5. (10 points) Derive the order of the error with respect to the sin and cosine approximations.

Sin and Cos Error at x=1/N		
Series	sin(x)=x–$x^3/3!+x^5/5!+\dots$	cos(x)=1–$x^2/2!+x^4/4!$-...
Approx	$sin(x) \approx x$	$cos(x) \approx 1 - x^2/2$
Error Big O()	(sin series - approx) at x=1/N	(cos series - approx) at x=1/N

For sine:

- Error = sin(x) – x = (x - $\frac{x^3}{6}$ + $\frac{x^5}{120}$ - ...) - x
- Error = - $\frac{x^3}{6}$ + $\frac{x^5}{120}$ - ...
- Substituting x = = $\frac{1}{N}$
- Error = $-\frac{\frac{1}{N^3}}{6} + \frac{\frac{1}{N^5}}{120}$ - ...
- Error = - $\frac{1}{6N^3} + \frac{1}{120N^5}$ - ...
- When N is large, $\frac{1}{6N^3}$ is much larger than $\frac{1}{120N^5}$, so:
- Error ≈ - $\frac{1}{6N^3}$
- This means the error is of order o ($\frac{1}{N^3}$)

For cosine:

- Error = cos(x) - (1 - $\frac{x^2}{2}$) = (1 - $\frac{x^2}{2}$ + $\frac{x^4}{24}$) - ...) - (1 - $\frac{x^2}{2}$)
- Error = $\frac{x^4}{24}$ - ...
- Substituting x = $\frac{1}{N}$
- Error = $\frac{\frac{1}{N^4}}{24} - \frac{\frac{1}{N^6}}{720}$ + ...
- Error = $\frac{1}{24N^4} - \frac{1}{720N^6}$ + ...
- When N is large, $\frac{1}{24N^4}$ is much larger than $\frac{1}{720N^6}$ so:
- Error ≈ $\frac{1}{24N^4}$
- This means the error is of order o ($\frac{1}{N^4}$)