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Assignment-2

Question-1

Convolution of $W * I$

Using commutative property $W * I = I * W$

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \text{After } 180^\circ \text{ Rotation} \quad \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$m \times n = 3 \times 3$

Padding $\rightarrow$ $m-1$ & $n-1$ $\Rightarrow (2, 2)$.

Padding image I with 2 row of zeros on top and bottom, similarly for column.

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now doing $I * W$.

When W center is at $(1,1)$.

$$\begin{bmatrix} 0_{-1} & 0_0 & 0_0 & 0 & 0 & 0 \\ 0_0 & 0_0 & 0_0 & 0 & 0 & 0 \\ 0_0 & 0_0 & 1_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

$1 \times 1 + 0 = 1$ $\rightarrow$ Replacing $(1,1)$ in final output with 1.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow A_1$$

When W center is at $(2,1)$ of original image.

$$\begin{bmatrix} 0 & 0_{-1} & 0_0 & 0_0 & 0 & 0 \\ 0 & 0_0 & 0_0 & 0_0 & 0 & 0 \\ 0 & 0_0 & 1_0 & 0_1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

replacing $(2,1)$ with 0 in final output

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow A_2$$

Now for (3,1).

replacing (3,1) with 0.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \quad 0 \times 0 - 0$$

Now for (4,1).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

All zero, replacing (4,1) with 0.

Now for (1,2).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \quad 1 \times 0 + -1 \times 0 = 0$$

replacing (1,2) with 0.

Now ~~for (1,3)~~ (2,2).

$$-1 \times 0 + +1 \times 1 = +1$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

replacing (2,2) with +1.

Rest all zero upto (3,3).

$$-1 \times 1 + 1 \times 0 + 1 \times 0 = -1$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

replacing (3,3) with −1.

Rest all zero upto (4,4).

$$1 \times -1 + 1 \times 0 = -1$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

replacing (4,4) with −1.

Final output matrix

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \xrightarrow{\text{Cropping } 4\times4} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Q-3 b

Image $\rightarrow f(x,y)$.

3x3 Box filter = $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = B_f$

Obtaining high frequency details and adding that mask to original image.

$$f(x,y) + \left(f(x,y) - f(x,y) * B_f\right) \quad \text{--- (1)}$$

we know $f(x,y) * \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow f(x,y)$.

$\rightarrow$ Impulse fnc. (δ)

Putting this in eq (1).

$$2f(x,y) * \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - f(x,y) * B_f$$

$\Rightarrow f(x,y) * \Delta + f(x,y) * \Delta$

$\Rightarrow f(x,y) * (2\Delta)$

Using Distributive property.

$$f(x,y) * \left[\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - B_f} \right]$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

$$= \begin{bmatrix} -1/9 & -1/9 & -1/9 \\ -1/9 & 17/9 & -1/9 \\ -1/9 & -1/9 & -1/9 \end{bmatrix} \Rightarrow \frac{1}{9}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 17 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

On convolving $f(x,y) * \frac{1}{9}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 17 & -1 \\ -1 & -1 & -1 \end{bmatrix}$ we will get unsharp masked image

Q-4

Q-3 b

Image = f(x,y)

7x7 Box filter =

$$\frac{1}{49}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = B_F$$

Performing unsharp masking

$$f(x,y) + f(x,y) - f(x,y) * B_F \quad — (1)$$

using Delta function

$$f(x,y) * \delta(x,y) = f(x,y)$$

$$\begin{bmatrix} 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix}$$

using this in eq^n (1), replacing f(x,y) with $f(x,y) * \delta(x,y)$

$$2 f(x,y) * \delta(x,y) - f(x,y) * B_F$$

$$f(x,y) * \delta + f(x,y) \delta$$

$$f(x,y) * [2\delta]$$

Using Distributive property

$$f(x,y) * \left[\underbrace{2\delta(x,y) - B_F}\right] \quad — (ii)$$

$$\begin{bmatrix} 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&2&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix} - \frac{1}{49}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$2 - \frac{1}{49}$$

$$= \left.\begin{bmatrix} -1/49 & -1/49 & \cdots & -1/49 \\ \vdots & & & \\ -1/49 \ -1/49 \ -1/49 & \frac{97}{49} & -1/49 \ -\frac{1}{49} \ -\frac{1}{49} \\ \vdots & & & \\ -1/49 & \cdots & & -1/49 \end{bmatrix}\right\} 7$$

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$$= \frac{1}{49}\underbrace{\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 97 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}}_{7} \left.\vphantom{\begin{bmatrix}1\\1\\1\\1\\1\\1\\1\end{bmatrix}}\right\} 7$$

Comparing eq (4) with $f(x,y) * w(x,y)$

$w(x,y)$ Defined above

Q-4 Inverse Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$X(\omega) = \delta(\omega - k\omega_0) + \delta(\omega + k\omega_0)$$

Put $X(\omega)$ in above eq^n.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\delta(\omega - k\omega_0) + \delta(\omega + k\omega_0)\right] e^{j\omega t}\, d\omega$$

$$x(t) = \frac{1}{2\pi}\left[\int_{-\infty}^{\infty}\delta(\omega - k\omega_0)\, e^{j\omega t}\, d\omega + \int_{-\infty}^{\infty}\delta(\omega - k\omega_0)\cdot e^{j\omega t}\, d\omega\right]$$

~~$x(t) = \frac{1}{2\pi}$~~ we know $\delta(t) = 1$ when $t = 0$

$\omega + k\omega_0 = 0 \quad \omega = k\omega_0$

$$x(t) = \frac{1}{2\pi}\left[e^{jk\omega_0 t} + e^{-jk\omega_0 t}\right]$$

$$x(t) = \frac{1}{2\pi}\left[\cos k\omega_0 t + \cancel{j\sin k\omega_0 t} + \cos k\omega_0 t - \cancel{j\sin k\omega_0 t}\right]$$

$$x(t) = \frac{1}{\cancel{2}\pi}\cancel{2}\cos k\omega_0 t = \frac{\cos k\omega_0 t}{\pi}$$