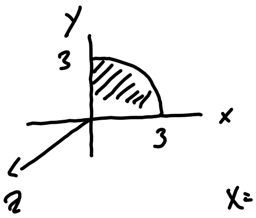
6.10.1 a) $I = \iiint_A x \, dx\,dy\,dz$, $x, y \geq 0$, $x^2+y^2 \leq 9$, $0 \leq z \leq 2$.



Sylinderkoord:

$x = r\cos t$, $y = r\sin t$, $z = z$.

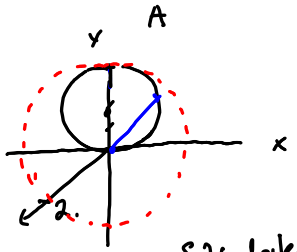
$$I = \int_0^{\pi/2}\int_0^3\int_0^2 r^2\cos t \, dz\,dr\,dt$$

$$= 2\int_0^{\pi/2}\int_0^3 r^2\cos t \, dr\,dt$$

$$= 2\int_0^{\pi/2} \left[\tfrac{1}{3}r^3 \cdot \cos t\right]_0^3 dt$$

$$= 2\cdot 9 \cdot \int_0^{\pi/2} \cos t \, dt = \underline{\underline{18}}$$

(c) $\iiint_A z\sqrt{x^2+y^2} \, dx\,dy\,dz$, $x^2+(y-1)^2 \leq 1$, $0 \leq z \leq 2$.



(Prøvde først: $x = r\cos t$, $y = 1 + r\sin t$, $z = z$)

Sylinderkoord: $x = r\cdot\cos t$
$y = r\cdot\sin t$
$z = z$.

$0 \leq t \leq \pi$

Må finne grensene til r: Se for hvilken r vi har at $(r\cos t, r\sin t)$ ligger på sirkelen

$$x^2+(y-1)^2 = 1.$$

$$r^2\cos^2 t + (r\sin t - 1)^2 = 1$$

$$\underbrace{r^2\cos^2 t + r^2\sin^2 t}_{= r^2} - 2r\sin t + \cancel{1} = \cancel{1}$$

$$r^2 = 2r\sin t$$

$$\underline{r = 2\sin t}$$

Så: $0 \leq r \leq 2\sin t$.

apr 9-10:16

$$I = \iiint_A z\sqrt{x^2+y^2}\,dx\,dy\,dz.$$

$$I = \int_0^{\pi}\int_0^{2\sin t}\int_0^{2} z\cdot r^2\,dz\,dr\,dt.$$

$$= \int_0^{\pi}\int_0^{2\sin t}\left[\tfrac{1}{2}z^2 r^2\right]_0^2 dr\,dt$$

$$= 2\int_0^{\pi}\int_0^{2\sin t} r^2\,dr\,dt$$

$$= 2\int_0^{\pi}\left[\tfrac{1}{3}r^3\right]_0^{2\sin t} dt$$

$$= 2\int_0^{\pi}\tfrac{1}{3}\cdot 8\sin^3 t\,dt$$

$$= \frac{16}{3}\int_0^{\pi}\sin^3 t\,dt.$$

$$= \frac{16}{3}\int_0^{\pi}\sin t\,(1-\cos^2 t)\,dt$$

$$= \frac{16}{3}\int_0^{\pi}\sin t - \sin t\cdot\cos^2 t\,dt.$$

$$= \frac{16}{3}\cdot\left[-\cos t + \tfrac{1}{3}\cos^3 t\right]_0^{\pi}$$

$$= \frac{16}{3}\cdot\left[1-\tfrac{1}{3}+1-\tfrac{1}{3}\right] = \underline{\underline{\frac{16}{3}\cdot\frac{4}{3}}}.$$

② (a) $I = \iiint_A (x^2+y^2)\,dx\,dy\,dz$, A e kula med radius 1.

<u>Kulekoordinater:</u>

$$x = \rho \sin\phi \cdot \cos\theta$$
$$y = \rho \sin\phi \cdot \sin\theta$$
$$z = \rho \cdot \cos\theta$$

$$0 \le \rho \le 1, \quad 0 \le \phi \le \pi, \quad 0 \le \theta \le 2\pi.$$

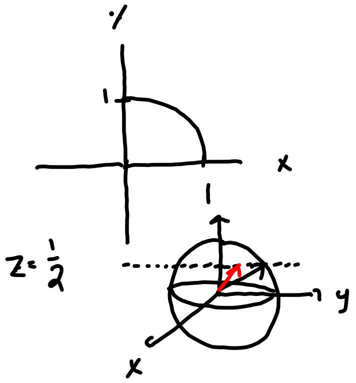
$$I = \int_0^1 \int_0^\pi \int_0^{2\pi} \left(\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2\theta\right) \cdot \rho^2 \sin\phi \; d\theta\, d\phi\, d\rho$$

$$= \int_0^1 \int_0^\pi \int_0^{2\pi} \rho^4 \sin^3\phi \cdot d\theta\, d\phi\, d\rho$$

$$= 2\pi \int_0^1 \int_0^\pi \rho^4 \sin^3\phi \; d\phi\, d\rho$$

$$= \frac{2\pi}{5} \int_0^\pi \sin^3\phi \; d\phi = \underline{\underline{\frac{2\pi}{5} \cdot \frac{4}{3}}}.$$

6.10 2 b) $I = \iiint_A x \, dx\,dy\,dz$, $x,y \geq 0$, $z \geq 1/2$, $x^2+y^2+z^2 \leq 1$.



$0 \leq \theta \leq \frac{\pi}{2}$.

$0 \leq \phi \leq \frac{\pi}{3}$

$z \geq \frac{1}{2}$ gir

$\rho \cdot \cos\phi \geq \frac{1}{2}$

$\rho \geq \frac{1}{2\cos\phi}$

$x=\rho \sin\phi \cos\theta$

$y=\rho\cdot \sin\phi \sin\theta$

$z=\rho\cdot\cos\theta$

$\frac{1}{2\cos\phi} \leq \rho \leq 1$.

$$I = \int_0^{\pi/3}\int_0^{\frac{\pi}{2}}\int_{\frac{1}{2\cos\phi}}^{1} \rho\cdot\sin\phi\cdot\cos\theta\cdot\rho^2\sin\phi \, d\rho\, d\theta\, d\phi$$

$$\int_0^{\pi/3}\int_0^{\pi/2}\int_{\frac{1}{2\cos\phi}}^{1} \rho^3\sin^2\phi\cdot \underline{\cos\theta}\, d\rho\, d\theta\, d\phi.$$

$$\int_0^{\pi/3}\int_{\frac{1}{2\cos\phi}}^{1} \rho^3\sin^2\phi \left[\sin\theta\right]_0^{\pi/2} d\rho\, d\phi$$

$$= \int_0^{\pi/3}\int_{\frac{1}{2\cos\phi}}^{1} \rho^3\sin^2\phi \, d\rho\, d\phi$$

$$= \frac{1}{4}\int_0^{\pi/3} \left[\rho^4\right]_{\frac{1}{2\cos\phi}}^{1} \sin^2\phi \, d\phi$$

$$= \frac{1}{4}\int_0^{\pi/3} \sin^2\phi - \left(\frac{1}{2\cos\phi}\right)^4\cdot\sin^2\phi \, d\phi$$

$$= \frac{1}{4}\int_0^{\pi/3} \sin^2\phi - \frac{\sin^2\phi}{16\cos^4\phi}\, d\phi$$

b

$$I = \frac{1}{4}\int_0^{\pi/3} \underbrace{\sin^2\phi}_{} - \underbrace{\frac{\sin^2\phi}{16\cos^4\phi}\, d\phi}_{I_1} .$$

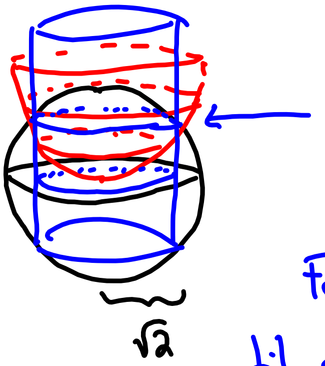
$$\int_0^{\pi/3} \frac{1-\cos 2\phi}{2}\, d\phi$$

$$I_1 = \frac{1}{16}\int_0^{\pi/3} \frac{\sin^2\phi}{\cos^2\phi}\cdot\frac{1}{\cos^2\phi}\, d\phi$$

$$= \frac{1}{16}\int_0^{\pi/3} \tan^2\phi \cdot \frac{1}{\cos^2\phi}\, d\phi$$

$$= \frac{1}{16}\left[\frac{1}{3}\tan^3\phi\right]_0^{\pi/3} = \ldots\ldots \; .$$

③ (a) $I = \iiint_A z\,dx\,dy\,dz$, A er området over $z = x^2+y^2$ og under $x^2+y^2+z^2 = 2$.



Snittet mellom paraboloiden og kula.

Forsøker å finn (x,y)-koord. til dette snittet, så vi kan bruke sylinderkoord.

$$x^2+y^2+\underbrace{z^2}_{(x^2+y^2)^2} = 2 \iff (\underbrace{x^2+y^2}_{1})+(\underbrace{x^2+y^2}_{1})^2 = 2.$$

Ser at $x^2+y^2 = 1$.

Sylinderkoord: $x = r\cos t$, $y = r\sin t$, $z = z$.

$0 \le t \le 2\pi$, $0 \le r \le 1$

$$x^2+y^2 \le z \le \sqrt{2-(x^2+y^2)} \Bigg\} \quad I = \int_0^1 \int_0^{2\pi} \int_{r^2}^{\sqrt{2-r^2}} z\cdot r\,dz\,dt\,dr.$$

$$= 2\pi \int_0^1 \int_{r^2}^{\sqrt{2-r^2}} z\cdot r\,dz\,dr$$

$$= 2\pi \int_0^1 \left[\tfrac{1}{2}z^2\right]_{r^2}^{\sqrt{2-r^2}} r\,dr$$

$$= \pi \int_0^1 \left((2-r^2)-r^4\right) r\,dr = \dots = \underline{\underline{\frac{7\pi}{12}}}.$$

3 (c) $I = \iiint_A e^{-\sqrt{x^2+y^2+z^2}}\,dx\,dy\,dz$, A er kula med radius 1.

Kulekoordinater:

$x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \cdot \sin\theta$, $z = \rho \cdot \cos\phi$.

$$I = \int_0^{\pi}\int_0^{2\pi}\int_0^{1} e^{-\rho} \cdot \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$$

integrer ϕ og θ

$$= \cdots\cdots = 2\pi \cdot 2 \underbrace{\int_0^1 e^{-\rho} \cdot \rho^2 d\rho}_{I_1}.$$

$u = \rho^2 \quad v' = e^{-\rho}$

$u' = 2\rho \quad v = -e^{-\rho}$

$$I_1 = \left[-\rho^2 e^{-\rho}\right]_0^1 + 2\int_0^1 \rho e^{-\rho} d\rho.$$

$$= -e^{-\rho} + 2 \cdot \underbrace{\int_0^1 \rho e^{-\rho} d\rho}_{I_2}.$$

$u = \rho \quad v' = e^{-\rho}$

$u' = 1 \quad v = -e^{-\rho}$

$$I_2 = \left[-\rho e^{-\rho}\right]_0^1 + \int_0^1 e^{-\rho} d\rho$$

$$= -e^{-\rho} + \int_0^1 e^{-\rho} d\rho \qquad \underline{\underline{\text{ok}}}$$

$$= \underline{4\pi(2 - 5e^{-1})}.$$