The Impact of 2D and 3D BAO Measurements on the Cosmic Distance Duality Relation with HII Galaxies

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Abstract. The cosmic distance duality relation (CDDR) is a fundamental and practical condition in observational cosmology that connects the luminosity distance and angular diameter distance. Testing its validity offers a powerful tool to probe new physics beyond the standard cosmological model. In this work, for the first time, we present a novel consistency test of CDDR by combining HII galaxy data with a comprehensive set of Baryon Acoustic Oscillations (BAO) measurements. The BAO measurements include two-dimensional (2D) BAO and three-dimensional (3D) BAO from the Sloan Digital Sky Survey (SDSS), as well as the latest 3D BAO data from the Dark Energy Spectroscopic Instrument (DESI) Data Release 2 (DR2). We adopt four different parameterizations of the distance duality relation parameter, $\eta(z)$, to investigate possible deviations and their evolution with cosmic time. To ensure accurate redshift matching across datasets, we reconstruct the distance measures through a model-independent Artificial Neural Network (ANN) approach. We find no significant deviation from the CDDR (less than 68% confidence level) among four parameterizations. Furthermore, our results show that the constraints on $\eta(z)$ obtained separately from 2D and 3D BAO measurements are consistent at the 68% confidence level. This indicates that there is no significant tension between the two datasets under the four parameterizations considered. Our ANN reconstruction of HII galaxies could provide constraints on the CDDR at redshifts beyond the reach of Type Ia supernovae. Finally, the consistency of our results supports the standard CDDR and demonstrates the robustness of our analytical approach.

Keywords: Bayesian reasoning, baryon acoustic oscillations, high redshift galaxies.

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1 Introduction

The cosmic distance duality relation (CDDR), also known as the Etherington relation [1], is a cornerstone of modern observational cosmology. It provides a direct connection between the luminosity distance (d_L) and the angular diameter distance (d_A) through the expression $d_L(z) = d_A(z)(1+z)^2$, where z denotes the redshift. This relation, first proposed by Etherington, relies on three fundamental assumptions [2]: i) spacetime is described by a metric theory of gravity, ii) photons travel along unique null geodesics, and iii) photon number is conserved. Under these conditions, the CDDR is expected to hold at all redshifts within the standard cosmological framework, corresponding to a theoretical prediction of $\eta(z) = d_L(z)/\left(d_A(z)(1+z)^2\right) = 1$. Therefore, any significant deviation from the standard CDDR may indicate the breakdown of one or more of these assumptions, pointing toward possible new physics, such as photon–axion conversions [3, 4], cosmic opacity induced by intergalactic dust or exotic interactions [5, 6], and modifications to general relativity [7–9].

The increasing tensions between cosmological parameters inferred from early- and late-Universe observations [10–17] have raised growing concerns about the robustness of the standard cosmology. The most significant one is so-called the "Hubble tension", which has been observed by various cosmological probes [18, 19], reaching a statistical significance exceeding 5σ and indicating either the existence of new physics beyond the standard model or the need to re-examine some of its foundational assumptions [10, 20–22]. One of these assumptions is the validity of the CDDR. Recent studies [23–26] suggest that a violation of the CDDR could introduce inconsistencies in the calibration of distance measurements from different cosmological probes, potentially contributing to the observed tensions. Therefore, it is essential to test the validity of the CDDR in the context of the current cosmological tensions.

A considerable amount of research has focused on testing the CDDR employing a variety of cosmological observations, which requires simultaneous measurements of luminosity distances $(d_L(z))$ and angular diameter distances $(d_A(z))$. Type Ia supernovae (SNIa) are commonly used to determine luminosity distances, while angular diameter distances have been provided by different cosmological probes, such as the Sunyaev–Zeldovich effect and gas mass fraction measurements in galaxy clusters [27–29], baryon acoustic oscillations (BAO) [30–32], strong gravitational lensing systems [33–39], and the angular size of ultra-compact radio sources [40, 41]. The combination of these diverse probes has enabled multifaceted examinations of the CDDR across different cosmic epochs and distance scales [42–48].

Despite these numerous studies, several recent developments make a renewed investigation both timely and necessary. First, the Dark Energy Spectroscopic Instrument (DESI) collaboration has released its second dataset (DR2, hereafter 3D-DESI), including observations from the first three years of operation [49]. This release delivers the most precise BAO measurements to date. However, analyses of this dataset have reported potential tensions with the ΛCDM model [49–57], prompting further investigation into the underlying assumptions of standard cosmology. Since the CDDR relies on these assumptions, testing its validity with the latest BAO measurements may shed light on the origin of the deviations. Moreover, several studies have reported the disagreements between BAO measurements

obtained from the two-dimensional (2D, transverse or angular) BAO and the three-dimensional (3D, or anisotropic) BAO [58–63], raising the possibility that CDDR tests based on different BAO types may yield different results. Recently, [63] investigated how such tensions could affect the CDDR by performing a model-independent analysis that used SNIa to provide the luminosity distances, finding no significant violation of the relation. In this work, we revisit this issue by combining the latest HII galaxy data with different BAO datasets, and perform a parameterized analysis with four different forms of $\eta(z)$. This strategy enables us to probe potential deviations from the CDDR while directly assessing how the 2D and 3D BAO tension might affect the inferred behavior of $\eta(z)$.

We choose HII galaxy data as the luminosity distance probe in this analysis for several reasons. First, while SNIa are the most commonly used standard candles, an important consideration is their dependence on the absolute peak magnitude M_B , which is traditionally assumed to be constant. However, recent studies suggest that M_B may evolve with redshift [36, 59, 64–68], potentially introducing additional uncertainties in the luminosity distance measurements and affecting the robustness of CDDR tests [65, 66, 69, 70]. By contrast, HII galaxy data and giant extragalactic HII regions (GEHR) data provide a viable alternative. Their redshift coverage extends up to $z \sim 2.5$, overlapping well with BAO datasets. In addition, they exhibit a robust correlation between the H β luminosity $L(H\beta)$ and the ionized gas velocity dispersion σ , enabling an independent determination of luminosity distances. Furthermore, their sensitivity to photon-number nonconservation makes them particularly well-suited for model-independent CDDR tests.

One of the main challenges in testing the CDDR is obtaining matched luminosity and angular diameter distances at the same redshifts. In our case, we reconstruct the luminosity distance–redshift relation from HII galaxy data using the Artificial Neural Networks (ANN) method. Compared to traditional non-parametric techniques, the ANN method offers several distinct advantages. It is fully data-driven, imposes no assumptions on the statistical distribution or functional form of the underlying relationship, and can flexibly capture complex non-linear patterns in the data. Moreover, as a universal approximator, the ANN method can accurately model any continuous function, provided that the hidden layer contains a sufficient number of neurons. This approach has been demonstrated to be effective in cosmological research [39, 53, 65, 71–79]. Hence, the ANN method is particularly suitable for our work, since its strengths allow for a model-independent and robust reconstruction of luminosity distances.

In this work, we perform the first systematic CDDR test that jointly uses HII galaxy data and various BAO datasets. This approach not only enables a robust parameterized determination of $\eta(z)$, but also allows us to assess whether the tensions between different BAO measurements would result in the deviation of $\eta(z)$. The outline of the paper is as follows: In section 2, we discuss the Data and Methodology. The analysis and results are explained in section 3. Finally, the discussions and conclusions are presented in section 4.

2 Data and Methodology

In this section, we present the details of the observational datasets (BAO and HII galaxies) and our methodology adopted for CDDR validation.

2.1 The BAO datasets

The clustering of matter imprinted by BAO serves as a "standard ruler" in cosmology, with its length set by the sound horizon at the drag epoch, denoted as r_d . During the drag epoch, baryons decoupled from photons and the BAO scale was "frozen in" at the sound horizon, $r_d = r_s(z_d)$, where z_d is the redshift of the drag epoch. The sound horizon is given by

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz, \tag{2.1}$$

where $c_s(z)$ is the sound speed and H(z) is the Hubble parameter. When using BAO measurements for cosmological studies, it is crucial to know the length of this standard ruler, as it enables the exploration of dark energy and the Universe's expansion history.

Galaxy surveys have succeeded in determining the angular BAO scale, $\theta_{\rm BAO}$, defined by

$$\theta_{\text{BAO}} = \frac{r_d}{(1+z)d_A(z)},\tag{2.2}$$

where $d_A(z)$ is the angular diameter distance and the comoving distance is $d_M(z) = (1+z)d_A(z)$. In this work, we use two types of BAO datasets: the angular (2D) BAO data, consisting of 15 measurements of $\theta_{\rm BAO}$ at various redshifts (see Table. 1), and the anisotropic (3D) BAO data, presented as $d_A(z)/r_d$ (see Table. 2). The 2D-BAO data are derived from SDSS data releases DR7, DR10, DR11, DR12, and DR12Q [80–85], obtained without assuming a fiducial cosmological model. For the 3D-BAO analysis, we consider two datasets: one from DES Y6 and BOSS/eBOSS [86–91], and another from recent DESI DR2 results [49]. To ensure model and calibrator independence, only the angular components of the 3D BAO measurements are used, with radial and dilation scale data excluded.

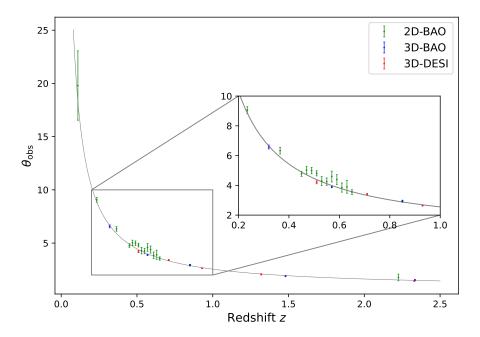


Figure 1: The 2D-BAO, 3D-BAO and 3D-DESI measurements of $\theta(z) = r_d/d_M(z)$. The grey line corresponds to the theoretical values of $\theta(z)$ from the Λ CDM model with $\Omega_m = 0.3$ and $H_0 = 70$ km/s/Mpc

As BAO measurements fundamentally rely on the value of r_d , the adopted treatment of r_d can affect both the estimated angular diameter distances $d_A(z)$ and the subsequent reconstruction of the $\eta(z)$ function. To avoid making any assumption, we treat r_d as a nuisance parameter in our analysis and numerically marginalize over it with broad, non-informative priors. Unless otherwise stated, all constraints reported below are marginalized over r_d .

2.2 The HII galaxy sample

To estimate the luminosity distance, we analyze a full sample of 181 HII galaxies (HIIGx) in the redshift range 0.01 < z < 2.6 [92]. The sample consists of 74 high-redshift HIIGx observed in the range 0.5 < z < 2.6 [93], and 107 local HIIGx with redshifts in the interval 0.01 < z < 0.2 [94].

HIIGx are compact systems undergoing intense starburst episodes that dominate their total luminosity. Their optical spectra are characterized by strong Balmer emission lines, especially $H\alpha$ and $H\beta$, which result from the recombination of hydrogen ionized by young, massive stellar populations [95–98]. These systems share physical properties with giant extragalactic HII regions (GEHR), although GEHR are typically located in the outer disks of late-type spiral galaxies. A strong empirical correlation has been established between the $H\beta$ luminosity, $L(H\beta)$, and the velocity dispersion of the ionized gas, $\sigma(H\beta)$. This correlation, known as the L- σ relation, shows a small intrinsic scatter and enables the use of HIIGx and GEHR as standard candles in cosmological analyses [76, 99, 100].

Survey	z	$\theta_{\mathrm{BAO}} [\mathrm{deg}]$	References	
SDSS DR12	DSS DR12 0.11 19.8 ± 3.26 de Carvalho et al.		de Carvalho et al. (2021)	
SDSS DR7	0.235	9.06 ± 0.23	Alcaniz et al. (2017)	
	0.365	6.33 ± 0.22		
	0.45	4.77 ± 0.17		
	0.47	5.02 ± 0.25		
SDSS DR10	0.49	4.99 ± 0.21	Carvalho et al. (2016)	
SDSS DICTO	0.51	4.81 ± 0.17		
	0.53	4.29 ± 0.30		
	0.55	4.25 ± 0.25		
SDSS DR11	0.57	4.59 ± 0.36		
	0.59	4.39 ± 0.33	Carvalho et al. (2020)	
	0.61	3.85 ± 0.31		
	0.63	3.90 ± 0.43		
	0.65	3.55 ± 0.16		
BOSS DR12Q	2.225	1.77 ± 0.31	de Carvalho et al. (2018)	

Table 1: List of the 15 2D BAO data points used in this work, with $\theta_{\text{BAO}}(z)$ [rad] = $r_d/[(1+z)d_A(z)]$. The values in the third column are given in degrees. See the quoted references for details.

Survey	z	$d_A(z)/r_d$	References	
BOSS DR12	0.32	6.5986 ± 0.1337	Gil-Marín et al. (2017)	
DOSS DICIZ	0.57	9.389 ± 0.103		
DES Y6	0.85	2.932 ± 0.068	Abbott et al. (2024a)	
eBOSS DR16Q	1.48	12.18 ± 0.32	Hou et al. (2020)	
eBOSS DR16 Ly α -F	2.334	$11.25^{+0.36}_{-0.33}$	du Mas des Bourboux et al. (2020)	
DESI DR2 LRG1	0.510	8.998 ± 0.112		
DESI DR2 LRG2	0.706	10.168 ± 0.106		
DESI DR2 LRG3+ELG1	0.934	11.155 ± 0.080	Abdul Karim et al. (2021)	
DESI DR2 ELG2	1.321	11.894 ± 0.138	Abdul Karini et al. (2021)	
DESI DR2 QSO	1.484	12.286 ± 0.305		
DESI DR2 Ly α	2.330	11.708 ± 0.159		

Table 2: Summary of 3D BAO measurements used in this work, with $d_A(z)/r_d$. See the quoted references for details. As explained in section 2, we employ two alternative 3D BAO datasets: the first five rows correspond to the BOSS/eBOSS data points, while the remaining rows include measurements from the DESI DR2.

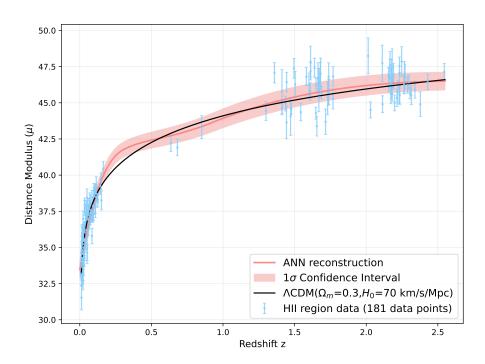


Figure 2: The observed values of $\log d_{L,\rm HII}(z)$ from HII galaxy measurements are shown as cyan data points, with error bars representing the 68% confidence level. The red line represents the reconstructed function, and the pink shaded region indicates the 68% confidence level obtained using the ANN method. The black line corresponds to the theoretical prediction of $\log d_{L,\rm HII}(z)$ from the ΛCDM model with $\Omega_m=0.3$ and $H_0=70$ km/s/Mpc

The L- σ correlation [94, 101] is given by

$$\log_{10} \left[\frac{L(H\beta)}{\text{erg s}^{-1}} \right] = \alpha \log_{10} \left[\frac{\sigma(H\beta)}{\text{km s}^{-1}} \right] + \beta, \tag{2.3}$$

where α and β are empirical constants representing the slope and intercept of the relation, respectively. Using the definition of luminosity distance, the corresponding expression for the distance modulus becomes

$$\mu = 5\log_{10}\left(\frac{d_L}{\text{Mpc}}\right) + 25 = 2.5\left[\alpha\log_{10}\left(\frac{\sigma(H\beta)}{\text{km s}^{-1}}\right) - \log_{10}\left(\frac{F(H\beta)}{\text{erg s}^{-1}\text{ cm}^{-2}}\right) + \beta\right] - 100.2, \quad (2.4)$$

where d_L is the luminosity distance and $F(H\beta)$ is the observed flux in the H β emission line.

Although the parameters α and β are, in principle, nuisance parameters that should be fit jointly with cosmological parameters to avoid circularity, studies have shown that they are largely insensitive to the choice of cosmology. Therefore, we adopt the values $\alpha = 33.268 \pm 0.083$ and $\beta = 5.022 \pm 0.058$, as obtained in previous analyses [76, 92, 93, 102].

The corresponding uncertainty in the distance modulus derived from Eq. 2.4 is given by

$$\sigma_{\mu}^{2} = 6.25 \left(\sigma_{\log_{10} F}^{2} + \beta^{2} \sigma_{\log_{10} \sigma}^{2} + \sigma_{\beta}^{2} (\log_{10} \sigma)^{2} + \sigma_{\alpha}^{2} \right), \tag{2.5}$$

where $\sigma_{\log F}$ and $\sigma_{\log \sigma}$ represent the uncertainties in the logarithmic flux and velocity dispersion, respectively, and σ_{α} and σ_{β} are the uncertainties associated with the fitted parameters.

We note that recent studies suggest the L- σ relation is not sufficiently precise for use as a cosmological distance indicator at the highest redshifts. For example, Ref. [103] analyzed HII galaxies up to $z \sim 7$ with Giant Extragalactic HII Regions (GEHRs) and JWST data. Their work, along with related studies, reports that high-redshift samples show significantly larger scatter in the L- σ relation. To avoid these less-standardized regimes, where systematics become more severe, we impose a redshift cutoff. Since the DESI DR2 dataset extends only to z < 2.4, we restrict our HII galaxy sample to the same range. This excludes the GEHR and JWST high-z data, which contribute additional scatter

and deviations in the relation. Within this redshift interval, HII galaxies have been consistently used as standard candles in cosmological analyses, with their reliability supported by multiple independent studies [76, 104–106].

2.3 Reconstruction Method: Artificial Neural Network

The ANN method provides a fully data-driven and non-parametric framework for reconstructing functions from observational data. In particular, it avoids imposing prior assumptions on the cosmological model or the statistical distribution of the measurements [39, 53, 65, 71–79]. This flexibility allows ANNs to capture complex, non-linear relationships in observational datasets, even when the noise properties deviate from Gaussianity. Furthermore, as universal approximators, the ANN method can represent a wide range of functions to arbitrary precision given sufficient neurons and a suitable network design. These characteristics make ANNs particularly suitable for reconstructing the distance modulus $\mu(z)$ from HII galaxy data, for which we use the REFANN Python package [71].

The ANN is designed to learn a nonlinear mapping between the input and output based solely on observational measurements. In our case, the input variable is the redshift z, and the output to be learned is the distance modulus $\mu(z)$. The adopted network consists of one input neuron (for z), a single hidden layer with 4096 neurons, and one output neuron corresponding to the predicted $\mu(z)$ and its uncertainty σ_{μ} . Each neuron performs a linear transformation followed by a non-linear activation, where we use the Exponential Linear Unit (ELU) [107]:

$$f(x) = \begin{cases} x, & x > 0 \\ \alpha(e^x - 1), & x \le 0 \end{cases}, \quad \alpha = 1.$$
 (2.6)

The training process minimizes the least absolute deviation loss [71], which is more robust to non-Gaussian errors and outliers than mean-squared error, thus keeping our implementation consistent with REFANN:

$$L = \frac{1}{mp}|\hat{Y} - Y|,\tag{2.7}$$

where m is the batch size, p is the number of output parameters, \hat{Y} denotes the network's prediction, and Y is the corresponding ground-truth values. Following the standard training configuration of REFANN [71], we use gradient descent with the Adam optimizer (initial learning rate 0.01) and a batch size equal to 50% of the dataset. The network is trained over 30,000 iterations to ensure convergence for the HII galaxy $\mu(z)$ reconstruction. For each reconstruction, a single realization of the observational dataset is used for training, and the uncertainty in the reconstructed $\mu(z)$ is estimated by propagating the measurement errors through the trained network, following the standard REFANN procedure.

By training the ANN on the HII galaxy data, we obtain a smooth, model-independent reconstruction of $\mu(z)$ that captures the nonlinear relation between redshift and distance modulus (see in figure 2). This allows us to directly derive the luminosity distance $d_L(z)$ at the BAO redshifts. Hence, we can make a model-independent comparison with the angular diameter distances $d_A(z)$ from BAO measurements and test the CDDR. While the ANN approach provides flexibility and robustness, its performance can be influenced by hyperparameter choices such as network architecture and training methodology. These factors may affect the flexibility and generalization of the model, and thus the reconstructed $\mu(z)$ and its confidence intervals.

2.4 Parameterizations of CDDR

To explore the possibility of violation of the standard CDDR, we rewrite the relationship between angular diameter distance $d_A(z)$ and luminosity distance $d_L(z)$ at redshift z as

$$\eta(z) = \frac{d_L(z)}{(1+z)^2 d_A(z)},\tag{2.8}$$

where $\eta(z) = 1$ holds if the standard relation is valid, and any deviation of $\eta(z)$ from unity implies the violation of the CDDR. In this work, we examine four parameterizations of $\eta(z)$ [25, 36, 42, 108–111], namely:

- A linear parameterization, P1: $\eta(z) = 1 + \eta_1 z$,
- A modified linear parameterization, P2: $\eta(z) = 1 + \eta_1 \frac{z}{1+z}$,
- A logarithmic parameterization, P3: $\eta(z) = 1 + \eta_1 \ln(1+z)$,
- A power-law parameterization, P4: $\eta(z) = (1+z)^{\eta_1}$,

where the parameter $\eta_1 = 0$ corresponds to the standard CDDR.

In our analysis, we focus on two parameters: the cosmic distance duality relation (η_1) and the sound horizon (r_d) . For η_1 , we adopted a wide flat prior $\mathcal{U}[-2,2]$ to minimize prior-driven effects on the posterior distribution. Further, we performed marginalization over r_d by integrating the posterior with a uniform prior $\mathcal{U}[10,300]$. This approach allows us to constrain η_1 robustly without imposing strong assumptions on the sound horizon. To perform the parameter estimation, we use Bayesian inference with the Python module $emcee^1$, an affine-invariant Markov chain Monte Carlo (MCMC) sampler [112]. The MCMC analysis uses 40 walkers and 40,000 steps for each walker to provide a thorough exploration of the parameter space. The initial 20% of the samples from each chain are discarded as burn-in to eliminate potential biases from initial conditions. The remaining samples are used to construct the posterior distributions. We check the convergence of the chains in two ways. First, we visually inspected the trace plots for each parameter to confirm proper mixing and stability around the best-fit values. Second, we calculated the integrated auto-correlation time τ_f using the *autocorr.integrated_time* function from the *emcee* package. This procedure ensures that the correlated nature of ensemble samplers and provides statistically consistent estimates of the posteriors.

3 Results

We present the first test of the CDDR with HII galaxy data serving as luminosity distance indicators, providing an alternative to SNIa for measuring $d_L(z)$. Moreover, we consider three types of BAO datasets, 2D-BAO, 3D-BAO, and 3D-DESI, to quantify whether the tension between 2D angular and 3D anisotropic BAO datasets would affect the constraints on the parameterized CDDR, $\eta(z)$. Here, we show the constraints obtained on η_1 with four different parameterizations: $\eta(z) = 1 + \eta_1 z$ (P1), $\eta(z) = 1 + \eta_1 \frac{z}{1+z}$ (P2), $\eta(z) = 1 + \eta_1 \ln(1+z)$ (P3), and $\eta(z) = (1+z)^{\eta_1}$ (P4), as summarized in table 3 and shown in figure 3. In particular, we treat r_d as a nuisance parameter, sampling from a prior uniform in [10, 300] and marginalizing over it.

For the **P1** model, defined as $\eta(z)=1+\eta_1z$, our analysis yields the following constraints: $\eta_1=-0.054^{+0.220}_{-0.172}$ from 2D-BAO, $\eta_1=0.008^{+0.246}_{-0.162}$ from 3D-BAO, and $\eta_1=0.129^{+0.379}_{-0.214}$ from 3D-DESI. The constraints on η_1 are consistent with zero at the 68% confidence level, supporting the validity of the CDDR. In the case of the **P2** model, where $\eta(z)=1+\eta_1\frac{z}{1+z}$, we obtain the following constraints: $\eta_1=-0.327^{+0.686}_{-0.486}$ from 2D-BAO, $\eta_1=-0.066^{+1.106}_{-0.572}$ from 3D-BAO, and $\eta_1=-0.047^{+0.584}_{-0.516}$ from 3D-DESI. Similar to the P1 model, the results show no strong evidence for a violation of the CDDR. For the **P3** model, where $\eta(z)=1+\eta_1\ln(1+z)$, our analysis yields the following constraints: $\eta_1=-0.166^{+0.409}_{-0.293}$ from 2D-BAO, $\eta_1=-0.004^{+0.598}_{-0.331}$ from 3D-BAO, and $\eta_1=0.037^{+0.344}_{-0.308}$ from 3D-DESI. These results yield η_1 values consistent with zero within a 68% confidence level. Besides, for the **P4** model, where $\eta(z)=(1+z)^{\eta_1}$, we obtain the following constraints: $\eta_1=-0.414^{+0.437}_{-0.463}$ from 2D-BAO, $\eta_1=-0.289^{+0.435}_{-0.455}$ from 3D-BAO, and $\eta_1=-0.059^{+0.452}_{-0.480}$ from 3D-DESI. The constraints on η_1 remain consistent with zero within approximately 1σ , continuing the trend observed in the previous models. Moreover, the power-law form of this parameterization provides additional evidence in support of the CDDR and complements the findings from the other parameterizations.

For all four parameterizations, the constraints obtained from 2D-BAO, 3D-BAO, and 3D-DESI are consistent and show no significant deviation from $\eta_1 = 0$. These results support the validity of the CDDR. Importantly, this consistency across different types of BAO data suggests that the

¹https://emcee.readthedocs.io/en/stable/

potential tension between 2D and 3D BAO measurements does not have a significant impact on the parameterized CDDR tests. Nevertheless, it should be noted that the current uncertainties are relatively large, leading to rather weak constraints.

For comparison, the results of other studies using model-independent approaches are presented in Ref. [113], where the CDDR was tested using SNIa data in combination with both low- and high-redshift BAO measurements. In contrast, our analysis combines HII galaxy data with both 2D- and 3D-BAO measurements, including the latest DESI DR2 dataset. This not only extends the redshift coverage, but also allows for a more systematic comparison among different types of BAO data. Compared to Ref. [114] and Ref. [31], who considered only a single type of BAO measurement, our work systematically compares different BAO measurements. Hence, we can assess whether potential tensions between 2D and 3D BAO measurements could affect the parameterized tests of the CDDR.

Our analysis provides a new and independent way to test the CDDR using HII galaxy and BAO data, providing complementary insights that go beyond the conventional SNIa and BAO data framework. The relatively large uncertainties mainly originate from the larger intrinsic scatter of the L- σ relation compared to the calibrated SNIa luminosities, as well as additional corrections for dust extinction and metallicity that must be marginalized over. Moreover, the current HII galaxy data are smaller in size and have limited signal-to-noise at high redshift, which further broadens the posterior distributions. Nevertheless, the constraints remain centered around $\eta_1 = 0$, showing no strong evidence for deviations from CDDR. This demonstrates that HII galaxy data provide a valuable and independent probe of the CDDR, with the unique potential to extend distance measurements to $z \geq 2$ compared to SNIa.

Data	P1	P2	Р3	P4
2D-BAO	$-0.054_{-0.172}^{+0.220}$	$-0.327^{+0.686}_{-0.448}$	$-0.166^{+0.409}_{-0.293}$	$-0.414^{+0.437}_{-0.463}$
3D-BAO	$0.008^{+0.246}_{-0.162}$	$-0.066^{+1.106}_{-0.572}$	$-0.004^{+0.598}_{-0.331}$	$-0.289^{+0.435}_{-0.455}$
3D-DESI	$0.129^{+0.379}_{-0.214}$	$-0.047^{+0.584}_{-0.516}$	$0.037^{+0.344}_{-0.308}$	$-0.059^{+0.452}_{-0.480}$

Table 3: The best-fit values and its 68% confidence level uncertainties for the parameter η_1 obtained from the combination of HII galaxy data with 2D-BAO, 3D-BAO, and 3D-DESI BAO measurements, following the procedure described in section 2

4 Discussions and Conclusions

Motivated by the existing tension between 2D and 3D BAO measurements, we perform a comprehensive test of the CDDR by combining HII galaxy data with three BAO datasets (2D-BAO, 3D-BAO, and 3D-DESI). This work represents the first robust study of the CDDR using HII galaxies together with BAO data. We aim to quantify whether the existing tension between the angular and anisotropic BAO data would affect the validity of the CDDR. By considering four broad and representative parameterizations of potential CDDR violations, we test the validity of this fundamental relation of cosmology.

Our analysis obtains several valuable conclusions:

• Across the four parameterizations (P1-P4), we find no statistically significant evidence for a violation of the CDDR. The best-fit values of η_1 from 2D-BAO, 3D-BAO, and 3D-DESI BAO differ slightly, but their 68% intervals largely overlap. These results are stable and do not rely on any single functional form. Moreover, the potential tension between 2D and 3D BAO measurements does not significantly affect the constraints of CDDR. This is consistent with the model–independent assessment of Ref. [63]. The minor deviation from the standard CDDR cannot be excluded at the present precision, and forthcoming high-precision BAO data will tighten the constraints on this test.

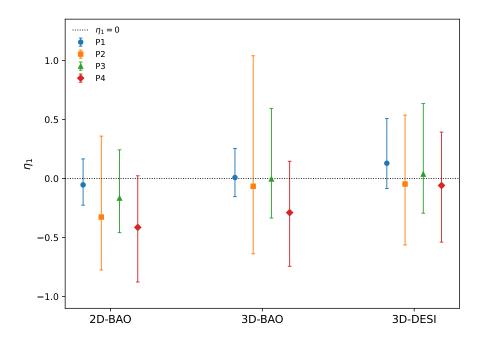


Figure 3: The normalized posterior distributions of η obtained from HII galaxy data combined with 2D-BAO, 3D-BAO, and 3D-DESI BAO datasets. The vertical dashed line at $\eta = 0$ marks the value predicted by the standard CDDR.

• In this work, we present that the HII galaxy data provide an independent perspective on testing the CDDR. Our results provide consistent support for the validity of the CDDR, despite the uncertainties remaining large due to the intrinsic scatter of the $L-\sigma$ relation and the limited size of the HII sample. This highlights the importance of exploring alternative distance indicators in cosmological tests.

Future improvements in HII galaxy and BAO observations, especially at higher redshifts, will provide valuable tools for testing fundamental cosmological principles. It will be crucial to reassess the robustness of CDDR tests as the high-precision BAO data from DESI and the Euclid Space Telescope become available [115, 116]. Ideally, both 2D and 3D BAO measurements would be derived from the same collaborations, i.e., DESI ² and eBOSS ³. It would be particularly useful for determining whether any existing 2D–3D BAO tensions affect the validity of the CDDR. With the forthcoming data from additional and complementary Stage-IV dark energy surveys, such as the Vera C. Rubin Observatory, new opportunities will arise in the next decade. These data will enable tighter constraints on possible deviations from the CDDR and provide insights into its potential redshift evolution. Moreover, exploring alternative parameterizations and combining other cosmological probes will further strengthen our understanding of the CDDR and its implications for fundamental physics.

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²https://www.desi.lbl.gov/

³https://www.sdss4.org/surveys/eboss/

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References

- [1] I.M.H. Etherington, On the Definition of Distance in General Relativity., Philosophical Magazine 15 (1933) 761.
- [2] G.F.R. Ellis, On the definition of distance in general relativity: I. M. H. Etherington (Philosophical Magazine ser. 7, vol. 15, 761 (1933)), General Relativity and Gravitation 39 (2007) 1047.
- [3] B.A. Bassett and M. Kunz, Cosmic distance-duality as a probe of exotic physics and acceleration, Phys. Rev. D 69 (2004) 101305 [astro-ph/0312443].
- [4] B.A. Bassett and M. Kunz, Cosmic Acceleration versus Axion-Photon Mixing, Astrophys. J. 607 (2004) 661 [astro-ph/0311495].
- [5] S. More, J. Bovy and D.W. Hogg, Cosmic Transparency: A Test with the Baryon Acoustic Feature and Type Ia Supernovae, Astrophys. J. 696 (2009) 1727 [0810.5553].
- [6] R. Nair, S. Jhingan and D. Jain, Cosmic distance duality and cosmic transparency, J. Cosmol. Astropart. Phys. 12 (2012) 028 [1210.2642].
- [7] J.-P. Uzan, N. Aghanim and Y. Mellier, Distance duality relation from x-ray and Sunyaev-Zel'dovich observations of clusters, Phys. Rev. D 70 (2004) 083533 [astro-ph/0405620].
- [8] L.T. Santana, M.O. Calvão, R.R.R. Reis and B.B. Siffert, How does light move in a generic metric-affine background?, Phys. Rev. D 95 (2017) 061501 [1703.10871].
- [9] R.P.L. Azevedo and P.P. Avelino, Distance-duality in theories with a nonminimal coupling to gravity, Phys. Rev. D 104 (2021) 084079 [2104.01209].
- [10] A.G. Riess, S. Casertano and W.e.a. Yuan, A comprehensive measurement of the local value of the hubble constant, The Astrophysical Journal Letters 934 (2022) L7.
- [11] Planck Collaboration, N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi et al., *Planck 2018 results. VI. Cosmological parameters*, *Astron. Astrophys.* **641** (2020) A6 [1807.06209].
- [12] C. Heymans, L. Van Waerbeke, L. Miller, T. Erben, H. Hildebrandt, H. Hoekstra et al., CFHTLenS: the Canada-France-Hawaii Telescope Lensing Survey, Mon. Not. R. Astron. Soc. 427 (2012) 146 [1210.0032].
- [13] E. Di Valentino, A. Melchiorri and J. Silk, *Planck evidence for a closed Universe and a possible crisis for cosmology*, *Nature Astronomy* 4 (2020) 196 [1911.02087].
- [14] W. Handley, Curvature tension: Evidence for a closed universe, Phys. Rev. D 103 (2021) L041301 [1908.09139].
- [15] C. Heymans, T. Tröster, M. Asgari, C. Blake, H. Hildebrandt, B. Joachimi et al., KiDS-1000 Cosmology: Multi-probe weak gravitational lensing and spectroscopic galaxy clustering constraints, Astron. Astrophys. 646 (2021) A140 [2007.15632].
- [16] E. Di Valentino, A. Melchiorri and J. Silk, Investigating Cosmic Discordance, Astrophys. J. Lett. 908 (2021) L9 [2003.04935].
- [17] T.M.C. Abbott, M. Aguena, A. Alarcon, S. Allam, O. Alves, A. Amon et al., Dark Energy Survey Year 3 results: Cosmological constraints from galaxy clustering and weak lensing, Phys. Rev. D 105 (2022) 023520 [2105.13549].
- [18] C. Potter, J.B. Jensen, J. Blakeslee, P. Milne, P.M. Garnavich and P. Brown, Calibrating the Type Ia Supernova Distance Scale Using Surface Brightness Fluctuations, in American Astronomical Society Meeting Abstracts #232, vol. 232 of American Astronomical Society Meeting Abstracts, p. 319.02, June, 2018.
- [19] K.C. Wong, S.H. Suyu, G.C.F. Chen, C.E. Rusu, M. Millon, D. Sluse et al., H0LiCOW XIII. A 2.4 per cent measurement of H₀ from lensed quasars: 5.3σ tension between early- and late-Universe probes, Mon. Not. R. Astron. Soc. 498 (2020) 1420 [1907.04869].

- [20] E. Di Valentino, A. Melchiorri and J. Silk, Reconciling Planck with the local value of H₀ in extended parameter space, Physics Letters B **761** (2016) 242 [1606.00634].
- [21] D.M. Scolnic, D.O. Jones, A. Rest, Y.C. Pan, R. Chornock, R.J. Foley et al., The Complete Light-curve Sample of Spectroscopically Confirmed SNe Ia from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample, Astrophys. J. 859 (2018) 101 [1710.00845].
- [22] G. Efstathiou, A Lockdown Perspective on the Hubble Tension (with comments from the SH0ES team), arXiv e-prints (2020) arXiv:2007.10716 [2007.10716].
- [23] F. Renzi and A. Silvestri, Reconstructing cosmic distance duality violations and implications for the hubble tension, Physical Review D 108 (2023) 043518.
- [24] X. Li, W. Lin and Y. Hu, Cosmic distance-duality relation in light of hubble tension, Monthly Notices of the Royal Astronomical Society (2023).
- [25] E.M. Teixeira, W. Giarè, N.B. Hogg, T. Montandon, A. Poudou and V. Poulin, Implications of distance duality violation for the H₀ tension and evolving dark energy, arXiv e-prints (2025) arXiv:2504.10464 [2504.10464].
- [26] A.C. Alfano and O. Luongo, Cosmic distance duality after DESI 2024 data release and dark energy evolution, arXiv e-prints (2025) arXiv:2501.15233 [2501.15233].
- [27] R.F.L. Holanda, J.A.S. Lima and M.B. Ribeiro, Testing the Distance-Duality Relation with Galaxy Clusters and Type Ia Supernovae, Astrophys. J. Lett. 722 (2010) L233 [1005.4458].
- [28] Z. Li, P. Wu and H. Yu, Cosmological-model-independent Tests for the Distance-Duality Relation from Galaxy Clusters and Type Ia Supernova, Astrophys. J. Lett. 729 (2011) L14 [1101.5255].
- [29] N. Liang, Z. Li, P. Wu, S. Cao, K. Liao and Z.-H. Zhu, A consistent test of the distance-duality relation with galaxy clusters and Type Ia Supernovae, Mon. Not. R. Astron. Soc. 436 (2013) 1017 [1104.2497].
- [30] P. Wu, Z. Li, X. Liu and H. Yu, Cosmic distance-duality relation test using type Ia supernovae and the baryon acoustic oscillation, Phys. Rev. D 92 (2015) 023520.
- [31] F. Yang, X. Fu, B. Xu, K. Zhang, Y. Huang and Y. Yang, Testing the cosmic distance duality relation using Type Ia supernovae and BAO observations, European Physical Journal C 85 (2025) 186.
- [32] B. Xu and Q. Huang, New tests of the cosmic distance duality relation with the baryon acoustic oscillation and type Ia supernovae, European Physical Journal Plus 135 (2020) 447.
- [33] A. Rana, D. Jain, S. Mahajan and A. Mukherjee, Constraining cosmic curvature by using age of galaxies and gravitational lenses, J. Cosmol. Astropart. Phys. **03** (2017) 028 [1611.07196].
- [34] A. Rana, D. Jain, S. Mahajan, A. Mukherjee and R.F.L. Holanda, *Probing the cosmic distance duality relation using time delay lenses*, *J. Cosmol. Astropart. Phys.* **07** (2017) 010 [1705.04549].
- [35] D. Kumar, D. Jain, S. Mahajan, A. Mukherjee and N. Rani, Constraining cosmological and galaxy parameters using strong gravitational lensing systems, Phys. Rev. D 103 (2021) 063511 [2002.06354].
- [36] D. Kumar, A. Rana, D. Jain, S. Mahajan, A. Mukherjee and R.F.L. Holanda, A non-parametric test of variability of Type Ia supernovae luminosity and CDDR, J. Cosmol. Astropart. Phys. 01 (2022) 053 [2107.04784].
- [37] R. Nair, S. Jhingan and D. Jain, Observational cosmology and the cosmic distance duality relation, J. Cosmol. Astropart. Phys. 05 (2011) 023 [1102.1065].
- [38] H.-N. Lin, X. Li and L. Tang, Strongly lensed gravitational waves as probes to test the cosmic distance duality relation, Chinese Physics C 45 (2021) 015109 [2010.03754].
- [39] J.-Z. Qi, Y.-F. Jiang, W.-T. Hou and X. Zhang, Testing the Cosmic Distance Duality Relation Using Strong Gravitational Lensing Time Delays and Type Ia Supernovae, Astrophys. J. 979 (2025) 2 [2407.07336].
- [40] X. Li and H.-N. Lin, Testing the distance duality relation using Type Ia supernovae and ultracompact radio sources, Mon. Not. R. Astron. Soc. 474 (2018) 313 [1710.11361].
- [41] Y. He, Y. Pan, D.-P. Shi, S. Cao, W.-J. Yu, J.-W. Diao et al., Cosmological-model-independent tests of cosmic distance duality relation with Type Ia supernovae and radio quasars, Chinese Journal of Physics 78 (2022) 297 [2206.04946].
- [42] A. Avgoustidis, C. Burrage, J. Redondo, L. Verde and R. Jimenez, Constraints on cosmic opacity and beyond the standard model physics from cosmological distance measurements, J. Cosmol. Astropart. Phys. 10 (2010) 024 [1004.2053].

- [43] D. Stern, R. Jimenez, L. Verde, M. Kamionkowski and S.A. Stanford, Cosmic chronometers: constraining the equation of state of dark energy. I: H(z) measurements, J. Cosmol. Astropart. Phys. 02 (2010) 008 [0907.3149].
- [44] R.F.L. Holanda, R.S. Gonçalves and J.S. Alcaniz, A test for cosmic distance duality, J. Cosmol. Astropart. Phys. 06 (2012) 022 [1201.2378].
- [45] R.F.L. Holanda, J.C. Carvalho and J.S. Alcaniz, Model-independent constraints on the cosmic opacity, J. Cosmol. Astropart. Phys. 04 (2013) 027 [1207.1694].
- [46] K. Liao, Z. Li, J. Ming and Z.-H. Zhu, Testing cosmic opacity from SNe Ia and Hubble parameter through three cosmological-model-independent methods, Physics Letters B 718 (2013) 1166 [1212.6612].
- [47] X. Fu and P. Li, Testing the distance-duality relation from strong gravitational lensing, type Ia supernovae and gamma-ray bursts data up to redshift $z \sim 3.6$, International Journal of Modern Physics D 26 (2017) 1750097 [1702.03626].
- [48] B. Kanodia, U. Upadhyay and Y. Tiwari, Revisiting Cosmic Distance Duality with Megamasers and DESI DR2: Model Independent Constraints on Early-Late Calibration, arXiv e-prints (2025) arXiv:2507.11518 [2507.11518].
- [49] A.G. Adame, J. Aguilar, S. Ahlen, S. Alam, D.M. Alexander, M. Alvarez et al., DESI 2024 III: baryon acoustic oscillations from galaxies and quasars, J. Cosmol. Astropart. Phys. 03 (2025) 012 [2404.03000].
- [50] A.G. Adame, J. Aguilar, S. Ahlen, S. Alam, D.M. Alexander, M. Alvarez et al., DESI 2024 VI: cosmological constraints from the measurements of baryon acoustic oscillations, J. Cosmol. Astropart. Phys. 02 (2025) 021 [2404.03002].
- [51] J. Zheng, D.-C. Qiang and Z.-Q. You, Cosmological constraints on dark energy models using DESI BAO 2024, arXiv e-prints (2024) arXiv:2412.04830 [2412.04830].
- [52] A. Hernández-Almada, M.L. Mendoza-Martínez, M.A. García-Aspeitia and V. Motta, *Phenomenological emergent dark energy in the light of DESI Data Release 1, Physics of the Dark Universe* **46** (2024) 101668 [2407.09430].
- [53] Q. Wang, S. Cao, J. Jiang, K. Zhang, X. Jiang, T. Liu et al., New Tests of the Cosmic Distance Duality Relation with DESI 2024 Baryon Acoustic Oscillation Observations, Astrophys. J. 987 (2025) 58 [2506.12759].
- [54] T.-N. Li, Y.-M. Zhang, Y.-H. Yao, P.-J. Wu, J.-F. Zhang and X. Zhang, Is non-zero equation of state of dark matter favored by DESI DR2?, arXiv e-prints (2025) arXiv:2506.09819 [2506.09819].
- [55] G. Ye and S.-J. Lin, On the tension between DESI DR2 BAO and CMB, arXiv e-prints (2025) arXiv:2505.02207 [2505.02207].
- [56] M. Abedin, L.A. Escamilla, S. Pan, E. Di Valentino and W. Yang, When Dark Matter Heats Up: A Model-Independent Search for Non-Cold Behavior, arXiv e-prints (2025) arXiv:2505.09470 [2505.09470].
- [57] Y. Cai, X. Ren, T. Qiu, M. Li and X. Zhang, The Quintom theory of dark energy after DESI DR2, arXiv e-prints (2025) arXiv:2505.24732 [2505.24732].
- [58] S. Anselmi, P.-S. Corasaniti, A.G. Sanchez, G.D. Starkman, R.K. Sheth and I. Zehavi, Cosmic distance inference from purely geometric BAO methods: Linear point standard ruler and correlation function model fitting, Phys. Rev. D 99 (2019) 123515 [1811.12312].
- [59] D. Camarena and V. Marra, A new method to build the (inverse) distance ladder, Mon. Not. R. Astron. Soc. 495 (2020) 2630 [1910.14125].
- [60] A. Bernui, E. Di Valentino, W. Giarè, S. Kumar and R.C. Nunes, Exploring the H₀ tension and the evidence for dark sector interactions from 2D BAO measurements, Phys. Rev. D 107 (2023) 103531 [2301.06097].
- [61] Ruchika, 2d bao vs 3d bao: Hints for new physics?, Phys. Rev. D 112 (2025) 063503.
- [62] S. Dwivedi and M. Högås, 2D BAO vs. 3D BAO: Solving the Hubble Tension with Bimetric Cosmology, Universe 10 (2024) 406 [2407.04322].
- [63] A. Favale, A. Gómez-Valent and M. Migliaccio, Quantification of 2D vs 3D BAO tension using SNIa as a redshift interpolator and test of the Etherington relation, Physics Letters B 858 (2024) 139027 [2405.12142].

- [64] B.R. Dinda and N. Banerjee, Model independent bounds on type Ia supernova absolute peak magnitude, Phys. Rev. D 107 (2023) 063513 [2208.14740].
- [65] D. Benisty, J. Mifsud, J. Levi Said and D. Staicova, On the robustness of the constancy of the Supernova absolute magnitude: Non-parametric reconstruction & Bayesian approaches, Physics of the Dark Universe 39 (2023) 101160 [2202.04677].
- [66] R. von Marttens, J. Gonzalez and J. Alcaniz, Reconstructing the redshift evolution of Type Ia supernovae absolute magnitude, arXiv e-prints (2025) arXiv:2504.15127 [2504.15127].
- [67] D. Camarena and V. Marra, On the use of the local prior on the absolute magnitude of Type Ia supernovae in cosmological inference, Mon. Not. R. Astron. Soc. **504** (2021) 5164 [2101.08641].
- [68] D. Camarena and V. Marra, Local determination of the Hubble constant and the deceleration parameter, Physical Review Research 2 (2020) 013028 [1906.11814].
- [69] L. Kazantzidis, H. Koo, S. Nesseris, L. Perivolaropoulos and A. Shafieloo, Hints for possible low redshift oscillation around the best-fitting ΛCDM model in the expansion history of the Universe, Mon. Not. R. Astron. Soc. 501 (2021) 3421 [2010.03491].
- [70] L. Kazantzidis and L. Perivolaropoulos, Hints of a local matter underdensity or modified gravity in the low z Pantheon data, Phys. Rev. D 102 (2020) 023520 [2004.02155].
- [71] G.-J. Wang, X.-J. Ma, S.-Y. Li and J.-Q. Xia, Reconstructing Functions and Estimating Parameters with Artificial Neural Networks: A Test with a Hubble Parameter and SNe Ia, Astrophys. J. Suppl. Ser. 246 (2020) 13 [1910.03636].
- [72] K. Dialektopoulos, J.L. Said, J. Mifsud, J. Sultana and K. Zarb Adami, Neural network reconstruction of late-time cosmology and null tests, J. Cosmol. Astropart. Phys. **02** (2022) 023 [2111.11462].
- [73] J.-Z. Qi, P. Meng, J.-F. Zhang and X. Zhang, Model-independent measurement of cosmic curvature with the latest H (z) and SNe Ia data: A comprehensive investigation, Phys. Rev. D 108 (2023) 063522 [2302.08889].
- [74] L. Tang, H.-N. Lin and L. Liu, Deep learning method for testing the cosmic distance duality relation, Chinese Physics C 47 (2023) 015101 [2210.04228].
- [75] A. Mitra, I. Gómez-Vargas and V. Zarikas, Dark energy reconstruction analysis with artificial neural networks: Application on simulated Supernova Ia data from Rubin Observatory, Physics of the Dark Universe 46 (2024) 101706 [2402.18124].
- [76] Y. Yang, T. Liu, J. Huang, X. Cheng, M. Biesiada and S.-m. Wu, Simultaneous measurements on cosmic curvature and opacity using latest HII regions and H(z) observations, European Physical Journal C 84 (2024) 3 [2401.03413].
- [77] M. Abedin, G.-J. Wang, Y.-Z. Ma and S. Pan, In search of an interaction in the dark sector through Gaussian process and ANN approaches, Mon. Not. R. Astron. Soc. **540** (2025) 2253 [2505.04336].
- [78] Z. Huang, Z. Xiong, X. Luo, G. Wang, Y. Liu and N. Liang, Gamma-ray bursts calibrated from the observational H(z) data in artificial neural network framework, Journal of High Energy Astrophysics 47 (2025) 100377 [2502.10037].
- [79] S. Li and J.-Q. Xia, Testing General Relativity Using Large-scale Structure Photometric Redshift Surveys and the Cosmic Microwave Background Lensing Effect, Astrophys. J. Suppl. Ser. 276 (2025) 71 [2501.02852].
- [80] R.C. Nunes, S.K. Yadav, J.F. Jesus and A. Bernui, Cosmological parameter analyses using transversal BAO data, Mon. Not. R. Astron. Soc. 497 (2020) 2133 [2002.09293].
- [81] E. de Carvalho, A. Bernui, F. Avila, C.P. Novaes and J.P. Nogueira-Cavalcante, *BAO angular scale at* $z_{eff} = 0.11$ with the SDSS blue galaxies, Astron. Astrophys. **649** (2021) A20 [2103.14121].
- [82] G.C. Carvalho, A. Bernui, M. Benetti, J.C. Carvalho and J.S. Alcaniz, *Baryon acoustic oscillations from the SDSS DR10 galaxies angular correlation function*, *Phys. Rev. D* **93** (2016) 023530 [1507.08972].
- [83] J.S. Alcaniz, G.C. Carvalho, A. Bernui, J.C. Carvalho and M. Benetti, Measuring baryon acoustic oscillations with angular two-point correlation function, arXiv e-prints (2016) arXiv:1611.08458 [1611.08458].
- [84] G.C. Carvalho, A. Bernui, M. Benetti, J.C. Carvalho, E. de Carvalho and J.S. Alcaniz, The transverse baryonic acoustic scale from the SDSS DR11 galaxies, Astroparticle Physics 119 (2020) 102432 [1709.00271].

- [85] E. de Carvalho, A. Bernui, G.C. Carvalho, C.P. Novaes and H.S. Xavier, Angular Baryon Acoustic Oscillation measure at z=2.225 from the SDSS quasar survey, J. Cosmol. Astropart. Phys. **04** (2018) 064 [1709.00113].
- [86] H. Gil-Marín, W.J. Percival, L. Verde, J.R. Brownstein, C.-H. Chuang, F.-S. Kitaura et al., The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: RSD measurement from the power spectrum and bispectrum of the DR12 BOSS galaxies, Mon. Not. R. Astron. Soc. 465 (2017) 1757 [1606.00439].
- [87] DES collaboration, Dark Energy Survey: A 2.1% measurement of the angular baryonic acoustic oscillation scale at redshift zeff=0.85 from the final dataset, Phys. Rev. D 110 (2024) 063515 [2402.10696].
- [88] J. Hou, A.G. Sánchez, A.J. Ross, A. Smith, R. Neveux, J. Bautista et al., The completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: BAO and RSD measurements from anisotropic clustering analysis of the quasar sample in configuration space between redshift 0.8 and 2.2, Mon. Not. R. Astron. Soc. 500 (2021) 1201 [2007.08998].
- [89] H. du Mas des Bourboux, J. Rich, A. Font-Ribera, V. de Sainte Agathe, J. Farr, T. Etourneau et al., The Completed SDSS-IV Extended Baryon Oscillation Spectroscopic Survey: Baryon Acoustic Oscillations with Lyα Forests, Astrophys. J. 901 (2020) 153 [2007.08995].
- [90] S. Alam, M. Ata, S. Bailey, F. Beutler, D. Bizyaev, J.A. Blazek et al., The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample, Mon. Not. R. Astron. Soc. 470 (2017) 2617 [1607.03155].
- [91] S. Alam, M. Aubert, S. Avila, C. Balland, J.E. Bautista, M.A. Bershady et al., Completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: Cosmological implications from two decades of spectroscopic surveys at the Apache Point Observatory, Phys. Rev. D 103 (2021) 083533 [2007.08991].
- [92] A.L. González-Morán, R. Chávez, E. Terlevich, R. Terlevich, D. Fernández-Arenas, F. Bresolin et al., Independent cosmological constraints from high-z H II galaxies: new results from VLT-KMOS data, Mon. Not. R. Astron. Soc. 505 (2021) 1441 [2105.04025].
- [93] A.L. González-Morán, R. Chávez, R. Terlevich, E. Terlevich, F. Bresolin, D. Fernández-Arenas et al., Independent cosmological constraints from high-z H II galaxies, Mon. Not. R. Astron. Soc. 487 (2019) 4669 [1906.02195].
- [94] R. Chávez, R. Terlevich, E. Terlevich, F. Bresolin, J. Melnick, M. Plionis et al., The L-σ relation for massive bursts of star formation, Mon. Not. R. Astron. Soc. 442 (2014) 3565 [1405.4010].
- [95] J. Melnick, M. Moles, R. Terlevich and J.-M. Garcia-Pelayo, Giant H II regions as distance indicators -I. Relations between global parameters for the local calibrators., Mon. Not. R. Astron. Soc. 226 (1987) 849.
- [96] L. Searle and W.L.W. Sargent, Inferences from the Composition of Two Dwarf Blue Galaxies, Astrophys. J. 173 (1972) 25.
- [97] J. Bergeron, Characteristics of the blue stars in the dwarf galaxies I Zw 18 and II Zw 40., Astrophys. J. 211 (1977) 62.
- [98] R. Terlevich and J. Melnick, The dynamics and chemical composition of giant extragalactic H II regions, Mon. Not. R. Astron. Soc. 195 (1981) 839.
- [99] J. Melnick, R. Terlevich and E. Terlevich, Hii galaxies as deep cosmological probes, Mon. Not. R. Astron. Soc. 311 (2000) 629 [astro-ph/9908346].
- [100] E.R. Siegel, R. Guzmán, J.P. Gallego, M. Orduña López and P. Rodríguez Hidalgo, Towards a precision cosmology from starburst galaxies at z ¿ 2, Mon. Not. R. Astron. Soc. 356 (2005) 1117 [astro-ph/0410612].
- [101] R. Chávez, E. Terlevich, R. Terlevich, M. Plionis, F. Bresolin, S. Basilakos et al., Determining the Hubble constant using giant extragalactic H II regions and H II galaxies, Mon. Not. R. Astron. Soc. 425 (2012) L56 [1203.6222].
- [102] S. Cao and B. Ratra, $H_0 = 69.8 \pm 1.3 \ km \ s^{-1} \ Mpc^{-1}$, $\Omega_{m0} = 0.288 \pm 0.017$, and other constraints from lower-redshift, non-CMB, expansion-rate data, Phys. Rev. D 107 (2023) 103521 [2302.14203].
- [103] J. Melnick and E. Telles, *HII galaxies as standard candles: Evolutionary corrections*, *Astron. Astrophys.* **690** (2024) A157 [2407.18704].

- [104] J. Gao, Y. Chen and L. Xu, Optimizing the L-σ Relation of HII Galaxies for improving cosmological application, arXiv e-prints (2024) arXiv:2408.10560 [2408.10560].
- [105] J. Rincón, H. Martínez-Huerta, A. Huet, A. Hernández-Almada and M.A. García-Aspeitia, *Late-Time Cosmic Acceleration from QCD Confinement Dynamics*, arXiv e-prints (2025) arXiv:2506.13812 [2506.13812].
- [106] R. Sandoval-Orozco, C. Escamilla-Rivera, R. Briffa and J. Levi Said, Testing f(T) cosmologies with HII Hubble diagram and CMB distance priors, Phys. Dark Univ. 46 (2024) 101641 [2405.06633].
- [107] D.-A. Clevert, T. Unterthiner and S. Hochreiter, Fast and Accurate Deep Network Learning by Exponential Linear Units (ELUs), arXiv e-prints (2015) arXiv:1511.07289 [1511.07289].
- [108] N.B. Hogg, M. Martinelli and S. Nesseris, Constraints on the distance duality relation with standard sirens, J. Cosmol. Astropart. Phys. 12 (2020) 019 [2007.14335].
- [109] F. Renzi, N.B. Hogg, M. Martinelli and S. Nesseris, Strongly lensed supernovae as a self-sufficient probe of the distance duality relation, Physics of the Dark Universe 32 (2021) 100824 [2010.04155].
- [110] S. Gahlaut, Model—Independent Probe of Cosmic Distance Duality Relation, Research in Astronomy and Astrophysics 25 (2025) 025019 [2501.15086].
- [111] L. Tang, H.-N. Lin and Y. Wu, Cosmic distance duality relation in light of time-delayed strong gravitational lensing, Chinese Physics C 49 (2025) 015104 [2410.08595].
- [112] D. Foreman-Mackey, D.W. Hogg, D. Lang and J. Goodman, emcee: The mcmc hammer, PASP 125 (2013) 306 [1202.3665].
- [113] B. Xu, Z. Wang, K. Zhang, Q. Huang and J. Zhang, Model-independent Test for the Cosmic Distance-Duality Relation with Pantheon and eBOSS DR16 Quasar Sample, Astrophys. J. 939 (2022) 115 [2212.00269].
- [114] M. Wang, X. Fu, B. Xu, Y. Huang, Y. Yang and Z. Lu, Testing the cosmic distance duality relation with Type Ia supernova and transverse BAO measurements, European Physical Journal C 84 (2024) 702 [2407.12250].
- [115] J. Amiaux, R. Scaramella, Y. Mellier, B. Altieri, C. Burigana, A. Da Silva et al., Euclid mission: building of a reference survey, in Space Telescopes and Instrumentation 2012: Optical, Infrared, and Millimeter Wave, M.C. Clampin, G.G. Fazio, H.A. MacEwen and J.M. Oschmann, Jr., eds., vol. 8442 of Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, p. 84420Z, Sept., 2012, DOI [1209.2228].
- [116] M. Martinelli, C.J.A.P. Martins, S. Nesseris, D. Sapone, I. Tutusaus, A. Avgoustidis et al., Euclid: Forecast constraints on the cosmic distance duality relation with complementary external probes, Astron. Astrophys. 644 (2020) A80 [2007.16153].