Bubble Sort vs Selection Sort: Mathematical Analysis

Bubble Sort Analysis

Comparisons per pass:

Pass 1: (n-1) comparisons

Pass 2: (n-2) comparisons

Pass 3: (n-3) comparisons

Pass (n-1): 1 comparison

Total comparisons = (n-1) + (n-2) + (n-3) + ... + 2 + 1 = $\sum_{i=1}^{n-1} i$

Derivation of $\sum_{i=1}^{n-1} i$:

S = 1 + 2 + 3 + ... + (n-2) + (n-1)

S = (n-1) + (n-2) + ... + 3 + 2 + 1

Adding these equations:

2S = [1+(n-1)] + [2+(n-2)] + [3+(n-3)] + ... + [(n-2) +2] + [(n-1) +1]

2S = n + n + n + ... + n + n (n-1 terms)

2S = n(n-1)

$S = \frac{n(n-1)}{2}$

Therefore:

$$\text{Total comparisons} = \frac{n(n-1)}{2} = \frac{n^2-n}{2} = O(n^2)$$

Selection Sort Analysis

Comparisons per pass:

Pass 1: (n-1) comparisons

Pass 2: (n-2) comparisons

Pass 3: (n-3) comparisons

Pass (n-1): 1 comparison

Total comparisons = (n-1) + (n-2) + (n-3) + ... + 2 + 1 = $\sum_{i=1}^{n-1} i = \frac{n(n-1)}{2} = O(n^2)$

Both algorithms perform the same number of comparisons (O (n^2)) but Selection Sort performs fewer swaps (O (n)) compared to Bubble Sort (O (n^2))