TU: Logistic Regression

Industrial AI & Automation by Y.K.Kim

Mod: 2024-2

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Date: 24.09.20

Introduction

Binary linear classifier

`Method 1:`fitclinear()

- For two-class (binary) learning with **high-dimensional**, full or sparse predictor data.

`Method 2:`fitglm()

- For low- through medium-dimensional predictor data sets, see Alternatives for Lower-Dimensional Data

Examples

Example 1: Logistic Regression

Linear Regression

$$y = \beta_0 + \beta_1 x$$

Logistic Function

$$y = \frac{1}{1+e^{-x}}$$

Linear Regression + Logistic Function

$$P(y = 1) = \frac{1}{1+e^{-(\beta_0+\beta_1 x\ )}}$$

Data Acquisition

```
clear
load fisheriris
X = meas(:,1:2);           % Use two features (x1, x2) for fitting
sp = categorical(species);
Ystat=sp=='setosa';        % Binary classfication:  setosa vs no-setosa
```

Classification: Logistic Regression

```
Mdl = fitclinear(X,Ystat,'Learner','logistic')
```

```
Mdl =
  ClassificationLinear
      ResponseName: 'Y'
        ClassNames: [0 1]
    ScoreTransform: 'logit'
              Beta: [2×1 double]
              Bias: 8.3233
            Lambda: 0.0067
           Learner: 'logistic'


  Properties, Methods
```

Plot Logistic Regression of Train data

- z= b0+b1X1+b2X2
- y=1/(1+exp(-z)) = h(z)

```
% Plot z vs Y=sigmoid(z)

z=Mdl.Bias+X*Mdl.Beta;

% Define logistif function h(z)=y
Y= 1.0./(1.0+exp(-z));

% Index of data which is class setosa
Y1indx=find(sp=='setosa')
```

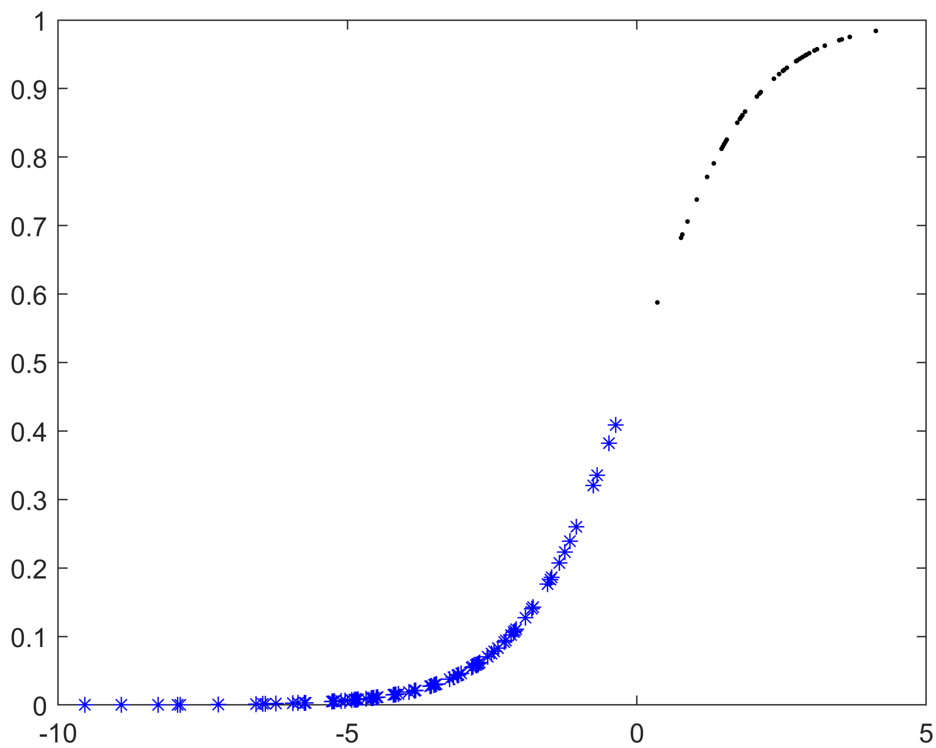
```
Y1indx = 50×1
     1
     2
     3
     4
     5
     6
     7
     8
     9
    10
     ⋮
```

```
% Index of data which is NOT in class setosa
Y0indx=find(sp~='setosa')
```

```
Y0indx = 100×1
    51
    52
    53
    54
    55
    56
    57
    58
    59
```

```
    60
     ⋮
```

```
figure
plot(z(Y1indx),Y(Y1indx),'k.')
hold on
plot(z(Y0indx),Y(Y0indx),'b*')
hold off
```



Cross-validation (k-fold)

Construct a cross-validated classifier from the model.

```
cvMdl = fitclinear(X,Ystat,'Learner','logistic', 'KFold',5)
```

```
cvMdl = 
  ClassificationPartitionedLinear
    CrossValidatedModel: 'Linear'
           ResponseName: 'Y'
        NumObservations: 150
                  KFold: 5
              Partition: [1×1 cvpartition]
             ClassNames: [0 1]
         ScoreTransform: 'none'
```

```
Properties, Methods
```

Examine the cross-validation loss, which is the average loss of each cross-validation model when predicting on data that is not used for training.

```
kloss = kfoldLoss(cvMdl)
```

```
kloss = 0.0067
```

Get the optimal paramter of theta from the training

b0, b1, b2 of   z= b0+b1X1+b2X2

```
% b_0
Mdl.Bias
```

```
ans = 8.3233
```

```
% b_1, b_2
Mdl.Beta
```

```
ans = 2×1
   -3.3883
    3.1645
```

Test

```
% an average flower feature values
flwr = mean(X);
flwr2 = mean(X(1:10,:));
Xtest=flwr2;

% Convert TestData as Logistic Function
ztest=Mdl.Bias+Xtest*Mdl.Beta;
Ytest=1./(1+exp(-ztest))
```

```
Ytest = 0.9114
```

```
flwrClass = predict(Mdl,Xtest)
```

```
flwrClass = logical
   1
```

Exercise

Exercise 1

Create a simple training of the logistic regression model using gradient descent.

Cost function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$= -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log\left(h_\theta\left(x^{(i)}\right)\right) + (1-y^{(i)}) \log\left(1 - h_\theta\left(x^{(i)}\right)\right)\right]$$

Learning: fit parameter θ

$$\min_\theta J(\theta)$$

Prediction: given new x

Output $h_\theta(x) = \frac{1}{1+e^{-\theta^\top x}}$

Slide credit: Andrew Ng

Gradient Descent

Gradient descent for Linear Regression

Repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right) x_j^{(i)} \quad \boxed{h_\theta(x) = \theta^\top x}$$

}

Gradient descent for Logistic Regression

Repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right) x_j^{(i)} \quad \boxed{h_\theta(x) = \frac{1}{1+e^{-\theta^\top x}}}$$

}

Slide credit: Andrew Ng

```
load fisheriris
X = meas(:,1:2);                % Use two features (x1, x2) for fitting
sp = categorical(species);
Y=double(sp=='setosa');         % Binary classfication:  setosa vs no-setosa
N=length(Y);

% Initialization of training
w1 = [-3, 3];
w0 = 8      ;
lamda=0.1   ; % learning rate
loss=1      ;
itrN=5000   ;
k=1;
while(loss>0.001 && k<itrN)

    % Define function h(x)
    h= 1 ./(1+exp(-(X * w1' + w0)));
    % Define gradient w.r.t theta_1 and theta_0
    dJt1 = sum((h - Y).*X)/N;
    dJt0 = sum(h-Y)/N;

    % Update w1, w0
    w1 =w1 - lamda * dJt1;
    w0 =w0 - lamda * dJt0;
```

```
    % Cost Function
    loss = - sum(Y.*log(h) + (1-Y).*log(1-h))/N;
    loss_hist(k) = loss;
    k = k+1;
end
```

Get the optimal paramter of theta from the training

b0, b1, b2 of   z= b0+b1X1+b2X2

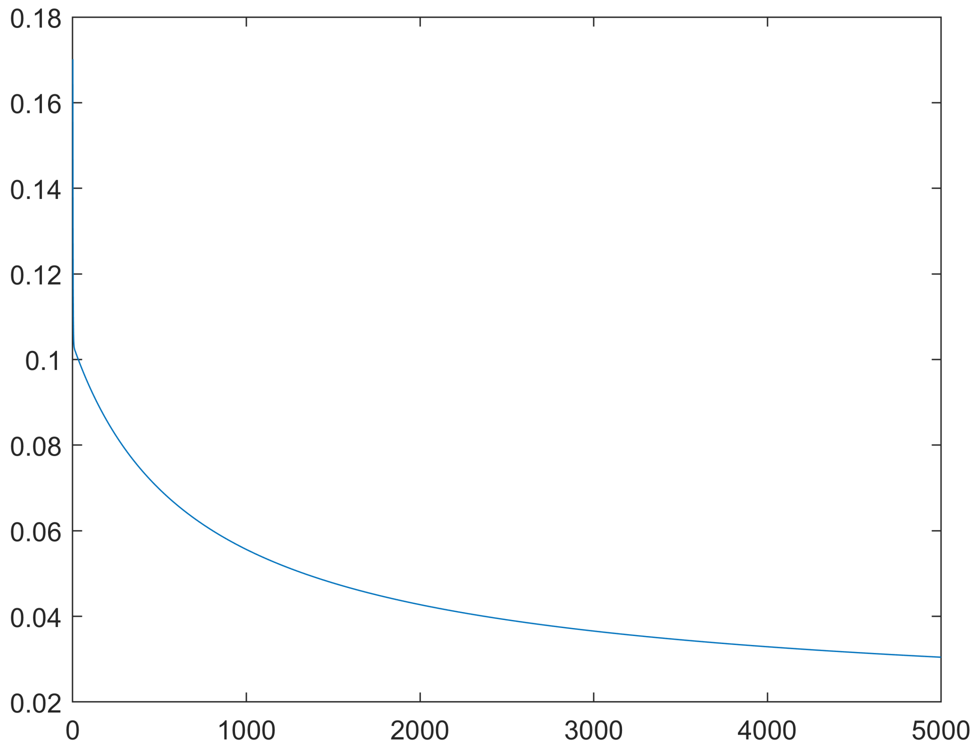
```
% b_0
w0
```

```
w0 = 8.6193
```

```
% b_1, b_2
w1
```

```
w1 = 1×2
   -5.7867    7.2528
```

```
% Plot loss vs iteration
figure
plot(loss_hist)
```

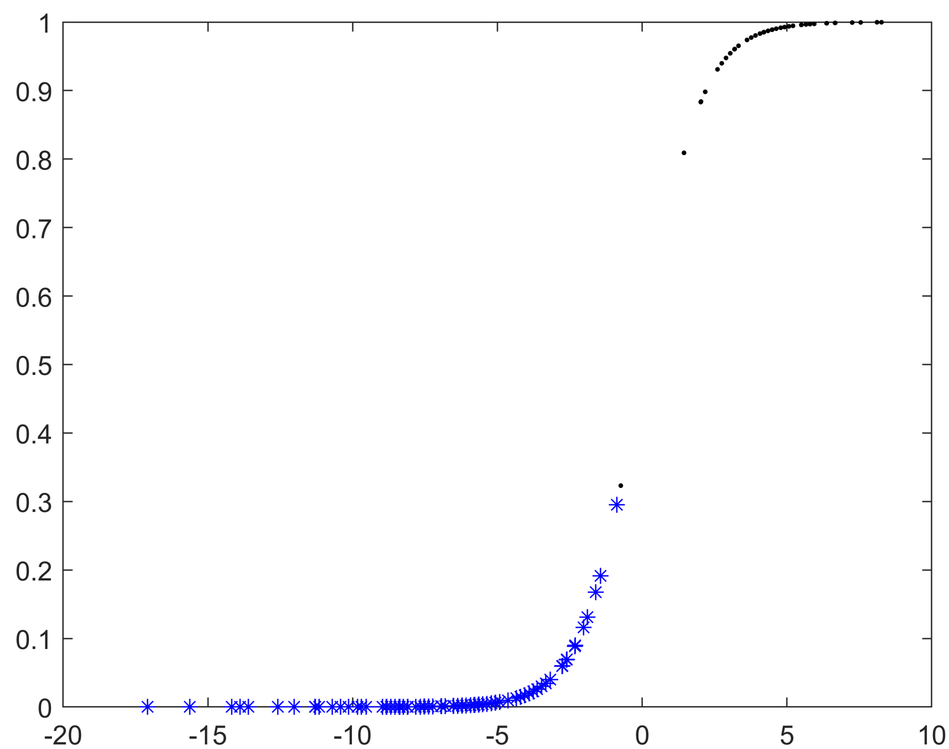



```
% Define z from (x, optimal w1 and w0)
z = X*w1' + w0;
```

```
% Calculate predicted Y from (x, optimal w1 and w0)
Y=1./(1+exp(-z));


% Index of data which is class setosa
Y1indx=find(sp=='setosa');
% Index of data which is NOT in class setosa
Y0indx=find(sp~='setosa');


% Plot the prediction output
figure
plot(z(Y1indx),Y(Y1indx),'k.') % class 1
hold on
plot(z(Y0indx),Y(Y0indx),'b*') % class 0
hold off
```



Exercise 2: CWRU dataset

Apply logistic regression to classifiy outer or inner bearing fault

Dataset

- Given dataset contains many features extracted from CWRU dataset
- For binary class: Class_Outer, Class_Inner Race Fault

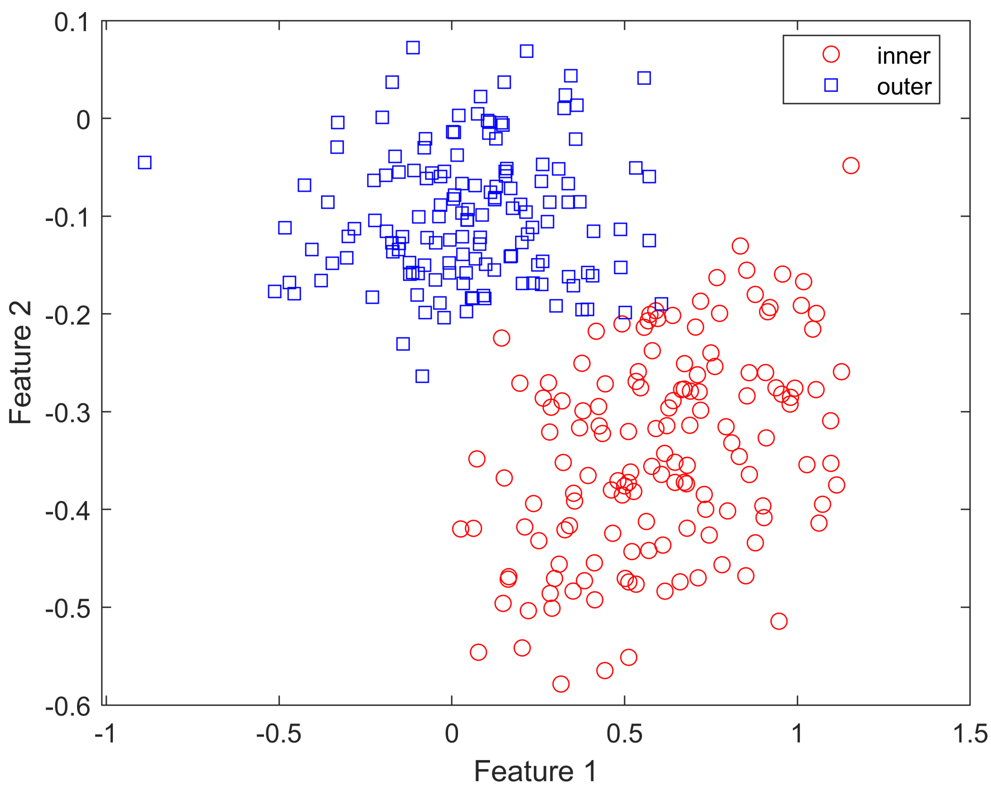
```
clear

% Load class 'Inner', 'Outer' dataset
load("../../Dataset/CWRU_selected_dataset/Feature_data/sample_train.mat");
feature1 = "sv";                % skewness feature
feature2 = "ipf";               % impulse factor feature

% Prepare X, Y for train
Xtrain = [glob_all_train.(feature1), glob_all_train.(feature2)];
Ytrain = class_cwru_train;              % fault class label

% we want to keep only inner and outer race faults
% eliminate class normal.
classKeep = ~strcmp(Ytrain,'normal');
X = Xtrain(classKeep,:);
Y = Ytrain(classKeep);

% Plot features
f = figure;
gscatter(X(:,1), X(:,2), Y,'rb','os');
xlabel('Feature 1');
ylabel('Feature 2');
```



Classify using Logistic Regression and Analyze

```
% Use k-fold 5 for logistic regression fit
```

```
Y = categorical(Y);

Mdl = fitclinear(X,Y,'Learner','logistic', 'KFold',5)
```

```
Mdl = 
  ClassificationPartitionedLinear
    CrossValidatedModel: 'Linear'
           ResponseName: 'Y'
        NumObservations: 288
                  KFold: 5
              Partition: [1×1 cvpartition]
             ClassNames: [inner    outer]
         ScoreTransform: 'none'


  Properties, Methods
```

```
kfoldloss = kfoldLoss(Mdl)
```

```
kfoldloss = 0.0556
```

Predict test data and Analyze

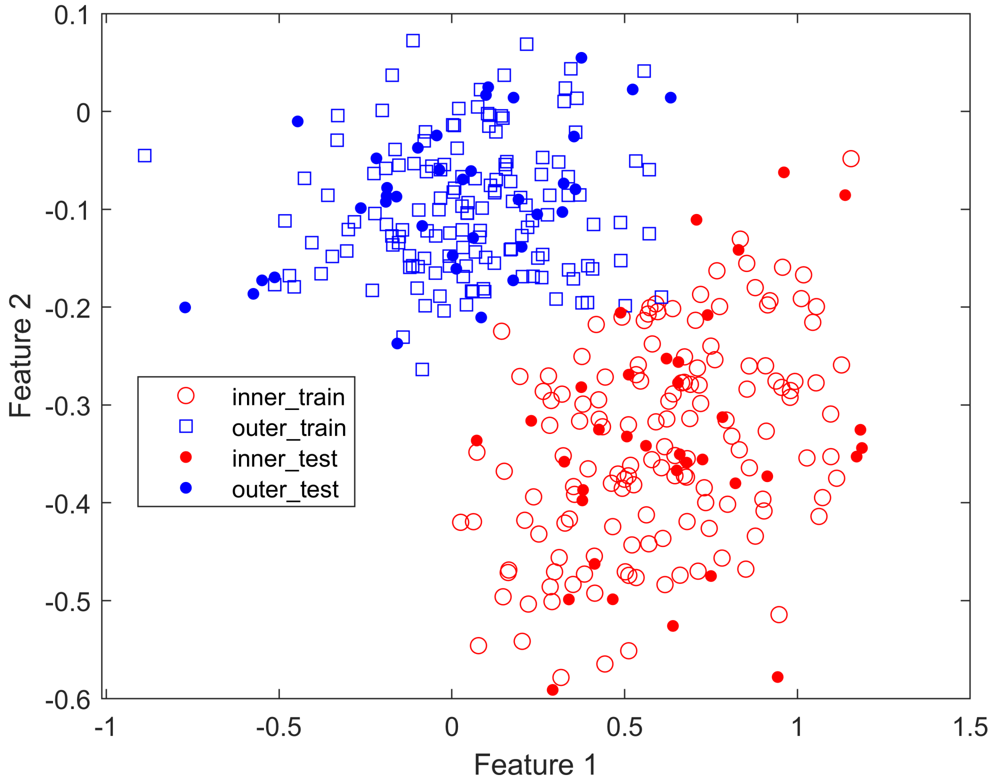
```
% Load Class 'Inner', 'Outer'
load("../../Dataset/CWRU_selected_dataset/Feature_data/sample_test.mat");

% Prepare X, Y for test
Xtest = [glob_all_test.(feature1), glob_all_test.(feature2)];
Ytest = class_cwru_test;

% we want to keep only inner and outer race faults
% eliminate class normal.
classKeep_test = ~strcmp(Ytest,'normal');
Xtest = Xtest(classKeep_test, :);
Ytest = Ytest(classKeep_test);

% Display Loss value
loss =  loss(Mdl.Trained{1},Xtest,Ytest);

% Plot the prediction
figure
gscatter(X(:,1), X(:,2), Y,'rb','os'); hold on;
gscatter(Xtest(:,1), Xtest(:,2), Ytest, 'rb', '..')
xlabel('Feature 1');
ylabel('Feature 2');
legend('inner\_train', 'outer\_train', 'inner\_test', 'outer\_test');
```


0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
Feature 2
-1
-0.5
0
0.5
1
1.5
Feature 1
inner_train
outer_train
inner_test
outer_test