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B 617 N.H. hus.

Prioriterte oppgaver :

1.9 : 1, 4, 8, 9, 10, 11

1.10 : 1, 2, 5a), 6

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1.9

1) $T: \mathbb{R}^3 \to \mathbb{R}^2$

$$T(x,y,z) = \begin{pmatrix} 2x - y + z \\ -x + y - 3z \end{pmatrix}$$

Finn matrisen til T.

$$T(e_1) = T(1,0,0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

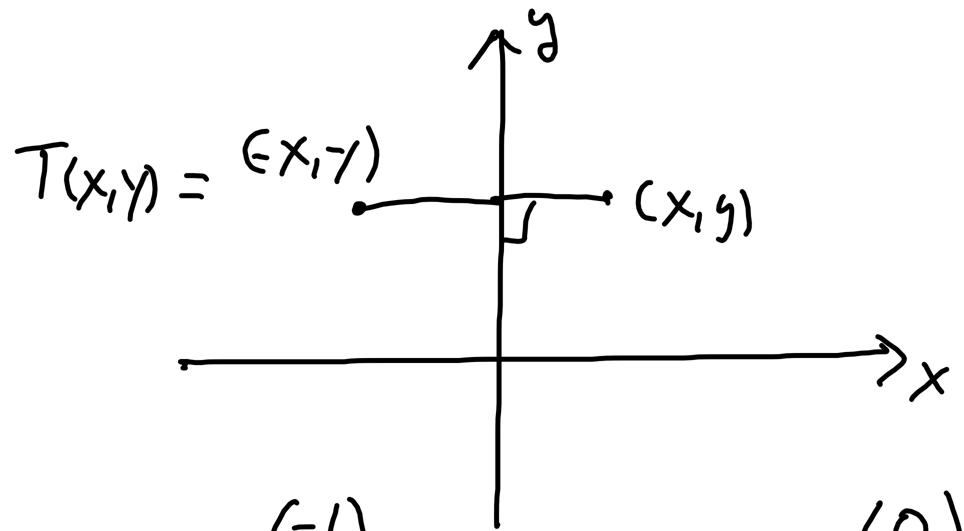
$$T(e_2) = T(0,1,0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$T(e_3) = T(0,0,1) = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

Matrisen til T blir da

$$\underline{\underline{A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & -3 \end{pmatrix}}}$$

4) Gitt $T: \mathbb{R}^2 \to \mathbb{R}^2$ speiling om y-aksen. Finn matrisen til T

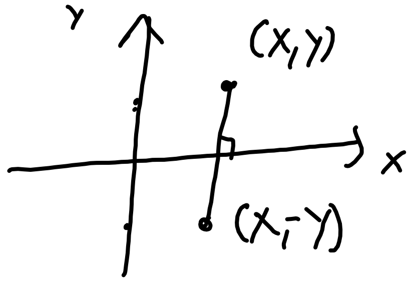


$$T(1,0) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \quad T(0,1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Matrisen $\underline{\underline{A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}}$

8) $T: \mathbb{R}^2 \to \mathbb{R}^2$

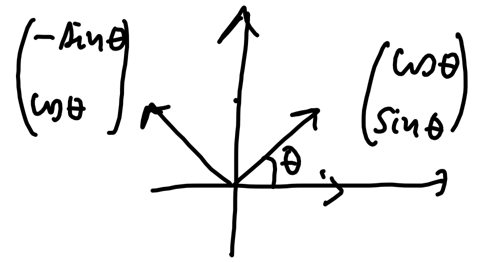
speiler vektorer om x-aksen for så å drei dem en positiv vinkel θ.



Matrise til speiling om x-akse

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = A$$

Dreing en vinkel θ har matrise (se boka)



$$A_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

T får da matrise

$$A_\theta A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$

S, T to lineæravbildninger

med matriser henholdsvis A og B

$$S \circ \underline{T(\vec{x})} = S(T(x))$$

$$= S(B\vec{x}) = A(B\vec{x}) = (AB)\vec{x}$$

Dvs matrisen til $S \circ T$ blir AB.

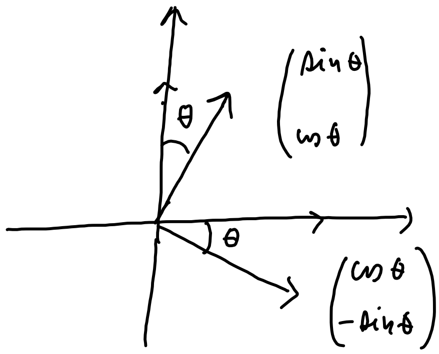
9) A_θ rotasjonsmatrisen

pos. vinkel θ

$$A_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Forklar at $A_{-\theta}$ er den inverse matrisen

$$A_{-\theta} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$



$A_{-\theta}$ svarer til til dreing en vinkel θ med urviseren (neg. retning)

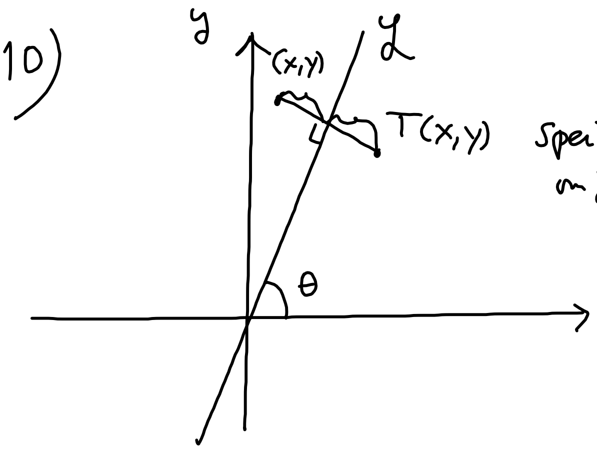
Dreier vi først en vinkel θ mot urviseren og så en vinkel θ med urviseren så „står vi stille" så vi får at sammensetningen blir identitets-avbildningen. (som har I som matrise)

dvs.

$I = A_{-\theta} A_\theta$. Tilsvarende

$I = A_\theta A_{-\theta}$

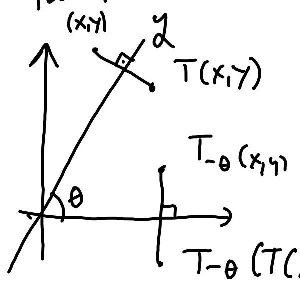
10)



Speiling om $\mathcal{L}$.

B matrisen til speiling om x-aksen

A_θ rotasjonsmatrisen. Matrisen C til T er da $C = A_\theta B A_{-\theta}$



La oss dreie en vinkel $-\theta$, $T_{-\theta}$

$T_{-\theta}(T(x,y))$

$T_{-\theta}(T(x,y))$ er speiling av $T_{-\theta}(x,y)$ om x-aksen

$T_{-\theta} \circ T$ har da matrise $B A_{-\theta}$, så $T = \underbrace{T_\theta \circ T_{-\theta}}_{id} \circ T$

får matrise $\underline{A_\theta B A_{-\theta}} = C$

$$C = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta - \sin^2\theta & 2\cos\theta\sin\theta \\ 2\cos\theta\sin\theta & \sin^2\theta - \cos^2\theta \end{pmatrix} = \underline{\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}}$$

11) $\vec{a} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

a) Finn x, y, z, u slik at

$e_1 = x\vec{a} + y\vec{b}, \quad e_2 = z\vec{a} + u\vec{b}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = x\begin{pmatrix} -2 \\ 1 \end{pmatrix} + y\begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad \begin{aligned} -2x + y &= 1 \\ x + 3y &= 0 \;\; \vert\, 2 \end{aligned} \quad \begin{aligned} 7y &= 1 \\ y &= \frac{1}{7} \end{aligned}$

$x = -3y = \frac{-3}{7}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = z\begin{pmatrix} -2 \\ 1 \end{pmatrix} + u\begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad \begin{aligned} -2z + u &= 0 \\ z + 3u &= 1 \;\; \vert\, 2 \end{aligned} \quad \begin{aligned} 7u &= 2 \\ u &= \frac{2}{7} \end{aligned}$

$z = \frac{u}{2} = \frac{1}{7}$

b) $T: \mathbb{R}^2 \to \mathbb{R}^2$ s.a

$T(\vec{a}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad T(\vec{b}) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$T(e_1) = T\left(-\frac{3}{7}\vec{a} + \frac{1}{7}\vec{b}\right) = \left(-\frac{3}{7}\right)T(\vec{a}) + \frac{1}{7}T(\vec{b})$

$= -\frac{3}{7}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{7}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -\frac{2}{7} \\ -\frac{4}{7} \end{pmatrix}$

$T(e_2) = T\left(z\vec{a} + u\vec{b}\right) =$

$= T\left(\frac{1}{7}\vec{a} + \frac{2}{7}\vec{b}\right) =$

$= \frac{1}{7}T(\vec{a}) + \frac{2}{7}T(\vec{b}) = \frac{1}{7}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{2}{7}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{7} \\ -\frac{1}{7} \end{pmatrix}$

c) Matrisen til T blir da:

$$\begin{pmatrix} -\frac{2}{7} & \frac{3}{7} \\ -\frac{4}{7} & -\frac{1}{7} \end{pmatrix}$$

1.10) Om affine avbildninger

$T: \mathbb{R}^n \to \mathbb{R}^m$

$T(\vec{x}) = A\vec{x} + \vec{c}$

1) $F(x,y,z) = \begin{pmatrix} 2x-3y+z-7 \\ -x+z-2 \end{pmatrix}$

affin avbildning. Finn A og $\vec{c}$ til denne

$$F(x,y,z) = \underbrace{\begin{pmatrix} 2 & -3 & 1 \\ -1 & 0 & 1 \end{pmatrix}}_{=A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \underbrace{\begin{pmatrix} -7 \\ -2 \end{pmatrix}}_{=\vec{c}}$$

2) Gitt $\mathcal{L}$;

$$\vec{r}(t) = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \vec{a} + t\vec{b}$$

og $F(x,y,z) = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Finn $F(\mathcal{L})$. $\quad F(\mathcal{L}) = F(\vec{a}) + tA\vec{b}$

(setning 1.10.2)

$$F(\vec{a}) = F\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

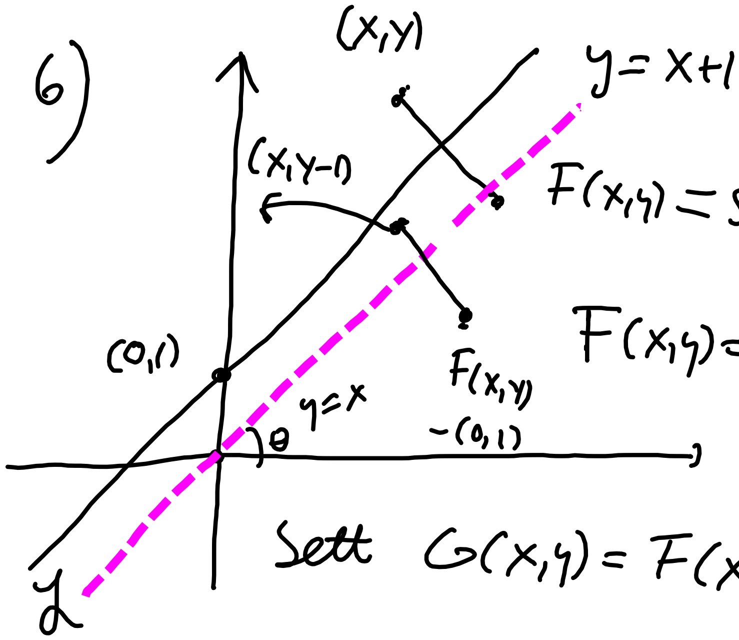
$$= \begin{pmatrix} 9 \\ -9 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 11 \\ -10 \end{pmatrix}$$

$$A\vec{b} = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

$$F(\vec{r}(t)) = F(\vec{a}) + tA\vec{b} =$$

$$= \begin{pmatrix} 11 \\ -10 \end{pmatrix} + t\begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

6)



$F(x,y) = \underset{?}{A}\begin{pmatrix} x \\ y \end{pmatrix} + \vec{c}$, $\vec{c} = ?$

Sett $G(x,y) = F(x,y) - (0,1)$

$G(L)$ er linja $y=x$, $G(x,y)$ er speiling av $(x, y-1)$ om linja $y=x$

Matrisen til speiling om linja $y=x$ blir (oppgave 1.9.10)

$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Her $\theta = \frac{\pi}{4}$

$$G(x,y) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y-1 \end{pmatrix} = \begin{pmatrix} y-1 \\ x \end{pmatrix}$$

$$F(x,y) = G(x,y) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} y-1 \\ x+1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$