Alternative Techniques for Rate Estimation

6

Jason Ng and Jeffrey J. Goldberger

6.1 Chapter Objectives

Chapter 5 discussed different algorithms that might be used to detect cardiology related waveforms. The sequence of time intervals between the detected waveforms can be used to calculate the rate of the cardiac cycle. Cardiac rates are typically expressed as the number of beats per minute or as an average cycle length – the time interval between consecutive beats. These are reciprocally related:

Rate (in beats/min) = 60 divided by average cycle length (in seconds)

The accuracy of determining rate depends on the ability to detect the waveforms accurately. However, this process may be complicated by various factors. Changing amplitude, polarity, and morphology of the waveforms can complicate automated detection as demonstrated in the chapter 5. Artifacts, noise, and baseline wander are sources of interference for detection. Sometimes waveforms blend into each other so that no identifiable onset or offset exists. In these cases, a proper baseline cannot be defined as a reference for the detection algorithm. In this chapter, we will discuss alternative techniques to estimate rates without detection of the individual waveforms.

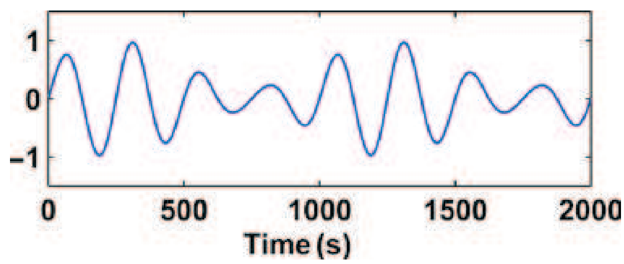
6.2 Autocorrelation

We will start with an example signal where rate estimation would be difficult with waveform detection. Figure 6.1 shows a 4 Hz sine wave with the amplitude modulated at 1 Hz. This signal has no definable baseline and the changing amplitude will confound most waveform detection algorithms. Yet, the 4 cycles per second rate or cycle length of 250 ms is apparent.

J. Ng (✉)
Department of Medicine, Division of Cardiology, Feinberg School of Medicine,
Northwestern University, Chicago, IL, USA
e-mail: jsnng@northwestern.edu

J.J. Goldberger and J. Ng (eds.),
Practical Signal and Image Processing in Clinical Cardiology,
DOI: 10.1007/978-1-84882-515-4_6, © Springer-Verlag London Limited 2010

Fig. 6.1 A 4 Hz sine wave with 1 Hz amplitude modulation



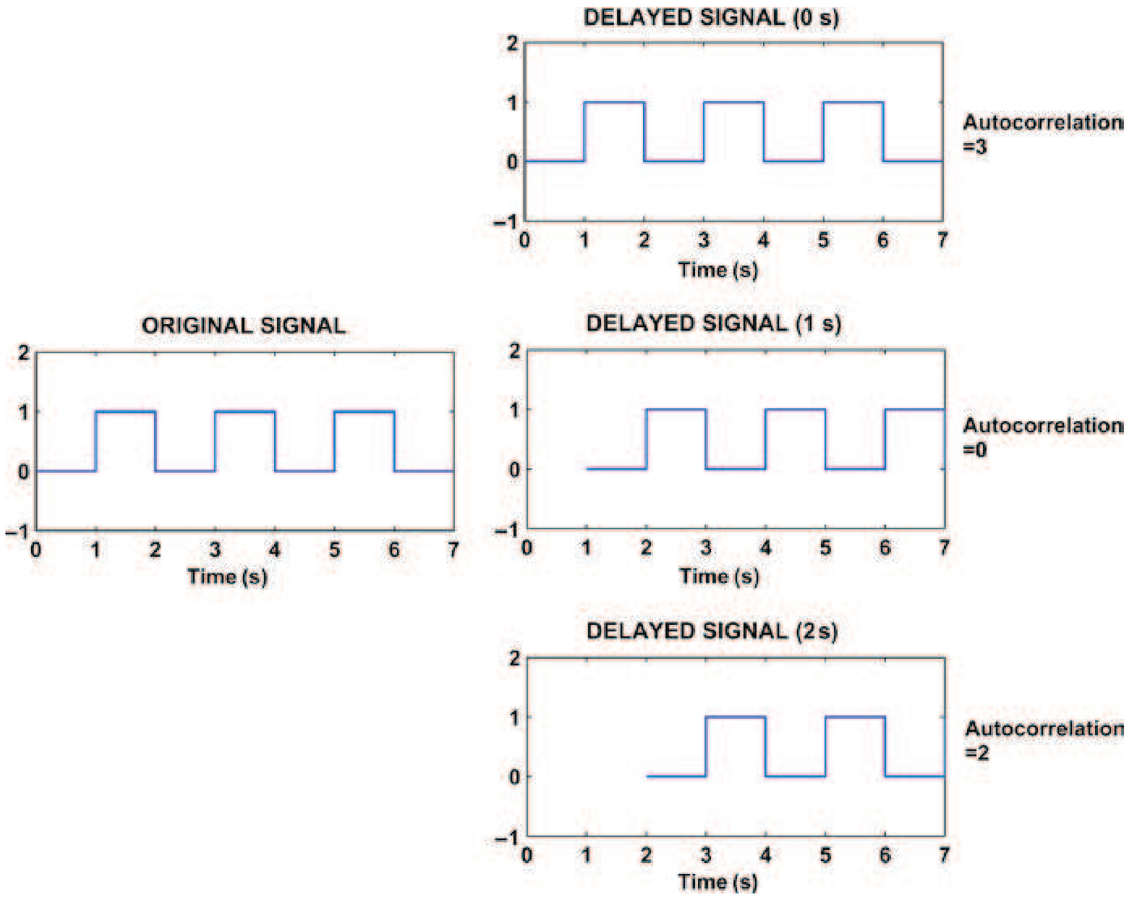


Fig. 6.2 Example of autocorrelation of a simple square wave delayed at 0, 1, and 2 s

The first method we will introduce is called autocorrelation. Autocorrelation is a time domain method to estimate cycle length that is similar to the template matching method discussed in chapter 5. Instead of aligning a template with different parts of the signal, in autocorrelation a copy of the signal is aligned with itself with different delays. Consider the simple square wave shown in Fig. 6.2. The value is 0 from 0–1, 2–3, and 4–5 s. The value is 1 from 1–2, 3–4, 5–6 s. When this signal is multiplied by itself with a time delay of 0, the autocorrelation value is 3. When the signal is multiplied by itself with a time delay of 1 s, the autocorrelation value is 0 because when the original signal has a value of 0, the delayed signal has a value of 1 and when the original signal has a value of 0, the delayed signal has a value of 0; thus the product is always 0. When the delay is 2 s the square waves overlap again and the value of the autocorrelation is 2. For each delay, the product of the two signals is summed. The nonzero delay that results in the next largest peak in the output is chosen as the estimate of the cycle length. The autocorrelation method applied to the more complex signal in Fig. 6.1 is illustrated in Fig. 6.3.

The autocorrelation starts out at its maximal value (the sum of square of all the points) when the delay is zero and the two signals are perfectly aligned. As the "test signal" is delayed, the autocorrelation declines in value. With a delay of 125 ms the peaks of the first signal line up with the troughs of the second signal and vice versa resulting in a negative autocorrelation value. At 250 ms the peaks are aligned with peaks and the troughs are aligned with troughs. At 375 ms the two signals are out of phase again. The complete autocorrelation is shown in Fig. 6.4. The second peak occurs at 250 ms corresponding to the 4 Hz sine wave (4 cycles/s = 1 cycle/250 ms).

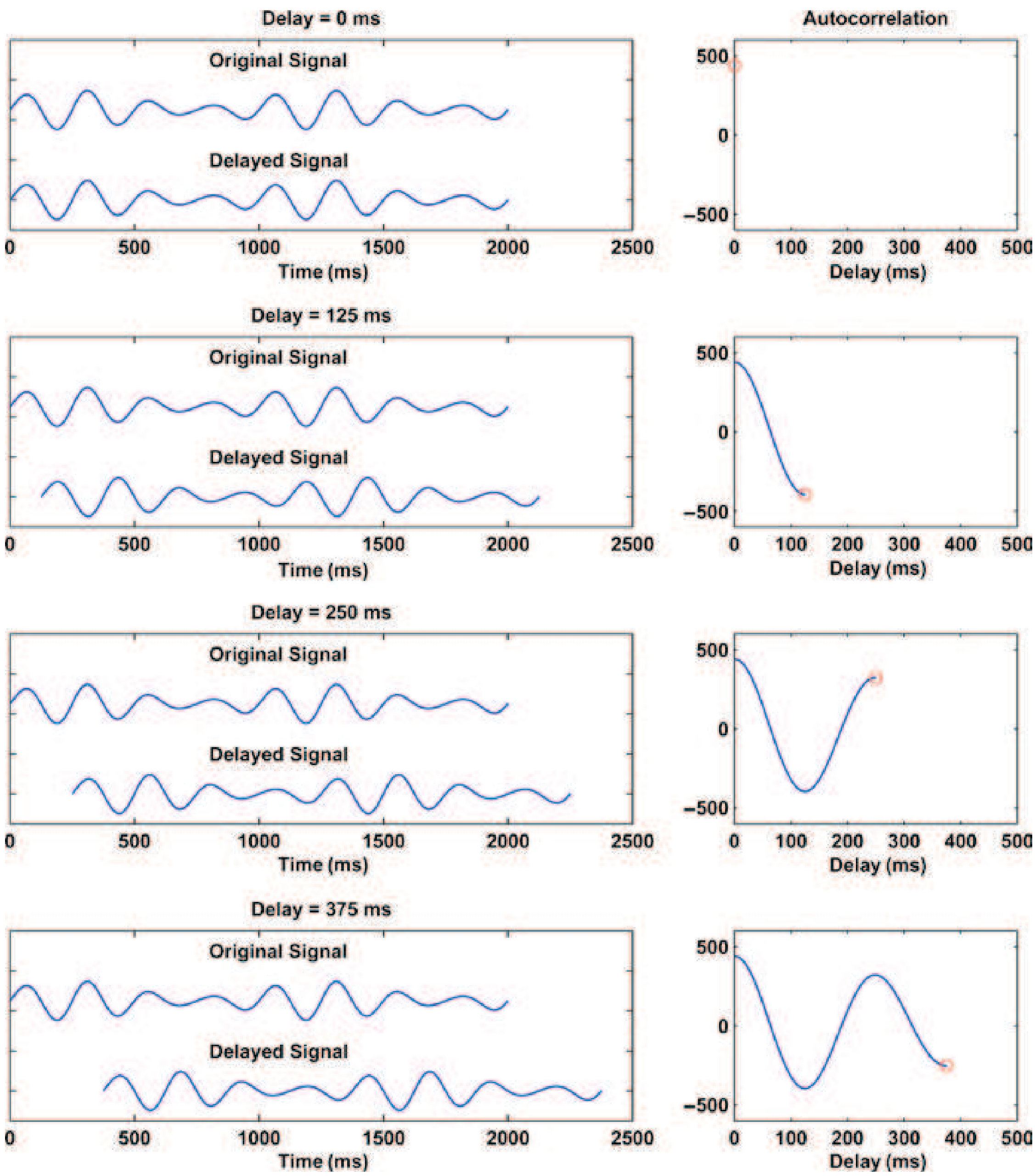


Fig. 6.3 Illustration of autocorrelation of an amplitude-modulated sine wave

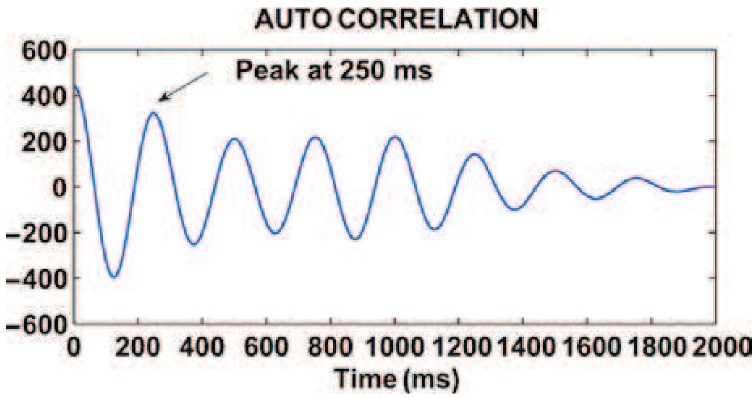


Fig. 6.4 The complete autocorrelation for the example in Fig. 6.3. The second peak occurs at 250 ms corresponding to the 4 Hz sine wave

6.3 Frequency Domain Estimation of Rate

A second common technique for rate estimation is through frequency domain analysis of the signals. As discussed in chapter 5, the premise of frequency domain analysis is that a signal can be decomposed to a set of sine waves, which when summed together will constitute the original signal. If a strong periodic element exists in the signal, in most cases there will be one sine wave component that will have an amplitude that is greater than that of the other sine waves that comprise the signal. The frequency of this sine wave is commonly referred to as the "dominant frequency." Like autocorrelation, dominant frequency analysis is used in situations where event detection is difficult because of amplitude variability, complex morphology, and noise.

Using the same example that was used for autocorrelation, the power spectrum of the 4 Hz sine wave with amplitude modulation was computed and is shown in Fig. 6.5. The peak of the power spectrum is located at 4 Hz as expected.

In the remainder of this chapter, we will examine some factors that may make event detection difficult and how they would affect rate estimation with autocorrelation and frequency domain analysis.

6.4 Noise

Noise was shown in chapter 5 to be a significant confounder in event detection, as a potential source of jitter, undersensing, and oversensing. In the setting of noise, template matching was shown in chapter 5 to be more robust than event detection techniques that attempt to detect a specific point. Autocorrelation similarly has advantages with noisy signals compared to computing intervals between waveforms. Because autocorrelation attempts to locate the strongest periodic element within a signal, it will be largely unaffected by white noise that has no dominant periodic elements. Autocorrelation of the amplitude modulated 4 Hz sine wave with added noise is shown in the middle row of Fig. 6.6. The estimated cycle length of the signal was found to be 231 ms, a 19 ms difference from the actual cycle length.