2.2.3

Vi må: For enhver $\varepsilon > 0$, finne en $\delta > 0$ s.a.

$$|(x_1, \dots, x_n) - (a_1, \dots, a_n)| < \delta$$

$$\Downarrow$$

$$|k_i(x_1, \dots, x_n) - k_i(a_1, \dots, a_n)| < \varepsilon \Leftrightarrow |x_i - a_i| < \varepsilon$$

(dette vil vise kontinuitet i $(a_1, \dots, a_n)$)

$$|k_i(x_1, \dots, x_n) - k_i(a_1, \dots, a_n)| = |x_i - a_i|$$

$$= \sqrt{(x_i - a_i)^2} \leq \sqrt{(x_1 - a_1)^2 + \dots + (x_n - a_n)^2}$$

$$= |(x_1, \dots, x_n) - (a_1, \dots, a_n)| < \delta = \varepsilon$$

Der jeg har valgt $\delta = \varepsilon$.

Derfor er k_i kontinuerlige.

2.2.4

a) Vet $|F(\vec{x})-F(\vec{y})| \leq M|\vec{x}-\vec{y}|$, $\vec{x},\vec{y}\in D_F$

Gitt ε, finn δ s.a. $|\vec{x}-\vec{a}|<\delta \Rightarrow |F(\vec{x})-F(\vec{y})|<\varepsilon$

$$|F(\vec{x})-F(\vec{a})| \leq M|\vec{x}-\vec{a}| \leq M\delta \leq \varepsilon$$

hvis vi setter $\delta = \dfrac{\varepsilon}{M}$

slik at F er kont.

b) Anta $F(\vec{x}) = A\vec{x}$ Vi har at

$|F(\vec{x})-F(\vec{y})| = |A\vec{x}-A\vec{y}| = |A(\vec{x}-\vec{y})|$

$\leq \|A\| |x-y|$ (setning 1.6.3)

slik at vi kan bruke a) med $M = \|A\|$

Derfor er F kont.

2.3.1

d) $$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy} \cdot \cos(x+y)$$

$$= \lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy} \lim_{(x,y)\to(0,0)} \cos(x+y)$$

$$= \lim_{h\to 0} \frac{\sin h}{h} \cos(0+0) = 1 \cdot 1 = \underline{\underline{1}}$$

2.4.6

$$f(v,h) = \frac{v}{h^2}$$

a) $$\Delta BMI = \Delta f = f(v+\Delta v, h+\Delta h) - f(v,h)$$

$$= \frac{v+\Delta v}{(h+\Delta h)^2} - \frac{v}{h^2} = \frac{(v+\Delta v)h^2 - v(h+\Delta h)^2}{h^2(h+\Delta h)^2}$$

$$= \frac{\cancel{vh^2} + \Delta v\, \underline{h^2} - \cancel{vh^2} - 2v\Delta h\, \underline{h} - v(\Delta h)^2}{h^2(h+\Delta h)^2}$$

$$= \frac{\Delta v}{(h+\Delta h)^2} - \frac{2v}{h(h+\Delta h)^2}\Delta h \underbrace{- \frac{v}{h^2(h+\Delta h)^2}(\Delta h)^2}_{\text{mye mindre enn to første}}$$

$$\approx \frac{\Delta v}{(h+\Delta h)^2} - \frac{2v}{h(h+\Delta h)^2}\Delta h$$

$$\approx \underline{\underline{\frac{\Delta v}{h^2} - \frac{2v}{h^3}\Delta h}}$$

b) Vi setter $\Delta h = 0.01$, $\Delta v = 1$

fra a) : $\Delta f = \frac{\Delta v}{h^2} - \frac{2v}{h^3} \Delta h$

$$= \frac{1}{h^2} - 0.02 \frac{v}{h^3}$$

når er $\Delta f = 0$? $\frac{1}{h^2} - 0.02 \frac{v}{h^3} = 0$

$$h = 0.02 v \Leftrightarrow v = 50h$$

$(h=2 \Rightarrow v = 100)$ $(h = 1.5 \Rightarrow v = 75)$

$$F\begin{pmatrix}-1\\0\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}\Rightarrow\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}\begin{pmatrix}-1\\0\end{pmatrix}+\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}$$

$$\begin{pmatrix}-a_{11}\\-a_{21}\end{pmatrix}+\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}\Rightarrow\begin{pmatrix}a_{11}\\a_{21}\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$$

$$\Rightarrow F\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}-1\\1\end{pmatrix}$$