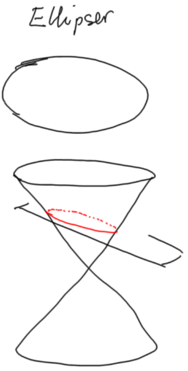
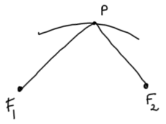
# Ellipser



- Gitt to punkter $F_1, F_2$ brennpunkter.

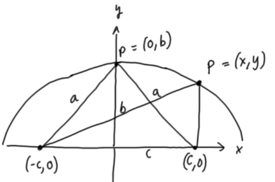
Ellipsen er gitt ved $\{P \mid |PF_1| + |PF_2| = K\}$ K konstant.

$\underline{K = 2a}$.




Ved Pytagoras:

$a^2 = b^2 + c^2$




Ligningen for ellipsen:

$|PF_1| + |PF_2| = 2a$

$|(x,y) - (-c,0)| + |(x,y) - (c,0)| = 2a$

$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$

$\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2} \quad |\,\cdot^2$

$(x+c)^2 + \cancel{y^2} = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + \cancel{y^2}$

$(x - \cancel{c} + x + \cancel{c})(\cancel{x} - c - \cancel{x} - c) = -4xc$

$4a\sqrt{(x-c)^2 + y^2} = 4a^2 + (x-c)^2 - (x+c)^2$

$\cancel{4}a\sqrt{(x-c)^2 + y^2} = \cancel{4}a^2 - \cancel{4}xc \quad |\,\cdot^2$

$a^2\left((x-c)^2 + y^2\right) = a^4 - 2a^2xc + x^2c^2$

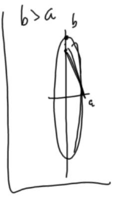
$a^2(x^2 - 2xc + c^2 + y^2)$

$a^2x^2 - \cancel{2a^2xc} + a^2c^2 + a^2y^2 = a^4 - \cancel{2a^2xc} + x^2c^2$

$x^2(a^2 - c^2) + a^2y^2 = a^4 - a^2c^2$

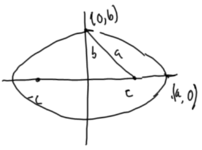
$x^2\underbrace{(a^2 - c^2)}_{b^2} + a^2y^2 = a^2\underbrace{(a^2 - c^2)}_{b^2}$

husk: $a^2 - c^2 = b^2$




$x^2b^2 + y^2a^2 = a^2b^2 \quad \left|\, \frac{1}{a^2b^2}\right.$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$




a store halvakse

b lille halvakse

Med brennpunkt

$y = 0, \; x = \pm\sqrt{a^2 - b^2}$

Med ~~E~~ center i $(m,n)$

$$\frac{(x-m)^2}{a^2} + \frac{(y-n)^2}{b^2} = 1.$$

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Eksempel:

$9x^2 + 4y^2 - 36x + 24y + 36 = 0.$ Vis at ellipse, akser, brennpunkt.

$9(x^2 - 4x) + 4(y^2 + 6y) + 36 = 0$

$9(x-2)^2 - 4\cdot 9 + 4(y+3)^2 - \cancel{9\cdot 4} + \cancel{9\cdot 4} = 0$

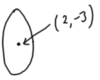
$9(x-2)^2 + 4(y+3)^2 = 9\cdot 4 \quad \left|\, \frac{1}{9\cdot 4}\right.$

$\frac{(x-2)^2}{2^2} + \frac{(y+3)^2}{3^2} = 1$

$a = 2, \; b = 3 \quad c = \sqrt{9-4} = \sqrt{5}$

Senter $(2,-3)$

Brennpunkter $(2, -3 \pm \sqrt{5})$

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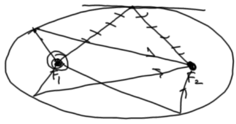
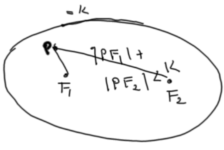
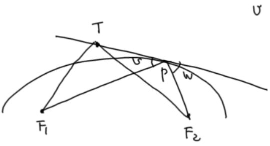
Refleksjonsegenskap



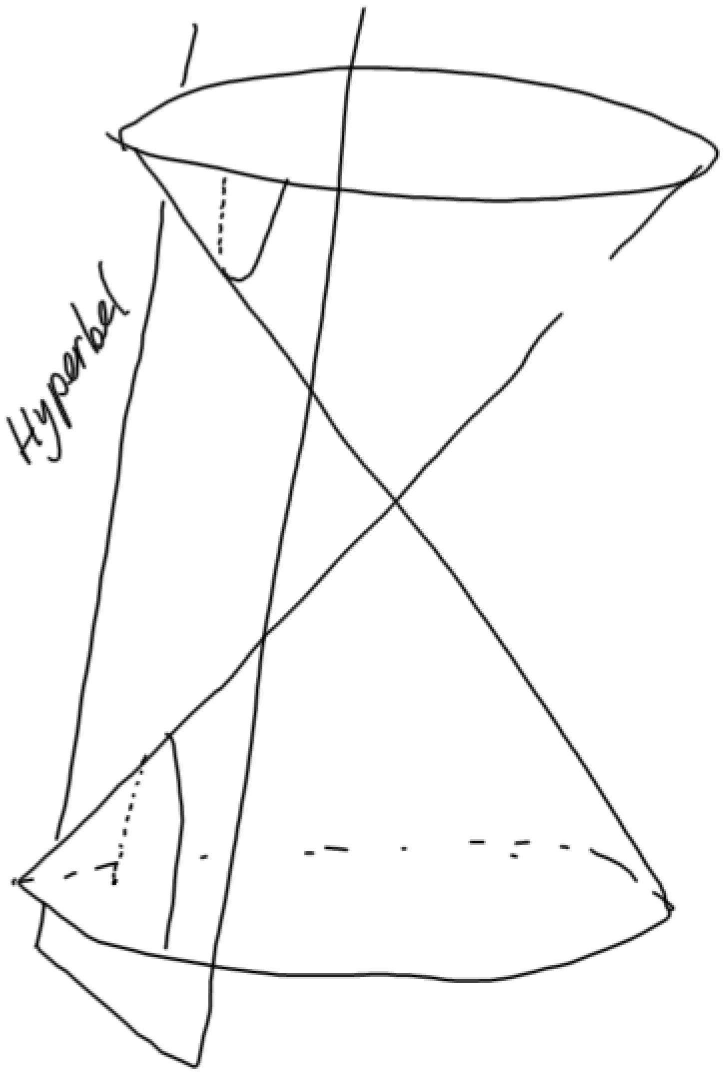




$v = w$ hvis $|F_1P| + |PF_2|$

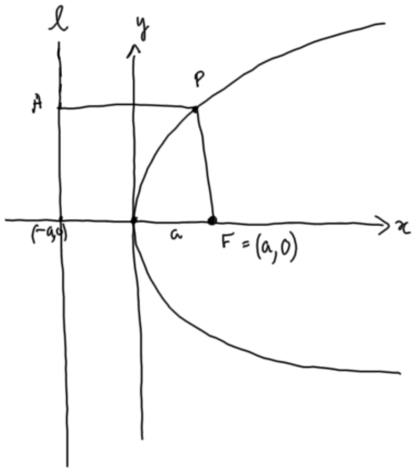
$\leq |F_1T| + |F_2T|$ for alle

punkter T i tangentplanet

Vet at T er utenfor ellipsen. $|TF_1| + |TF_2| > K = |PF_1| + |PF_2|$


Hyperbel

**Parabel** : {de punktene som ligger like langt fra F som fra $\ell$ }.




F : fokus, brennpunkt
$\ell$ : styringslinje

$|P\ell| = |PF|$.
$\overset{\shortparallel}{|PA|}$

$\ell$ gitt ved at $x = -a$.

Ligningen $|PA| = |PF|$ $\qquad P = (x,y)$

$$|PA| = x + a$$
$$|PF| = \sqrt{(x-a)^2 + y^2}$$
$$x + a = \sqrt{(x-a)^2 + y^2}$$
$$(x+a)^2 = (x-a)^2 + y^2$$
$$-y^2 = (x-a)^2 - (x+a)^2 = (x-a+x+a)(x-a-x-a)$$
$$= 2x \cdot (-2a)$$
$$y^2 = 4xa$$

Brennpunkt : $(a,0)$ Styringslinje $x = -a$
Toppunkt : $(0,0)$

"Flytter" ved å velge origo.

Toppunkt i $(m,n)$

$$4a(x-m) = (y-n)^2$$

$x = m - a$ styringslinje

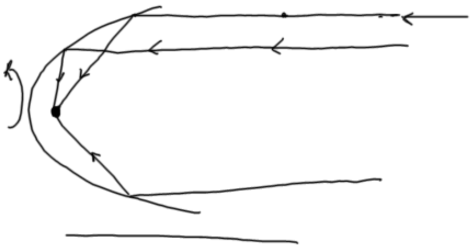
Eks $y^2 + 4y - 8x + 20 = 0$ : Vis at dette er en parabel. Topp, styringslinje ...

$\underbrace{y^2 + 4y + 4} - 4 - 8x + 20 \longrightarrow (y+2)^2 = 8x - 16 = 8(x-2)$

$n = -2 \quad m = 2 \quad 4a = 8 \quad a = 2$.

Topp $(2,-2)$ Styringslinje $x = 2 - 2 = 0$.

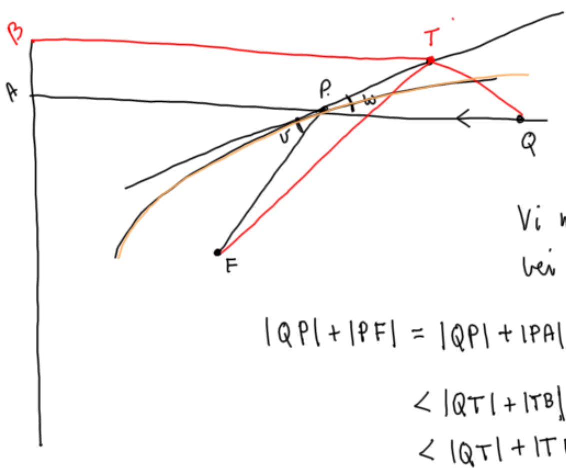
Refleksjonsegenskap.



① 



② Lys "velger" alltid korteste vei.




tangentplan i P. = $\Gamma$.

Hvis $v = w$ så treffer lyset fra Q F.

Vi må vise at $|QP| + |PF|$ er korteste vei mellom Q og F via $\Gamma$

$$|QP| + |PF| = |QP| + |PA| = |QA|$$
$$< |QT| + |TB|$$
$$< |QT| + |TF|$$

$|TB| < |TF|$ siden T ligger "utenfor" parabelen

+ Kjeglesnitt