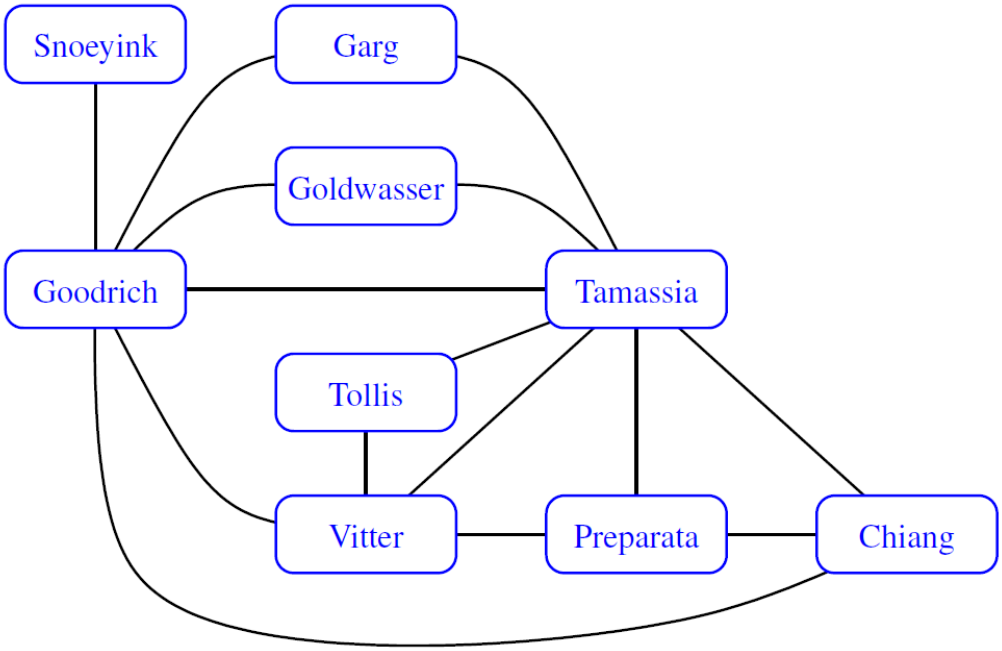
Data Structure Assignment 4

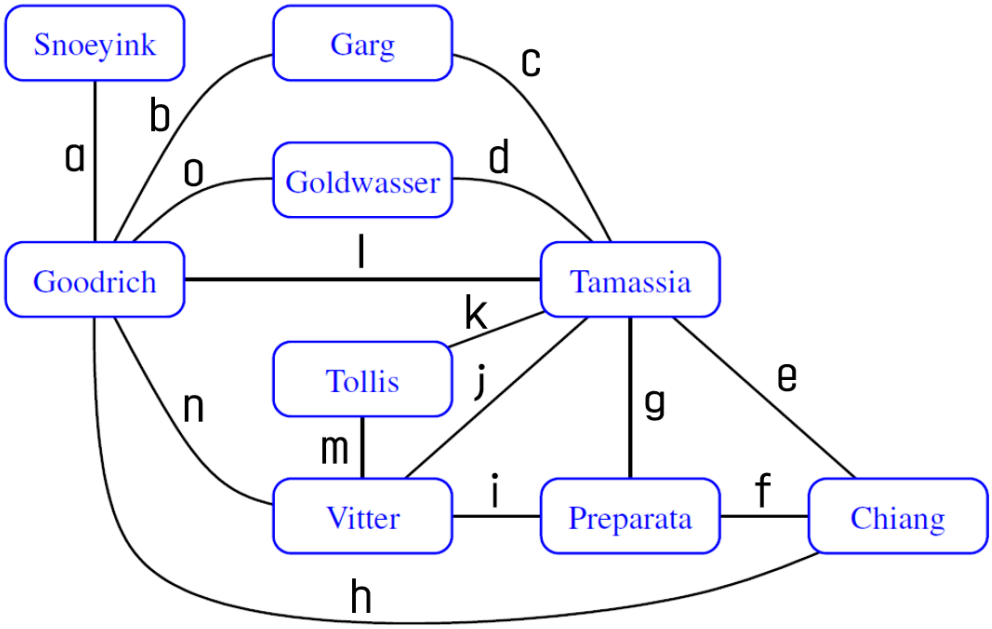
201911013 곽현우

A9 - Graphs (15+3 points)

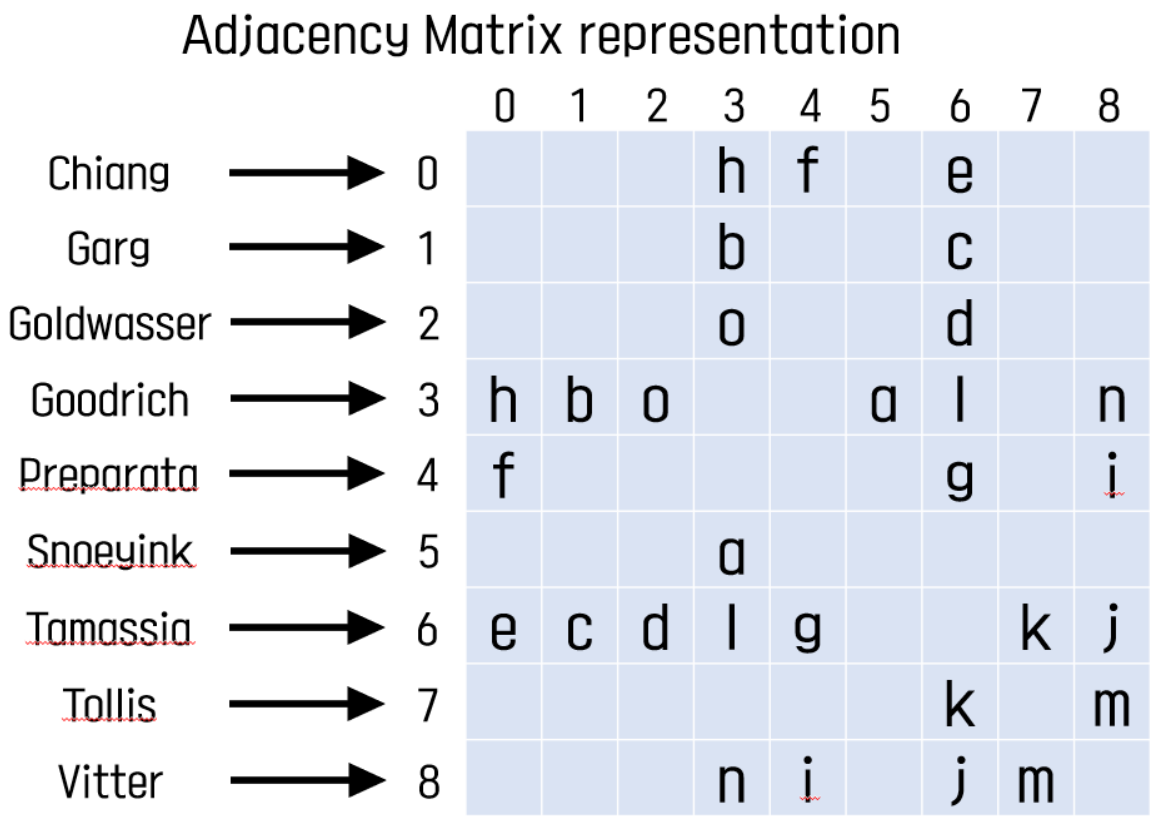
A9-1 (2 points) (Enumerate the vertices in alphabetical order.)



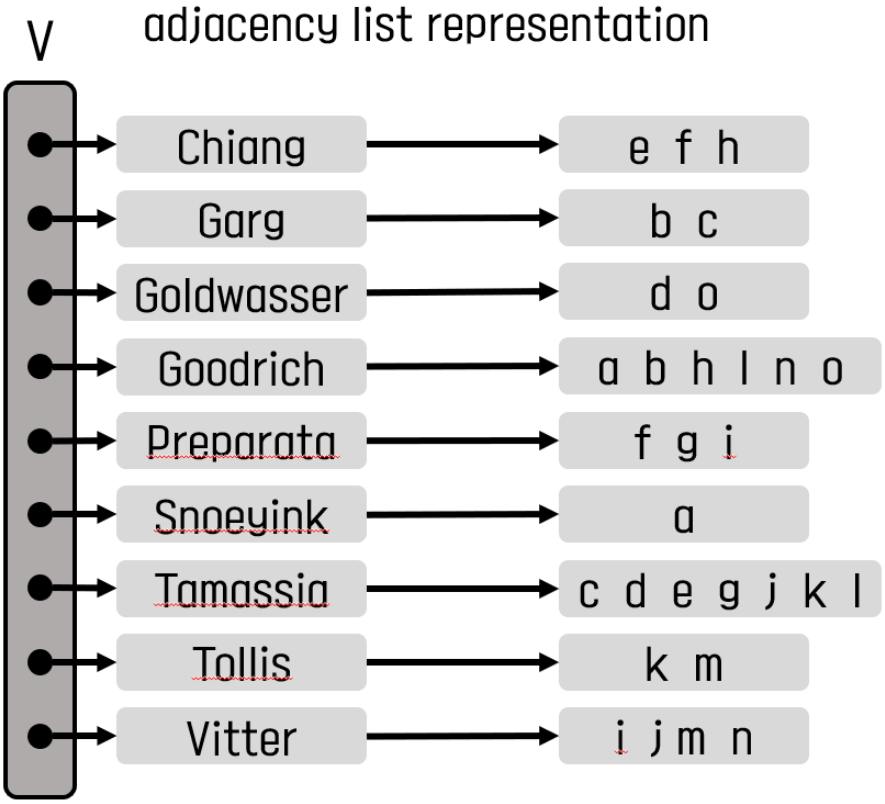
Edge 의 이름은 다음과 같이 하였다.



1. Draw an adjacency matrix representation of the undirected graph shown above.



2. Draw an adjacency list representation of the undirected graph shown above.



A9-2 (3 points)

Would you use the adjacency matrix structure or the adjacency list structure in each of the following cases? Justify your choice.

1. The graph has 10,000 vertices and 20,000 edges, and it is important to use as little space as possible.
2. The graph has 10,000 vertices and 20,000,000 edges, and it is important to use as little space as possible.
3. You need to answer the query get edge(u,v) as fast as possible, no matter how much space you use.

Vertices 의 개수를 n 개, Edge 의 개수를 m 개라 할 때, Adjacent matrix structure 의 저장공간은 $O(n^2)$, Adjacent List Structure 의 저장공간은 O(n+M)이므로 1 번의 경우에는 10000^2보다 10000+20000 이 더 작기 때문에 Adjacent List Structure 이 적절하고 2 번의 경우도 10000^2보다 20010000 가 더 작기 때문에 Adjacenct List Structure 가 더 적절하다.

3 번의 경우 가장 빠르게 Edge(u,v)를 찾기 위해서는 Adjacent Matrix Structure 을 사용하는 것이 효율적이다. 왜냐하면 u,v 각각에 대응되는 행과 열을 통해 Matrix 에 저장된 Edge 를 찾는데 걸리는 시간이 O(1)으로 Adjacent List 로 구현할 때 O(min(deg(u),deg(v)))에 비해 매우 빠르기 때문이다.

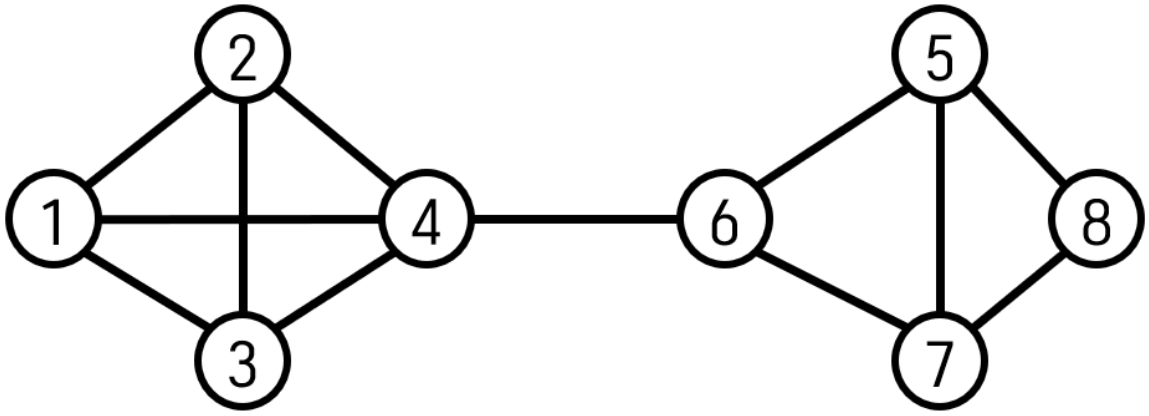
A9-3 (3 points)

Let G be an undirected graph whose vertices are the integers 1 through 8, and let the adjacent vertices of each vertex be given by the table below:

vertex	adjacent vertices
1	(2, 3, 4)
2	(1, 3, 4)
3	(1, 2, 4)
4	(1, 2, 3, 6)
5	(6, 7, 8)
6	(4, 5, 7)
7	(5, 6, 8)
8	(5, 7)

Assume that, in a traversal of G, the adjacent vertices of a given vertex are returned in the same order as they are listed in the table above.

1. Draw G.



2. Give the sequence of vertices of G visited using a DFS traversal starting at vertex 1.

   {1, 2, 4, 6, 5, 8 ,7, 3}

3. Give the sequence of vertices visited using a BFS traversal starting at vertex 1.

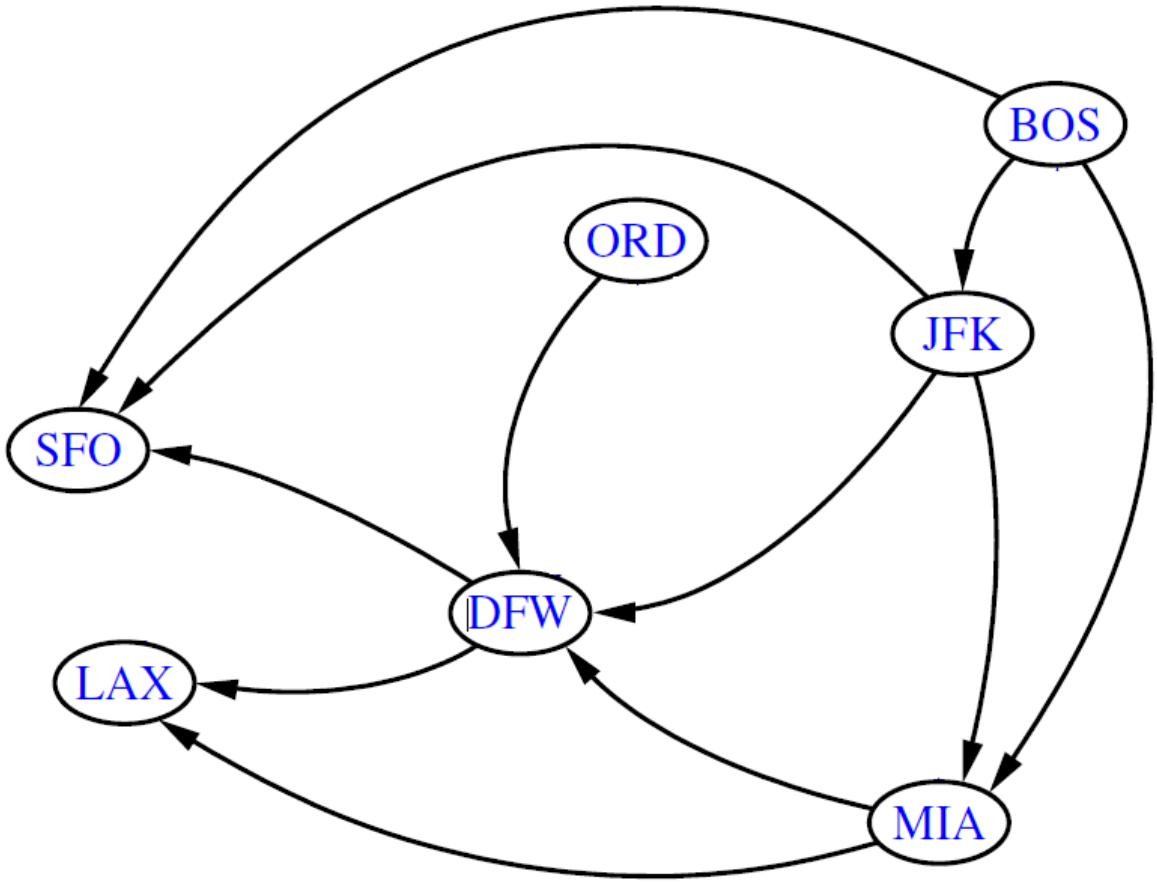
   {1, 2, 4, 3, 6, 5 ,7, 8}

A9-4 (2 point)

How many edges are in the transitive closure of a graph that consists of a simple directed path of n vertices? Explain why.

Vertex 1,2,…n 이 차례로 Simple directed path 를 이루는 경우를 생각해보자. Vertex 1 은 Incoming Edge 가 없고 Vertex n 은 Outgoing Edge 가 없기 때문에 나머지 Vertices 들에 대하여 생각해본다. Vertex2 는 1 과 3 을 연결해주기 때문에 1 과 3 을 연결하는 Transitive closure 가 생긴다. Vertex3 의 경우는 2 와 4, 1 과 4 를 연결해주기 때문에 2 개의 Transitive closure 가 생긴다. 이렇게 Vertex k 의 경우 Trasitive Closure 가 k-1 개 생긴다.(1<k<n) 따라서 모든 operation 이 수행된 후의 Edge 의 개수는 (1+2+…+n-2)(Transitive Closure)+n-1(원래있던 Edge 의 개수) = $\frac{n(n-1)}{2}$개 이다.

A9-5 (1 point)



Compute a topological ordering for this directed graph.(The Following operations are operating in the copy of above graph)

1) Start DFS From BOS

2) LAX is reached, and LAX has no outgoing edges. So LAX is no.7(the number of vertices is 7)

Delete incident edges.

3) Next, SFO is reached. So SFO is no.6. Then, Delete incident edges.

4) Next, DFW is reached. So DFW is no.5. Then, Delete incident edges.

5) Next, MIA is reached. So MIA is no.4. Then, Delete incident edges.

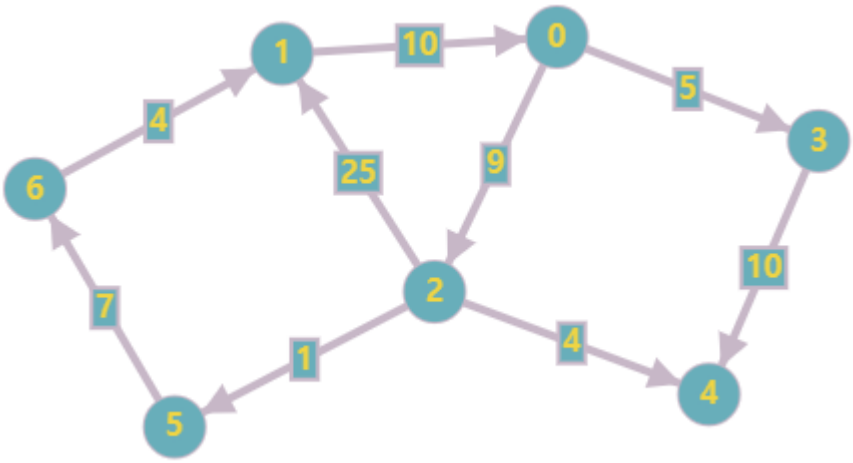
6) Next, JFK is reached. So JFK is no.3. Then, Delete incident edges.

7) Next, BOS is reached. So BOS is no.2. Then, Delete incident edges.

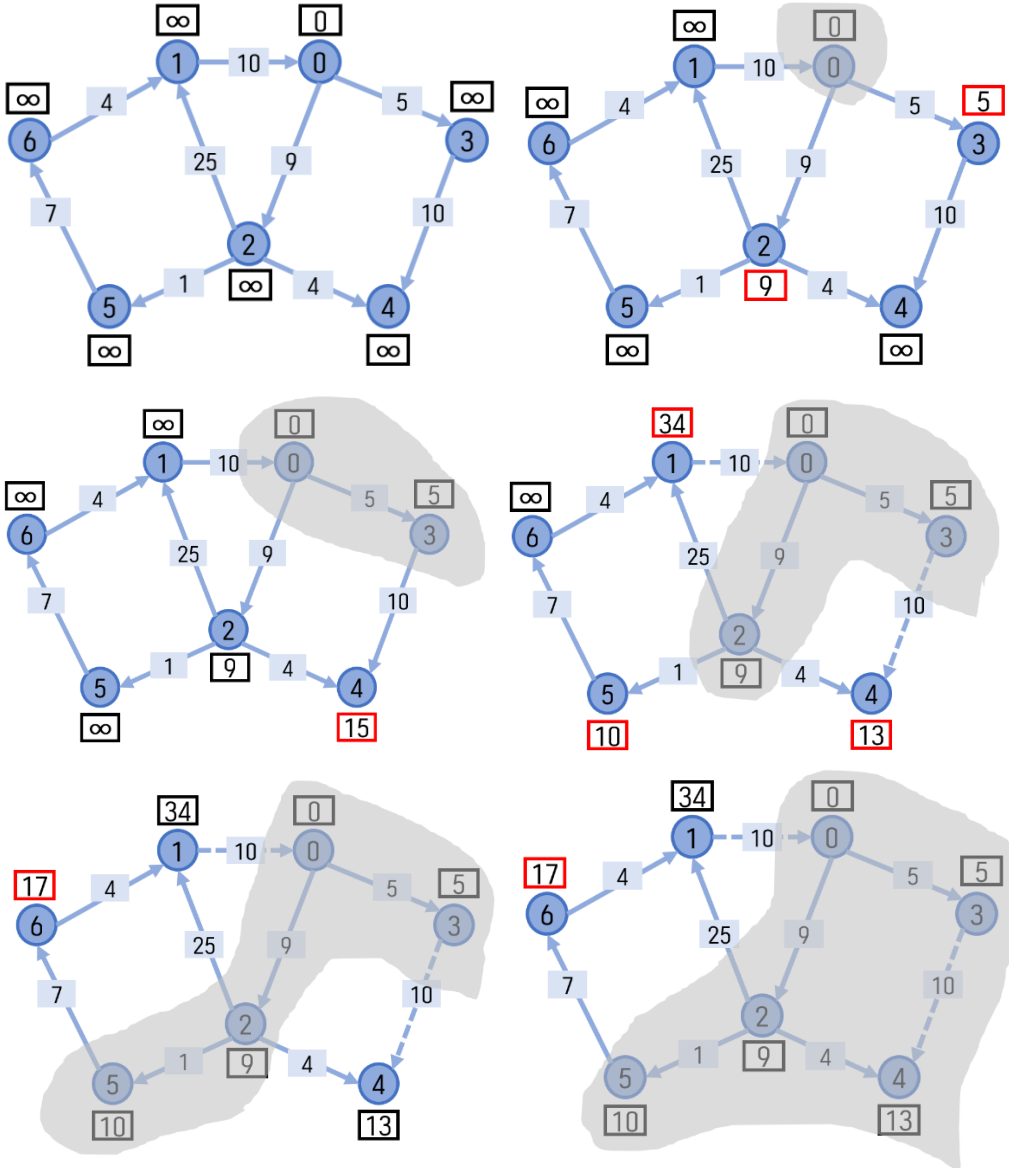
8) The Last vertex, ORD is no.1

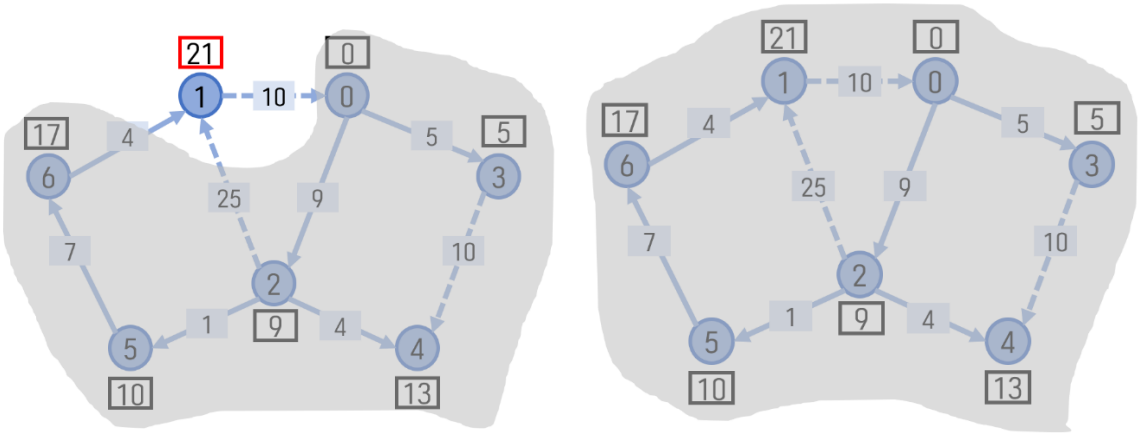
So a topological ordering in this directed graph is ORD, BOS, JFK, MIA, DFW, SFO, LAX

A9-6 (4 points)

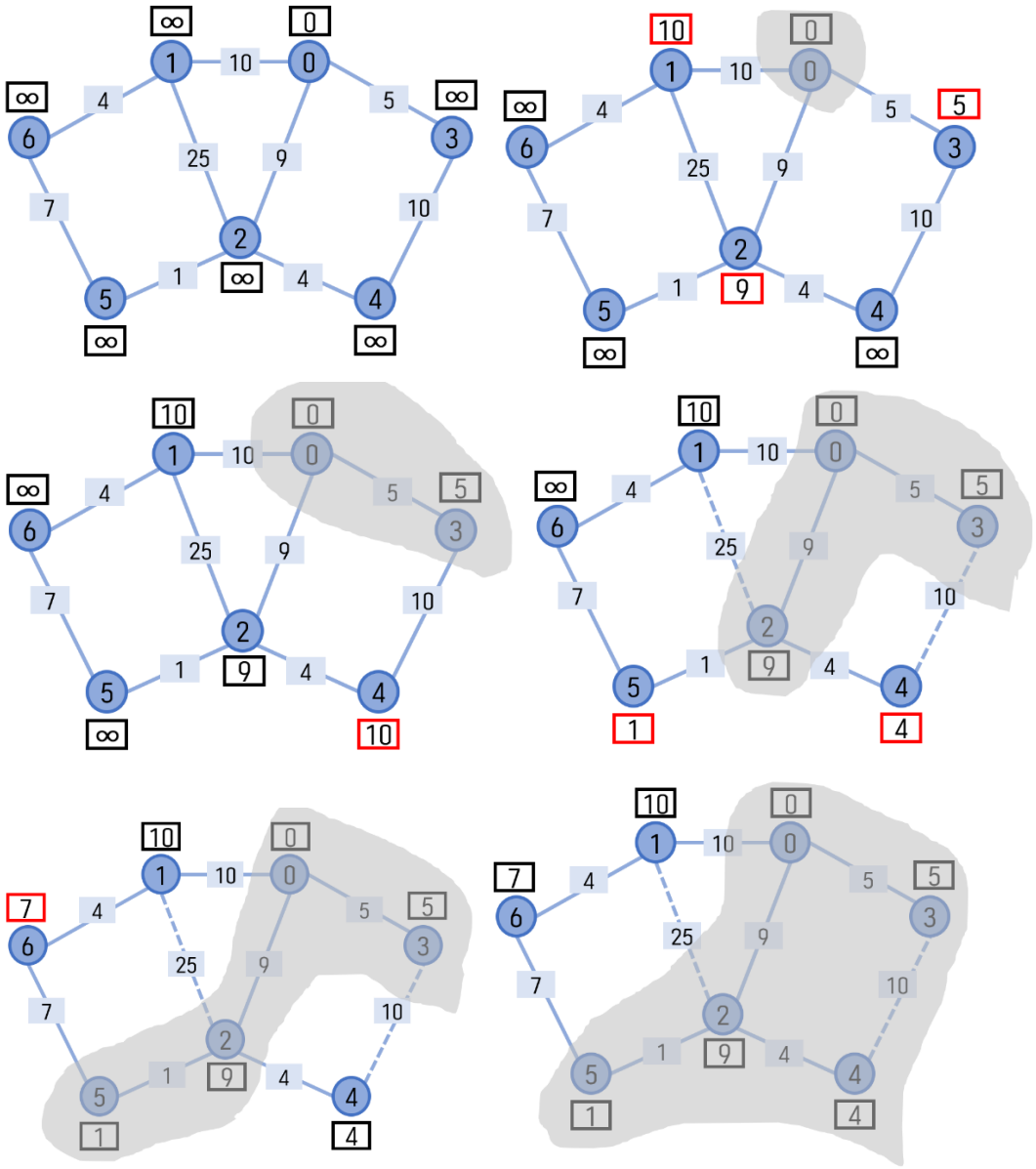


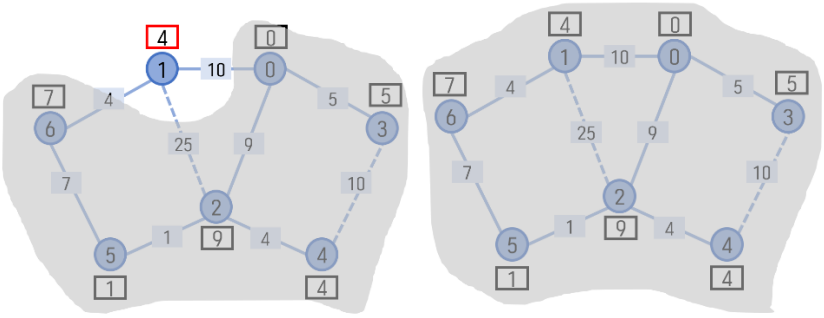
1. Illustrate a running of Dijkstra's algorithm on this graph, starting from the vertex 0.





2. Illustrate the execution of the Prim-Jarnik algorithm for computing the minimum spanning tree of this graph.(Assume all the edges are undirected.)





A9-Bonus (3 points)

Consider the following greedy strategy for finding a shortest path from vertex *start* to vertex *goal* in given connected graph.

1. Initialize *path* to *start*

2. Initialize *set* visited to {start}

3. If *start* = *goal*, return *path* and exit. Otherwise, continue.

4. Find the edge (*start*,v) of minimum weight such that v is adjacent to start and v is not in *visited*.

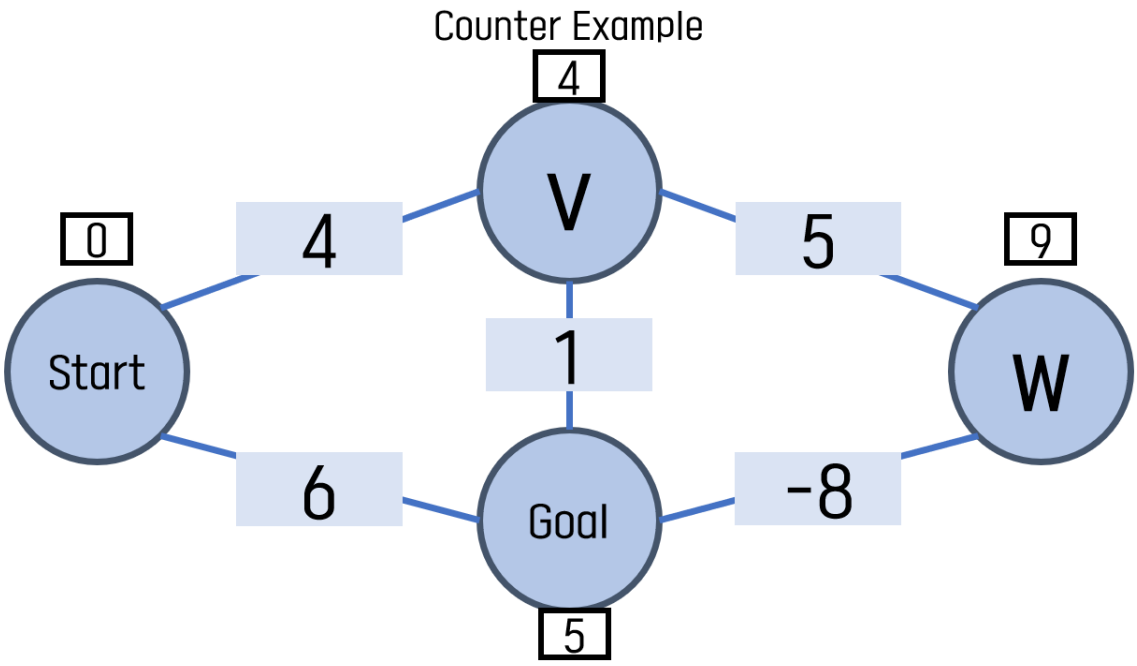
5. Add v to path

6. Add v to visited.

7.Set start equal to v and go to step 3.

Does this greedy strategy always find a shortest path from *start* to *goal*?

Either explain intuitively why it works, or give a counterexample.

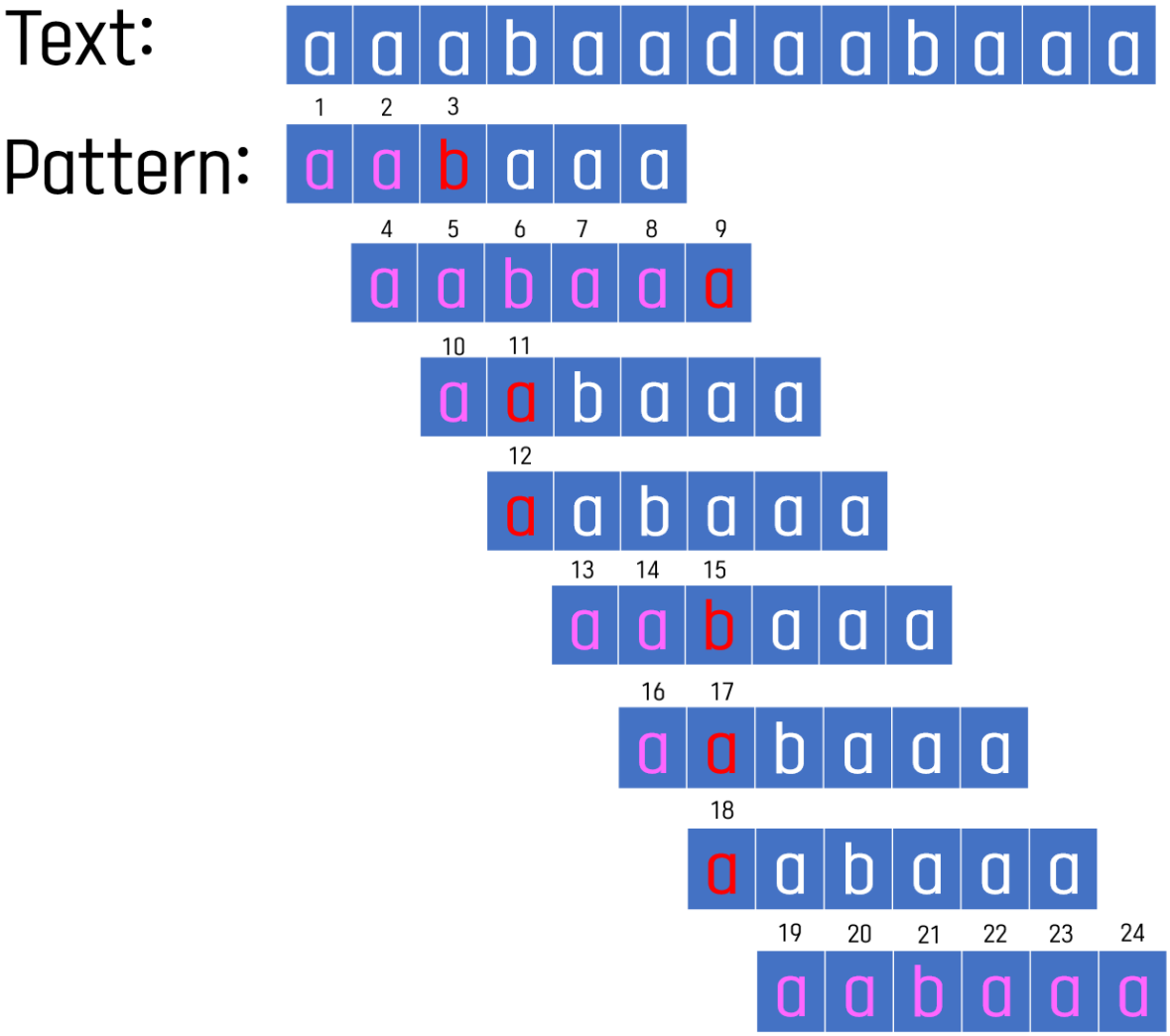


위 상황에서 알고리즘 대로 최단경로를 찾는다면 start – v – goal 이 되겠지만 실제로는 음의 값을 가지고 있는 edge 를 거치는 start – v – w – goal 이 가장 최단 경로가 된다. 따라서 위 알고리즘은 음의 경로가 있다면 최단경로를 잘 찾지 못하므로 항상 잘 작동한다고는 할 수 없을 것이다.

A10 - Text Processing (5 points)

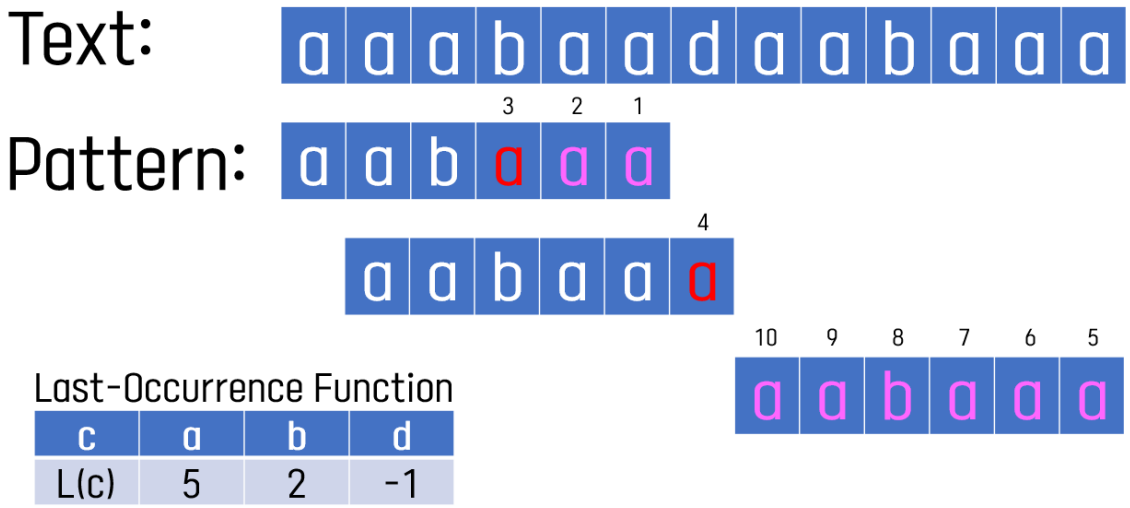
A10-1 (1 point)

Illustrate the comparisons done by brute-force pattern matching for the text `aaabaadaabaaa` and pattern `aabaaa`.



A10-2 (2 points)

Repeat A10-1 with Boyer-Moore algorithm. Present **last** function table as well.

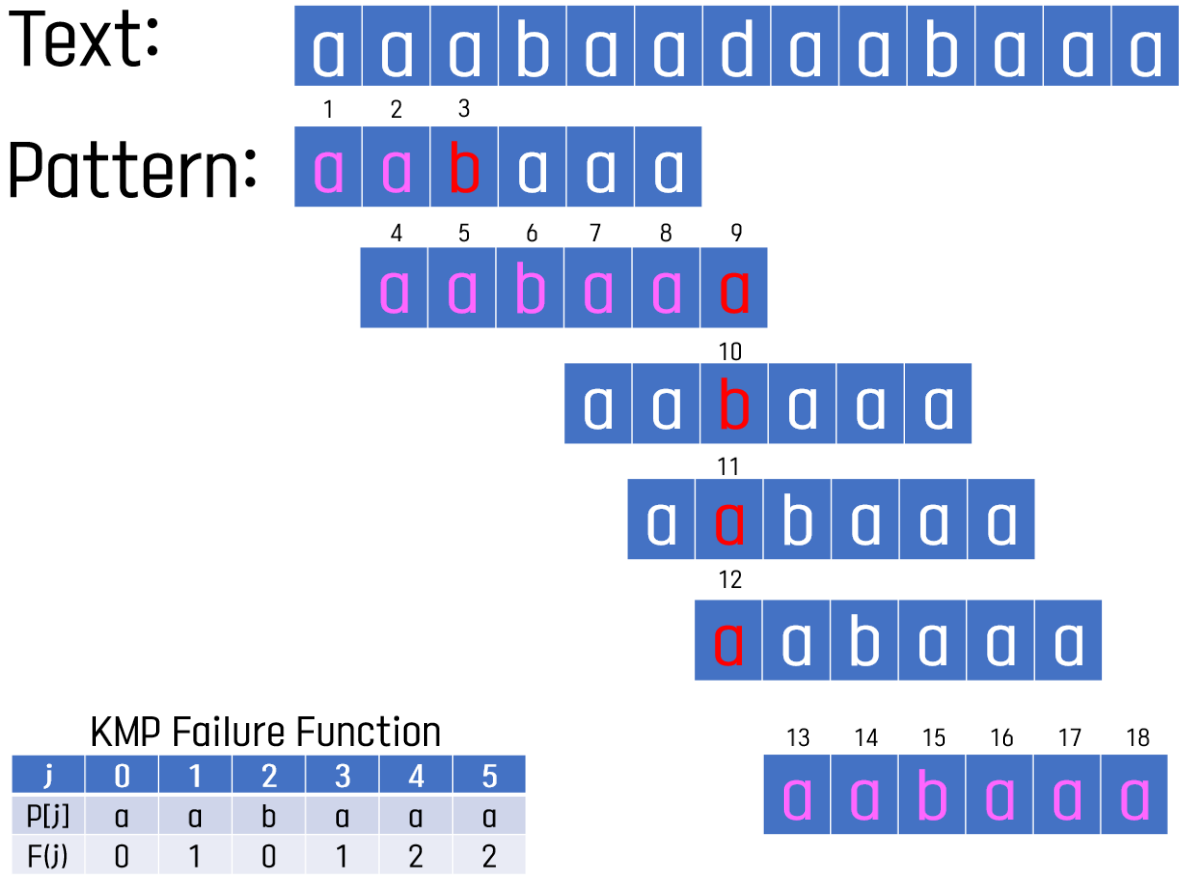


Last-Occurrence Function

c	a	b	d
L(c)	5	2	-1

A10-3 (2 points)

Repeat A10-1 with Knuth-Morris-Pratt algorithm. Present **failure** function table as well.



KMP Failure Function

j	0	1	2	3	4	5
P[j]	a	a	b	a	a	a
F(j)	0	1	0	1	2	2