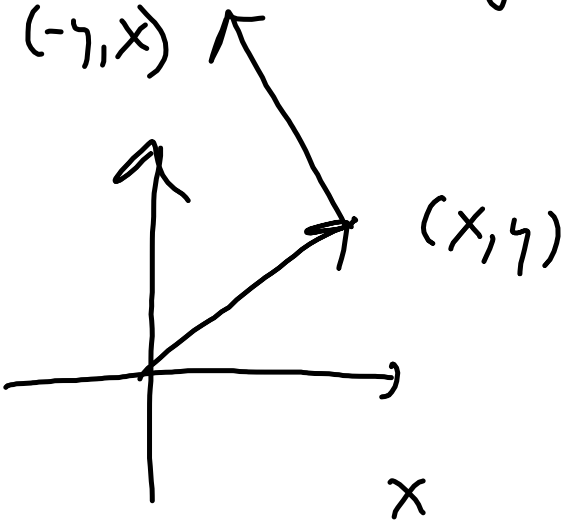
3.8.2

$$F(x,y) = \frac{-y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j}$$

b)



$$\begin{array}{ccc} & x & y \\ & \shortparallel & \shortparallel \end{array}$$

$$\boxed{\vec{r}(t) = \left(r_0 \cos\frac{t}{r_0^2},\ r_0 \sin\frac{t}{r_0^2}\right)}$$

$$\vec{r}\,'(t) = \left(-\frac{1}{r_0}\sin\frac{t}{r_0^2},\ \frac{1}{r_0}\cos\frac{t}{r_0^2}\right)$$

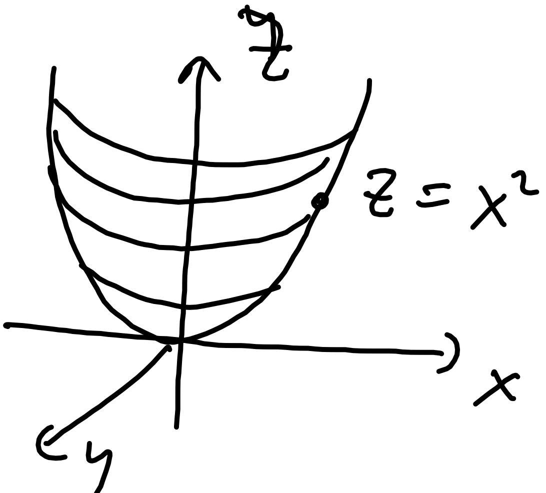
→ Strömning linjer Sirkel.

$$F(r(t)) = \frac{\left(-r_0 \sin\frac{t}{r_0^2},\ r_0\cos\frac{t}{r_0^2}\right)}{r_0^2}$$

$$= \vec{r}\,'(t).$$

3.9.1

$z = x^2 + y^2$



Flaten er grafen til $z = x^2$ rotert rundt z-aksen

$\vec{r}(x,y) = x\vec{i} + y\vec{j} + (x^2+y^2)\vec{k}$

Polarkoordinater:

$\vec{r}(r,\theta) = r\cos\theta\,\vec{i} + r\sin\theta\,\vec{j} + r^2\vec{k}$

3.9.2

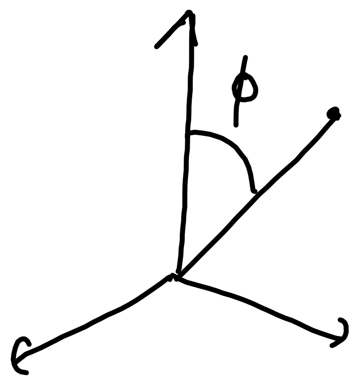
2) $x^2+y^2+z^2=4$ (kuleflate med radius 2)

$x \geq 0,\ y \geq 0,\ z \geq 0$

Bruker kulekoordinater med $\rho = 2$

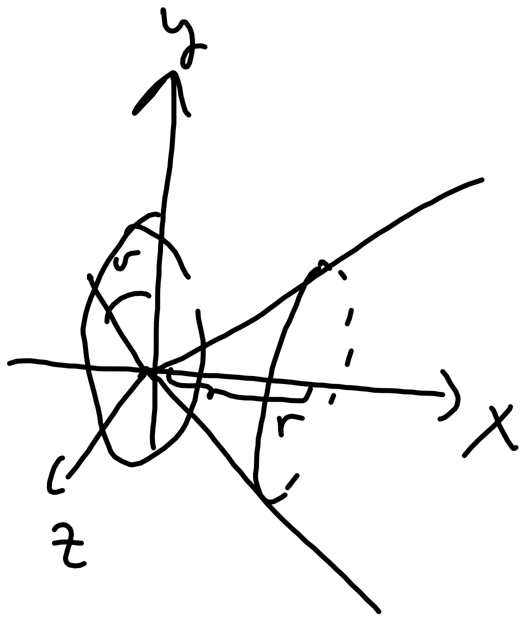
$$\vec{r}(\phi,\theta) = 2\cos\theta\sin\phi\,\vec{i} + 2\sin\theta\sin\phi\,\vec{j} + 2\cos\phi\,\vec{k}$$

$0 < \theta \leq \frac{\pi}{2},\ 0 < \phi < \frac{\pi}{2}$



3.9.5

$X = \sqrt{y^2 + z^2}$



Linja $X = \sqrt{y^2} = |y|$

rotert rundt x-aksen

(kjegle). Bruker polarkoordinater

i yz planet.

$\vec{r}(r, v) = r\vec{i} + r\cos v\vec{j} + r\sin v\vec{k}$

$r \geq 0, \quad v \in [0, 2\pi)$

Alternativ parametrisering.

$\vec{r}(y, z) = \sqrt{y^2 + z^2}\,\vec{i} + y\vec{j} + z\vec{k}$

3.9.8

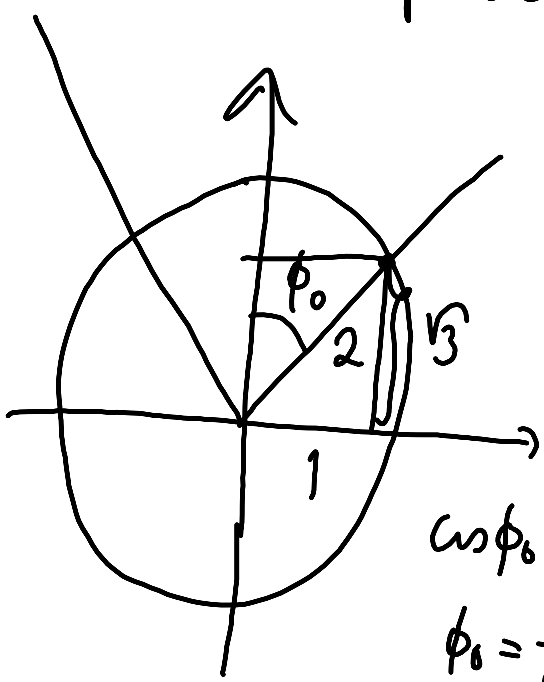
Den delen av kule

$$x^2+y^2+z^2=4 \quad \text{der } z>0$$

og som ligger inne i kjeglen

$$z^2=3(x^2+y^2)$$

snitt gjennom xz-planet



$z=\sqrt{3}|x|$

$\phi_0 = 3$

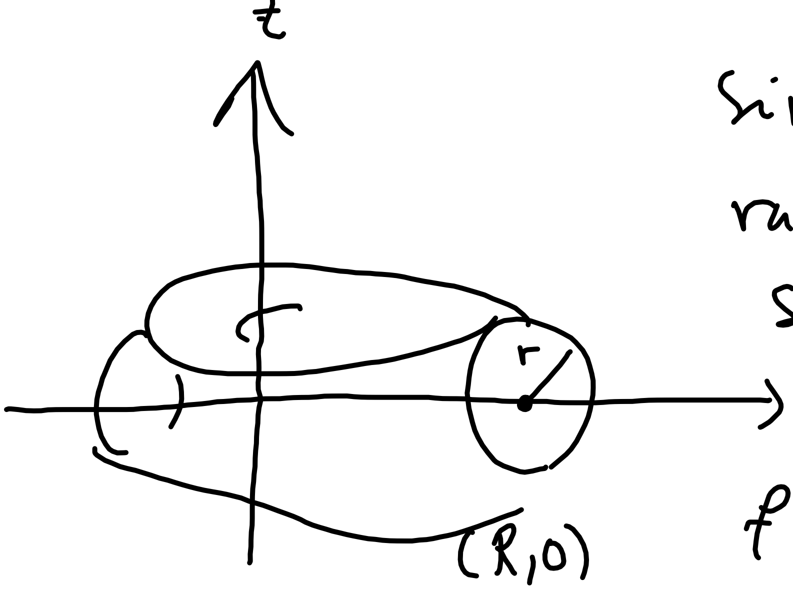
$\cos\phi_0 = \frac{\sqrt{3}}{2}$

$\phi_0 = \frac{\pi}{6}$

$$\vec{r}(\phi,\theta) = 2\cos\theta\sin\phi\,\vec{i} + 2\sin\theta\sin\phi\,\vec{j} + 2\cos\phi\,\vec{k}$$

$$0 \le \phi < \frac{\pi}{6}, \quad 0 \le \theta < 2\pi$$

3.9.14



Dreier sirkel med radius r sentrum (R,0) om z-aksen. får torus.

$r=3$, $R=5$. Matlab skal tegne.

$$x=(5+3\cos u)\cos v$$
$$y=(5+3\cos u)\sin v$$
$$z=3\sin u, \quad 0\le u\le 2\pi$$
$$0\le v\le 2\pi$$

$$R = [a, b] \times [c, d] \subset \mathbb{R}^2$$

$$\iint_R f(x,y)\,dx\,dy =$$

$$= \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

$$= \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy$$