DIP Assignment - 1

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Q-1

$$S_k = T(r_k) = (L-1)\sum_{j=0}^{k} P_r(r_j) \quad k = 0, 1 \dots L-1$$

$\sum P_r(r_j)$ ↳ CDF

Bit depth = 2 $\quad L = 2^{\text{Bit depth}}$

$L = 4 \Rightarrow L-1 = 3$

Given →

r	P(r)	CDF	T(rk)	T(rk)/s	Rounded value
0	0.4	0.4	3 × 0.4	1.2	1
1	0.2	0.6	3 × 0.6	1.8	2
2	0	0.6	3 × 0.6	1.8	2
3	0.4	1	3 × 1	3	3

$S_0 = (L-1)\, P_r(r_0)$

$= 3 \times 0.4 = 1.2 \simeq 1$

$S_1 = (L-1)\{P_r(r_0) + P_r(r_1)\}$

$= (3)\{0.6\} = 1.8 \simeq 2$

$S_2 = (L-1)\{P_r(r_0) + P_r(r_1) + P_r(r_2)\}$

$= 3\{0.6\} = 1.8 \simeq 2$

$S_3 = (L-1)\{P_r(r_0) + P_r(r_1) + P_r(r_2) + P_r(r_3)\}$

$S_3 = 3 \times 1 = 3$

Pixels with intensity 0 are changed to 1 and so on.

Mapping →

r		s		P(s)
0	→	1	→	0.4
1	→	2	→	0.2
2	→	2	→	0.2
3	→	3	→	0.4

Q-2

We know the Probability distribution remains same so equating the distribution we can find out the Transformation function.

$$P_s(s) = P_r(r)\left|\frac{dr}{ds}\right|$$

$$\int P_s(s)\, ds = \int P_r(r)\, dr$$

Integrating Both sides

Given $P_s(s) = s$

$$\int_0^s s\, ds = \int_0^r P_r(r)\, dr$$

$$\frac{s^2}{2} = \int_0^r P_r(r)\, dr$$

$$s^2 = 2 \int_0^r P_r(r)\, dr$$

$$s = \sqrt{2 \int_0^r P_r(r)\, dr}$$

We know ~~$s = T_r(r)$~~ $s = T(r)$

$$T(r) = \sqrt{2 \int_0^r P_r(w)\, dw}$$

$$\int_0^r P_r(w)\, dw = CDF = F(r)$$ This denotes CDF of the given PDF.

$$T(r) = \sqrt{2\, F(r)}$$

Q-3 Given initial & final coordinates; need to ~~for~~ find Affine Transform matrix.

Affine matrix.

$$T = \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

$X = VT$

$(V = UT)$

$X \cancel{=V} T$

$V = UT$

$U^T V = U^T U T$

$T = (U^T U)^{-1} U^T V$

using This eq, we can find the Transform matrix.

Scanned by CamScanner

~~A2~~

$$U = \begin{bmatrix} 10 & 15 & 1 \\ 8 & 3 & 1 \\ 11 & 17 & 1 \\ 5 & 11 & 1 \\ 6 & 13 & 1 \end{bmatrix} \qquad V = \begin{bmatrix} 33 & 20 & 1 \\ 18 & 7 & 1 \\ 37 & 22 & 1 \\ 20 & 13 & 1 \\ 23 & 16 & 1 \end{bmatrix}$$

$$U^T = \begin{bmatrix} 10 & 8 & 11 & 5 & 6 \\ 15 & 3 & 17 & 11 & 13 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Note→ calculations performed in rough via scientific cal.

$$U^T U = \begin{bmatrix} 346 & 494 & 40 \\ 494 & 813 & 59 \\ 40 & 59 & 5 \end{bmatrix}$$

$$(U^T U)^{-1} = \begin{bmatrix} 0.045 & -0.008 & \cancel{-0.0}\,0.26 \\ -0.008 & 0.010 & -0.051 \\ -0.264 & -0.051 & 2.913 \end{bmatrix}$$

Putting these values in the equation we get the final transformation matrix,

$$T = \begin{bmatrix} 1.945 & 0.5189 & 0 \\ 0.928 & 0.977 & 0 \\ -0.315 & -0.0872 & 1 \end{bmatrix}$$

Q-5

Scaling → $\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Translation → $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$

First scaling is performed, then Translation so

$$\cancel{T\,S} \quad \cancel{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix} \begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}} \Rightarrow \cancel{\begin{matrix} c_x & 0 & 0 \\ 0 & c_y & 0 \end{matrix}}$$

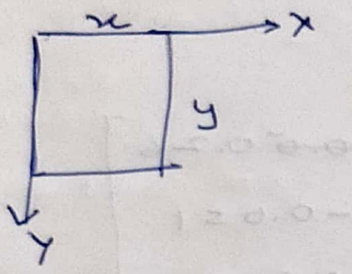
first scaling then translation → . S × T

$$\underbrace{\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{S} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}}_{t}$$

$$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ t_x & t_y & 1 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & 10 & 1 \end{bmatrix}$$

Scaling $c_x = 2$
$c_y = 1$

Translation → $T_x = 10$
$T_y = 10$



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