Lineære avbildninger

$T : \mathbb{R}^n \to \mathbb{R}^m$

$n = m = 1$

$T(x) = ax$

i) $T(c \cdot \bar{x}) = c \cdot T(\bar{x}) \quad c \in \mathbb{R}$

ii) $T(\bar{x} + \bar{y}) = T(\bar{x}) + T(\bar{y})$

Eks.

$(a, b \in \mathbb{R})$ $\{a \cdot \underbrace{\cos\varphi}_{e_1} + b \underbrace{\sin\varphi}_{e_2}\} \cong \mathbb{R}^2$

derivasjon $\frac{d}{d\varphi}$ lineær operator

Skrive på matriseform

$a(b+c) = ab + ac$

$$T: \begin{aligned} \frac{d}{dx} \cos\varphi &= -\sin\varphi \quad e_1 \mapsto -e_2 \\ \frac{d}{dx} \sin\varphi &= \cos\varphi \quad e_2 \mapsto e_1 \end{aligned}$$

Eks.

A $m \times n$-matrise

$T(\bar{x}) = A\bar{x} \quad \bar{x} \in \mathbb{R}^n$

$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$T(e_1) = -e_2 : \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$a_{11} \cdot 1 + a_{12} \cdot 0 = 0$

$a_{21} \cdot 1 + a_{22} 0 = -1$

$T(e_2) = e_1$

$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$a_{11} \cdot 0 + a_{12} \cdot 1 = 1$

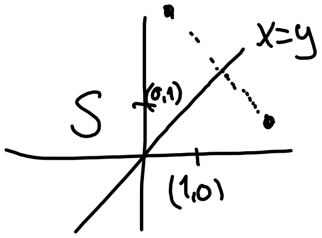
$a_{21} \cdot 0 + a_{22} \cdot 1 = 0$

dvs $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\{$Alle deriverbare funksjoner $f : \mathbb{R} \to \mathbb{R}\}$ $\xrightarrow{\frac{d}{dx}}$ $\{$Alle kontinuerlige funksjoner $f : \mathbb{R} \to \mathbb{R}\}$

$\frac{d}{dx}$ lineær

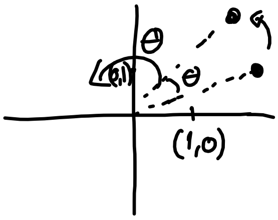
Eks.



lineær $S\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$ $S\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$

$$M(S)=\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

stive bevegelser



$$R\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix} \quad R\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-\sin\theta\\\cos\theta\end{pmatrix}$$

$$\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix}=0 \qquad \begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}\cdot\begin{pmatrix}-\sin\theta\\\cos\theta\end{pmatrix}=0$$

Affine avbildning

$$F(\bar{x})=A\cdot\bar{x}+\bar{c}$$

Egenverdi/egenvektor

$T: \mathbb{R}^n \to \mathbb{R}^n$, $\bar{x}$ er en egenvektor hvis det finnes et $\lambda \in \mathbb{R}$ slik at

$$T(\bar{x}) = \underset{\substack{\uparrow \\ \text{egenverdi}}}{\lambda} \cdot \bar{x}$$

Eks Egenvektor $\overset{y}{\vee}$ for $\frac{d}{dx}$ oppfylle $\frac{d}{dx}(y) = \lambda \cdot y$ dvs $y' = \lambda y$

Eks. Egenvektor for speiling om $x=y$: linja $x=y$

Eks Egenvektor for en rotasjon? ?