2\times 1} + \underbrace{\frac{d}{c}}_{\substack{1\times 1 \\ \vec{c}}}$$

slik at grafen til et plan kan skrives som en affin avbildning.

Omvendt: En affin avbildning fra $\mathbb{R}^2$ til $\mathbb{R}$ kan skrives

$$F(x,y) = \begin{pmatrix} a_{11} & a_{12} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + c$$

$$= a_{11}x + a_{12}y + c,$$

som er likningen for et plan.