METRIC RECTIFICATION AND ROBUST HOMOGRAPHY

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Preliminaries

In the initial phase of this assignment, two utility functions, namely *warp_image()* and *transform_homography()*, were developed. These functions aim to overlay a source image onto a destination image, where each point x (in homogeneous coordinates) in the source image is mapped to Hx in the destination image, utilizing a homography matrix H. Due to the discrete nature of images, this mapping may lead to points in the destination image not being mapped to or multiple points in the source image being mapped to a single point in the destination image. To address this, the inverse mapping H^{-1} is considered, ensuring each point in the destination image is mapped to one or no points in the source image. The implementation includes both *cv2.remap()* code and direct remapping code, with the latter being more explicit in handling cases where no points are mapped due to out-of-bounds.

Part 1

In this part two methods for metric rectification of an image were implemented. The first method uses stratification(affine rectification followed by metric rectification) while the second approach used a one step method. Both relied on theory and examples from Hartley and Zisserman 2003 which won't be elaborated on here. Only three things are to be noted - handling of more than two sets of parallel lines, the problem of the unidentified similarity transform, a comment on the one-step implementation.

For affine rectification we find points X that are on the line at infinity which we want to fit a line through. Although a well-known problem, it is not described in Hartley and Zisserman 2003 and a brief justification for the equation used is in place. Letting $X_i = (x_i, y_i, 1)^\top$ and $A = (a, b, c)^\top$ the problem is (least-squares)

$$\min_{a,b,c} \Sigma_{i=1}^{N}(ax_i + by_i + c)^2 = \min_{A\in\mathbb{R}^3} \Sigma A X_i X_i^\top A^\top = \min_{A\in\mathbb{R}^3} A\left(\Sigma_{i=1}^{N} X_i X_i^\top\right) A^\top \tag{1}$$

This is a QP problem that is clearly convex. Using the KKT conditions and setting up the KKT system as per Nocedal and Wright 2006 we find that the solution is given by

$$\Sigma_{i=1}^{N} X_i X_i^\top A = \mathbf{0} \tag{2}$$

In other words the nullspace of the matrix, which can be obtained as usual.

The second point is that rectification is up to a similarity transform. This means that the processed image often was rotated, translated and/or stretched. In order to solve this, manual tuning was mainly utilized. Also a function *warp_image_on_canvas* - a specialization of the already implemented function *warp_images_all* - was created guaranteeing that the mapped image would at least be visible on the canvas(It was included here to avoid circular inclusions).

Lastly it should be noted that the *compute_metric_rectification_one_step* used SVD to directly obtain the homography $H_A = U$ by $C_\infty^{*\prime} = USC_\infty^{*}SU^\top$(see Hartley and Zisserman 2003)and Cholesky decomposition was not used.

Part 2

In the last part a RANSAC algorithm for image stitching was to be implemented. Specifically two functions *compute_homography_error* and *compute_homography_ransac* were to be implemented. The only thing worth noting is that the implementation does not terminate once a model with a number of inliners above a certain threshold is found which is included in the description found in Hartley and Zisserman 2003.

References

Hartley, Richard and Andrew Zisserman (2003). *Multiple View Geometry in Computer Vision.* PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE.

Nocedal, Jorge and Stephen J. Wright (2006). *Numerical Optimization.* Springer.