3.1.9

$\vec{r}(t) = (t, \ln\cos t) \quad t \in [0, \frac{\pi}{4}]$

a/ $\vec{v}(t) = \left(1, \frac{-\sin t}{\cos t}\right) = (1, -\tan t)$

$v(t) = |\vec{v}(t)| = \sqrt{1+\tan^2 t} = \sqrt{\frac{\cos^2 t}{\cos^2 t} + \frac{\sin^2 t}{\cos^2 t}}$

$= \sqrt{\frac{1}{\cos^2 t}} = \frac{1}{|\cos t|} = \frac{1}{\cos t}$ (her)

b/ $L = \int_0^{\frac{\pi}{4}} v\,dt = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos t} = \int_0^{\frac{\pi}{4}} \frac{\cos t\,dt}{1-\sin^2 t}$

$u = \sin t \quad du = \cos t\,dt$

$\left(\frac{1}{1-u^2} = \frac{A}{1+u} + \frac{B}{1-u}\right) \quad = \int_0^{\frac{\sqrt{2}}{2}} \frac{du}{1-u^2} = \int_0^{\frac{\sqrt{2}}{2}} \frac{1}{2}\left(\frac{1}{1-u} + \frac{1}{1+u}\right) du$

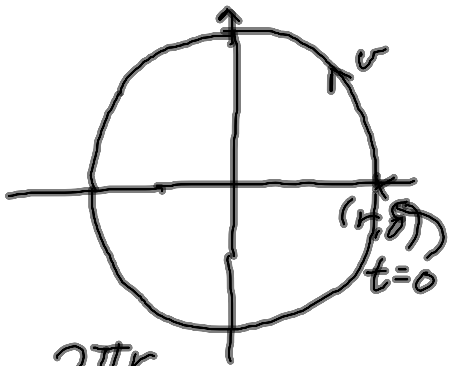
$= \frac{1}{2}\Big[-\ln(1-u) + \ln(1+u)\Big]_0^{\frac{\sqrt{2}}{2}} = \frac{1}{2}\Big[\ln\left(\frac{1+u}{1-u}\right)\Big]_0^{\frac{\sqrt{2}}{2}} = \frac{1}{2}\ln\left(\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}\right)$

$= \frac{1}{2}\ln\left(\frac{\left(1+\frac{\sqrt{2}}{2}\right)^2}{\frac{1}{2}}\right) = \frac{1}{2}\ln\left(\frac{\frac{3}{2}+\sqrt{2}}{\frac{1}{2}}\right) = \frac{1}{2}\ln(3+2\sqrt{2})$

3.184

tid t på en omdreining (omløpstid)

$$2\pi r = vt \Rightarrow t = \frac{2\pi r}{v}$$



kurven kan parametriseres ved

$$\vec{r}(t) = (r\cos(kt), r\sin(kt))$$

argumentet (kt) er lik 2π etter $t = \frac{2\pi r}{v}$

$$\Downarrow$$

$$k\frac{2\pi r}{v} = 2\pi \rightarrow k = \frac{v}{r}$$

$$\Rightarrow \vec{r}(t) = \left(r\cos\left(\frac{v}{r}t\right), r\sin\left(\frac{v}{r}t\right)\right)$$

b) $\vec{r}\,'(t) = \vec{v}(t) = \left(-r\sin\left(\left(\frac{v}{r}\right)t\right)\frac{v}{r}, r\cos\left(\frac{v}{r}t\right)\frac{v}{r}\right)$

$$\vec{r}\,'(t) = \vec{a}(t) = \left(-r\cos\left(\frac{v}{r}t\right)\left(\frac{v}{r}\right)^2, -r\sin\left(\frac{v}{r}t\right)\left(\frac{v}{r}\right)^2\right)$$

$$= -\left(\frac{v}{r}\right)^2\left(r\cos\left(\frac{v}{r}t\right), r\sin\left(\frac{v}{r}t\right)\right) = \underline{\underline{-\left(\frac{v}{r}\right)^2\vec{r}(t)}}$$

3.1.8

$\vec{r}(t) = (t^2, t^3) \quad t \in [0,10] \quad x'(t) = 2t \quad y'(t) = 3t^2$

$$L(0,10) = \int_0^{10} \underbrace{\sqrt{x'(t)^2 + y'(t)^2}}_{v} \, dt = \int_0^{10} \sqrt{4t^2 + 9t^4} \, dt$$

$$= \int_0^{10} t\sqrt{4 + 9t^2} \, dt \qquad u = 4 + 9t^2 \quad du = 18t \, dt$$

$$\frac{1}{18} du = t \, dt$$

$$= \frac{1}{18} \int_4^{904} \sqrt{u} \, du = \frac{1}{18}\left[\frac{2}{3} u^{\frac{3}{2}}\right]_4^{904} = \frac{1}{27}\left[u^{\frac{3}{2}}\right]_4^{904}$$

$$= \underline{\underline{\frac{904^{\frac{3}{2}} - 8}{27}}}$$

3.1.16

høyde $= y = -\frac{mg}{k}t + \left(\frac{mu_2}{k} + \frac{m^2 g}{k^2}\right)\left(1 - e^{-\frac{kt}{m}}\right)$

høyde$' = 0$:

$$-\frac{mg}{k} + \left(\frac{mu_2}{k} + \frac{m^2 g}{k^2}\right)\frac{k}{m} e^{-\frac{kt}{m}} = 0$$

$$e^{\frac{kt}{m}}\frac{mg}{k} = \left(\frac{\cancel{m}u_2}{\cancel{k}} + \frac{m^{\cancel{2}} g}{k^{\cancel{2}}}\right)\frac{\cancel{k}}{\cancel{m}} \qquad e^{-\frac{kt}{m}} = \frac{\frac{mg}{k}}{u_2 + \frac{mg}{k}}$$

$$e^{\frac{kt}{m}} = \frac{k}{mg}\left(u_2 + \frac{mg}{k}\right) \Rightarrow \underline{t = \frac{m}{k}\ln\left(\frac{ku_2}{mg} + 1\right)}$$

$\frac{m}{k}\left(u_2 + \frac{mg}{k}\right)$

høyde: setter inn:

$$-\frac{mg}{k}\left(\frac{m}{k}\ln\left(\frac{ku_2}{mg} + 1\right)\right) + \left(\frac{mu_2}{k} + \frac{m^2 g}{k^2}\right)\left(1 - \frac{\frac{mg}{k}}{u_2 + \frac{mg}{k}}\right)$$

$$= -\frac{m^2 g}{k^2}\ln\left(\frac{ku_2}{mg} + 1\right) + \frac{mu_2}{k} + \underline{\frac{m^2 g}{k^2} - \frac{m}{k}\frac{mg}{k}}$$

$$= \underline{-\frac{m^2 g}{k^2}\ln\left(\frac{ku_2}{mg} + 1\right) + \frac{mu_2}{k}}$$

3.2.8

$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} \qquad g(t) = f(\vec{r}(t)) \qquad f: \mathbb{R}^2 \to \mathbb{R}$

$$g'(t) = \underbrace{\frac{\partial f}{\partial x} x'(t)} + \frac{\partial f}{\partial y} y'(t) \qquad \left(\frac{\partial \left(\frac{\partial f}{\partial x}\right)}{\partial t} = \frac{\partial^2 f}{\partial x^2} x'(t) + \frac{\partial^2 f}{\partial x \partial y} y'(t) \right)$$

$$g''(t) = \left(\frac{\partial^2 f}{\partial x^2} x'(t) + \underline{\frac{\partial^2 f}{\partial y \partial x} y'(t)} \right) x'(t) + \frac{\partial f}{\partial x} x''(t)$$

$$\left(\underline{\frac{\partial^2 f}{\partial y \partial x}} x'(t) + \frac{\partial^2 f}{\partial y^2} y'(t) \right) y'(t) + \frac{\partial f}{\partial y} y''(t)$$

$$= \underline{\underline{\frac{\partial^2 f}{\partial x^2} (x'(t))^2 + 2 \frac{\partial^2 f}{\partial x \partial y} x'(t) y'(t) + \frac{\partial^2 f}{\partial y^2} (y'(t))^2 + \frac{\partial f}{\partial x} x''(t) + \frac{\partial f}{\partial y} y''(t)}}.$$

$$= \frac{\partial^2 f}{\partial x^2} \underline{\underline{(\vec{r}(t))}} (x'(t))^2 + \ldots$$

3.2.5

$$\vec{F}(x,y) = \begin{pmatrix} x^2 y \\ xy + x \end{pmatrix}$$

$$\vec{G}(t) = \vec{F}(\vec{r}(t))$$

$$\vec{F}'(x,y) = \begin{pmatrix} 2xy & x^2 \\ y+1 & x \end{pmatrix}$$

$$\vec{r}'(t) = \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix}$$

$$\vec{r}(t) = \sin t \, \vec{i} + \cos t \, \vec{j}$$

$$\vec{G}'(t) = \vec{F}'(\vec{r}(t)) \, \vec{r}'(t)$$

$$= \begin{pmatrix} 2xy & x^2 \\ y+1 & x \end{pmatrix} \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix}$$

$$= \begin{pmatrix} 2\sin t\cos t & \sin^2 t \\ \cos t + 1 & \sin t \end{pmatrix} \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix} 2\sin t\cos^2 t - \sin^3 t \\ \cos^2 t + \cos t - \sin^2 t \end{pmatrix} = \vec{G}'(t)}}$$