DIP Assignment – 4

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Q-1

Solve for W

$$\sum_k \sum_L |F(k,L) - \hat{F}(k,L)|^2 + \lambda |\hat{F}(k,l)\, L(k,L)|^2$$

$$\hat{F}(k,L) = W(k,l)\, G(k,l)$$

Substituting $\hat{F}$

$$\sum_k \sum_L |F(k,L) - W(k,l)\, G(k,l)|^2 + \lambda |W(k,l)\, G(k,l)\, L(k,l)|^2$$

Using Property →

I $\quad \sum_{k,l} |a-b|^2$

$$= \sum_{k,l} (a-b)(a-b)^*$$

II $\quad \frac{\partial}{\partial z} z z^* = 2 z^*$

$$\Rightarrow \quad (F - WG)(F - WG)^* + \lambda (WGL)(WGL)^*$$

Differentiating this with W

$$\frac{\partial}{\partial W} \Rightarrow (F - WG)(F - WG)^* + \lambda (WGL)(WGL)^*$$

equating to 0,

$$= \cancel{2}(F - WG)^*(-G) + \cancel{2}\lambda (WGL)^*(GL) = 0$$

$$\Rightarrow (WG - F)^* + \lambda L \; W^* G^* L^* = 0$$

$$W^* G^* - F^* + \lambda |L|^2 W^* G^* = 0$$

$$W^* = \frac{F^*}{G^*(1 + \lambda |L|^2)}$$

$$(W^*)^* = \frac{F}{G(1+\lambda|L|^2)}.$$

$$W = \frac{F}{G(1+\lambda|L|^2)}.$$

We know that $G = HF + N$.

Putting value of G.

$$W = \frac{F}{(HF+N)(1+\lambda|L|^2)}.$$

$$W = \frac{1}{\left(H+\frac{N}{F}\right)(1+\lambda|L|^2)}$$

$$\sum_u \sum_v W(u,v) = \frac{1}{\left(H(u,v) + \frac{N(u,v)}{F(u,v)}\right)\left(1+\lambda|L(u,v)|^2\right)}$$

Q-2 (a) given →

$$f(x,y) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 100 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

y
x

$\theta = \{0, \pi/4, \pi/2\}$. center at 100.

$\rho = \{0, -1, 1\}$

Radon transform →

$$g(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\cdot\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy$$

$$\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}$$

For all θ value. $\cos\theta$ & $\cos 0 = 1$ $\sin 0 = 0$

$\theta = 0$

$$g(P, 0) = \sum_{x} \sum_{y} f(x, y)\, \delta(x - P).$$

$\theta = \pi/4$

$$g(P, \pi/4) = \sum_{x} \sum_{y} f(x, y)\, \delta\left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} - P\right)$$

$$= \sum_{x} \sum_{y} f(x, y)\, \delta\left(\frac{x+y}{\sqrt{2}} - P\right)$$

$\theta = \frac{\pi}{2}$

$$g(P, \pi/2) = \sum_{x} \sum_{y} f(x, y)\, \delta(y - P)$$

For all P values

when $\theta = 0$

$P = 0$

$$g(0, 0) = \sum_{x} \sum_{y} f(x, y)\, \delta(x - 0)$$

$$= \sum_{x} \sum_{y} f(x, 0) \quad 2^{nd} \text{ column}$$

$$= 0 + 100 + 2 = 102$$

$P = -1$

$$g(-1, 0) = \sum_{x} \sum_{y} f(x, y)\, \delta(x + 1) \quad 1^{st} \text{ column}$$

$$= 1 + 0 + 1 = 2$$

$P = 1$

$$g(1, 0) = \sum_{x} \sum_{y} f(x, y)\, \delta(x - 1).$$

$$= 1 + 2 + 2 = 5$$

$$\boxed{g(P, 0) = \begin{pmatrix} 2 & 102 & 5 \\ P = -1 & 0 & 1 \end{pmatrix}}$$

$\theta = \frac{\pi}{2}$

$p = 0$

$$g(0, \tfrac{\pi}{2}) = \sum_x \sum_y f(x,y) \cdot \delta(y).$$

$$= 0 + 100 + 2 = 102.$$

$p = -1$

$$g(-1, \tfrac{\pi}{2}) = \sum_x \sum_y f(x,y) \cdot \delta(y+1).$$

$$= 1 + 2 + 2 = 5$$

$p = 1$

$$g(1, \tfrac{\pi}{2}) = \sum_x \sum_y f(x,y) \cdot \delta(y-1).$$

$$= 1 + 0 + 1 = 2.$$

$$\boxed{g(p, \tfrac{\pi}{2}) = \{\underset{p=-1}{5} \quad \underset{0}{102} \quad \underset{1}{2}\}}$$

$\theta = \pi/4.$

$p = 0$

$$g(0, \tfrac{\pi}{4}) = \sum_x \sum_y f(x,y) \, \delta\left(\frac{x+y}{\sqrt{2}} - p\right).$$

$$= 1 + 100 + 2 = 103$$

g x,y ~~does not have~~ integral value for: $P = -1, 1.$

$$g(-1, \tfrac{\pi}{4}) = g(1, \pi/4) = 0.$$

$$\boxed{g(p, \tfrac{\pi}{4}) = \{\underset{p=-1}{0} \quad \underset{0}{103} \quad \underset{1}{0}\}}$$

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Q.2 (b). $f_\theta(x,y) = g(x\cos\theta + y\sin\theta, \theta)$

for $\theta = 0$

$f_0(x,y) = \cancel{g(x,0)}\ g(x,0)$.

for $x = [-1, 0, 1]$.

$$f_0(x,y) = \begin{bmatrix} 2 & 102 & 5 \\ 2 & 102 & 5 \\ 2 & 102 & 5 \end{bmatrix}$$

$\theta = \frac{\pi}{2}$

$f_{\frac{\pi}{2}}(x,y) = g(y, \frac{\pi}{2})$

$\cancel{\text{For}}\ g(y, \frac{\pi}{2})$ for all values of y $= \{-1, 0, 1\}$

$$f_{\frac{\pi}{2}}(x,y) = \begin{bmatrix} 2 & 2 & 2 \\ 102 & 102 & 102 \\ 5 & 5 & 5 \end{bmatrix}$$

$\theta = \pi/4$

$f_{\pi/4}(x,y) = g\left(\frac{x+y}{\sqrt{2}}, \frac{\pi}{4}\right)$

for the value of $(x,y) \Rightarrow [-1, 0, 1]$

we get

$$f_{\pi/4}(x,y) = \begin{bmatrix} 103 & 0 & 0 \\ 0 & 103 & 0 \\ 0 & 0 & 103 \end{bmatrix}$$

Q-2 (c). Reconstructing $f(x,y)$ from $f_\theta(x,y)$.

$$f(x,y) = \sum_{\theta=(0,\frac{\pi}{2},\frac{\pi}{4})} f_\theta(x,y).$$

$$= \begin{bmatrix} 2 & 102 & 5 \\ 2 & 102 & 5 \\ 2 & 102 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 2 \\ 102 & 102 & 102 \\ 5 & 5 & 5 \end{bmatrix} + \begin{bmatrix} 102 & 0 & 0 \\ 0 & 102 & 0 \\ 0 & 0 & 103 \end{bmatrix}$$

$$= \begin{bmatrix} 107 & 104 & 7 \\ 104 & 307 & 107 \\ 7 & 107 & 113 \end{bmatrix}$$