6.1.1 e)

$$\iint_R xy\,e^{x^2y}\,dx\,dy \qquad R = [0,2]\times[1,2]$$

$$= \int_0^2\left[\int_1^2 xye^{x^2y}\,dy\right]dx = \int_1^2\Bigg[\underbrace{\int_0^2 xye^{x^2y}\,dx}\Bigg]dy$$

$$= \int_1^2\left[\frac{1}{2}e^{x^2y}\right]_0^2 dy = \int_1^2\left(\frac{1}{2}e^{4y}-\frac{1}{2}\right)dy \qquad u = x^2y \quad du = 2xy\,dx$$

$$= \left[\frac{1}{8}e^{4y}-\frac{1}{2}y\right]_1^2 = \frac{1}{8}e^8 - 1 - \frac{1}{8}e^4 + \frac{1}{2} = \underline{\underline{\frac{1}{8}e^8 - \frac{1}{8}e^4 - \frac{1}{2}}}$$

6.1.1

$$\int_0^1 \left[\int_{-1}^{1} x^2 e^y \, dx\right] dy = \int_0^1 \left[\frac{1}{3}x^3 e^y\right]_{-1}^{1} dy$$

$$= \int_0^1 \left(\frac{1}{3}e^y + \frac{1}{3}e^y\right) dy = \int_0^1 \frac{2}{3} e^y \, dy$$

$$= \left[\frac{2}{3}e^y\right]_0^1 = \frac{2}{3}e - \frac{2}{3} = \underline{\underline{\frac{2}{3}(e-1)}}$$

6.1.6 $f(x) = x^2$

For enhver ε, δ : Skal vise at det finnes x_0, x s.a. $|x - x_0| \leq \delta \Rightarrow |f(x) - f(x_0)| > \varepsilon$

finner faktisk x_0, x s.a. $\underline{|x - x_0| = \delta}$ $(x = x_0 + \delta)$

$$|f(x) - f(x_0)| = |x^2 - x_0^2| = |x - x_0||x + x_0| \overset{?}{>} \varepsilon$$

$$= \delta \, |x + x_0| \overset{?}{>} \varepsilon$$

$$|x + x_0| \overset{?}{>} \frac{\varepsilon}{\delta}$$

$$|x_0 + \delta + x_0| = |2x_0 + \delta| \overset{?}{>} \frac{\varepsilon}{\delta}$$

Det er klart at vi kan velge x_0 s.a. $|2x_0 + \delta| \overset{?}{>} \frac{\varepsilon}{\delta}$

(velg $x_0 > 0$, $2x_0 + \delta > 0$) $\Rightarrow 2x_0 + \delta > \frac{\varepsilon}{\delta}$ $\Rightarrow x_0 > \dfrac{\frac{\varepsilon}{\delta} - \delta}{2}$

6.1.7

$R = [a,b] \times [c,d]$

Siden f er kont., sett

$$m = \min_{(x,y)\in R} f(x,y)$$

$$M = \max_{(x,y)\in R} f(x,y) \quad . \quad \text{Da er}$$

$$\underbrace{m|R|}_{\text{nedre trappesum}} \leq \iint_R f(x,y)\,dx\,dy \leq \underbrace{M|R|}_{\text{øvre trappesum}}$$

$$\Downarrow$$

$$m \leq \frac{\iint_R f(x,y)\,dx\,dy}{|R|} \leq M$$

Skjæringssetningen: Enhver kont. funksjon antar alle verdier mellom min (m) og maks (M)

Derfor finnes $(\bar{x},\bar{y})$ s.a. $f(\bar{x},\bar{y}) = \dfrac{\iint_R f(x,y)\,dx\,dy}{|R|}$, siden

6.1.1 g

$$\iint_R \frac{1}{1+x^2y}\,dx\,dy = \int_1^{\sqrt{3}}\left[\int_0^1 \underbrace{\frac{1}{1+x^2y}\,dy}\right]dx$$

$$u = 1+x^2y \qquad du = x^2\,dy$$

$$= \int_1^{\sqrt{3}}\left[\frac{\ln(1+x^2y)}{x^2}\right]_0^1 dx = \int_1^{\sqrt{3}}\frac{\ln(1+x^2)}{x^2}\,dx$$

$$= \int_1^{\sqrt{3}}\frac{1}{x^2}\ln(1+x^2)\,dx = \left[-\frac{1}{x}\ln(1+x^2)\right]_1^{\sqrt{3}} + \int_1^{\sqrt{3}}\frac{2x}{x(1+x^2)}\,dx$$

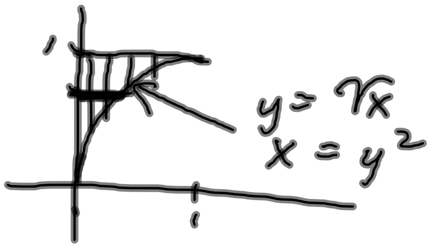
$$= -\frac{\ln 4}{\sqrt{3}} + \ln 2 + \int_1^{\sqrt{3}}\frac{2}{1+x^2}\,dx = -\frac{\ln 4}{\sqrt{3}} + \ln 2 + \left[2\arctan x\right]_1^{\sqrt{3}}$$

$$= -\frac{\ln 4}{\sqrt{3}} + \ln 2 + 2\frac{\pi}{3} - 2\frac{\pi}{4} = -\frac{2}{\sqrt{3}}\ln 2 + \ln 2 + \frac{2\pi}{3} - \frac{\pi}{2}$$

$$= \underline{\underline{\left(1-\frac{2}{3}\sqrt{3}\right)\ln 2 + \frac{\pi}{6}}}$$

6.2.3

c) $\displaystyle\int_0^1 \left[\int_{\sqrt{x}}^1 e^{\frac{x}{y^2}}\,dy \right] dx$



Området kan også skrives: $0 \le y \le 1$

$0 \le x \le y^2$

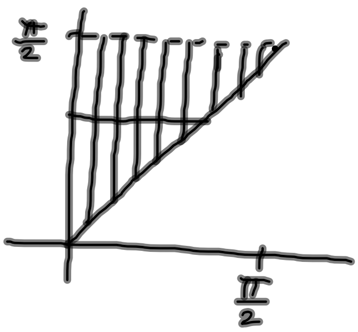
$$I = \int_0^1 \left[\int_0^{y^2} e^{\frac{x}{y^2}}\,dx \right] dy = \int_0^1 \left[y^2 e^{\frac{x}{y^2}} \right]_0^{y^2} dy$$

$$= \int_0^1 \left(y^2 e - y^2 \right) dy = \int_0^1 (e-1)y^2\,dy = \left[\frac{1}{3}(e-1)y^3 \right]_0^1$$

$$= \underline{\underline{\frac{1}{3}(e-1)}}$$

6.2.3

b) $$\int_0^{\frac{\pi}{2}} \left[\int_x^{\frac{\pi}{2}} \frac{\sin y}{y} \, dy \right] dx$$



Området kan også skrives:

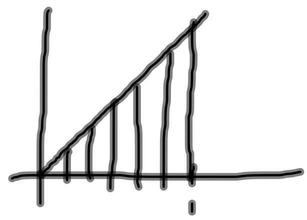
$0 \leq y \leq \frac{\pi}{2}$ $\qquad$ $0 \leq x \leq y$

$$I = \int_0^{\frac{\pi}{2}} \left[\int_0^{y} \frac{\sin y}{y} \, dx \right] dy = \int_0^{\frac{\pi}{2}} \left[\frac{\sin y}{y} x \right]_0^{y} dy$$

$$= \int_0^{\frac{\pi}{2}} \sin y \, dy = \left[-\cos y \right]_0^{\frac{\pi}{2}} = -(-1) = \underline{\underline{1}}$$

6.2.1

e) $I = \int_R e^{x^2}\,dx\,dy$



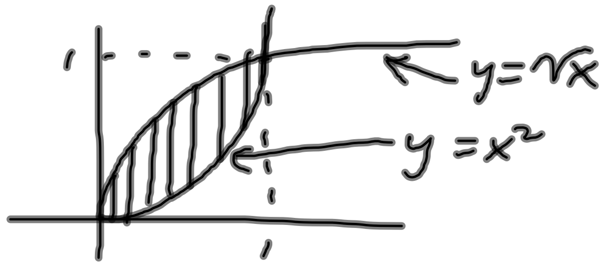
$0 \le x \le 1, \quad 0 \le y \le x$

$$I = \int_0^1 \left[\int_0^x e^{x^2}\,dy \right] dx = \int_0^1 \left[y e^{x^2} \right]_0^x dx$$

$$= \frac{1}{2}\int_0^1 2x e^{x^2}\,dx = \frac{1}{2}\left[e^{x^2} \right]_0^1 = \frac{1}{2}(e-1)$$

6.2.1

f) $I = \iint_R x^2 y \, dx \, dy$



$0 \le x \le 1 \quad x^2 \le y \le \sqrt{x}$

$$I = \int_0^1 \left[\int_{x^2}^{\sqrt{x}} x^2 y \, dy \right] dx = \int_0^1 \left[\frac{1}{2} x^2 y^2 \right]_{x^2}^{\sqrt{x}} dx$$

$$= \int_0^1 \left(\frac{1}{2} x^3 - \frac{1}{2} x^6 \right) dx = \left[\frac{1}{8} x^4 - \frac{1}{14} x^7 \right]_0^1 = \frac{1}{8} - \frac{1}{14} = \underline{\underline{\frac{3}{56}}}$$