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1 Setting

1.1 Header

```
#include<bits/stdc++.h>

using namespace std;
typedef long long ll;
typedef unsigned long long ull;
typedef pair<int, int> Pi;
typedef pair<ll,ll> Pll;

#define Fi first
#define Se second
#define pb(x) push_back(x)
#define sz(x) (int)x.size()
#define rep(i, n) for(int i=0;i<n;i++)
#define repp(i, n) for(int i=1;i<=n;i++)
#define all(x) x.begin(), x.end()

#define INF 987654321
#define IINF 654321987654321LL
```

1.2 vimrc

```
syntax on
set nu ai ci si nobk et ar ru nocp hls
set bs=2 ts=4 sw=4 sts=4
set cb=unnamed
set mouse=an
command PS vsp %:r.in|sp %:r.out|vert res 30|wa
command RIO wall|!g++ -O2 -std=c++14 -Wall -lm %:r.cpp && ./a.out < %:r.in > %:r.out
command RI wall|!g++ -O2 -std=c++14 -Wall -lm %:r.cpp && ./a.out < %:r.in
```

1.3 Sublime text

{

```

"shell_cmd": "g++ -O2 -std=c++11 \"${file}\" -o \"${file_path}/${file_base_name}\" && \"${file_path}/${file_base_name}\" < input.txt",
"working_dir": "${file_path}",
"selector": "source.c++",
}

```

## 2 String

### 2.1 KMP

```

vector<int> preprocess(string p){
    int m = p.size();
    vector<int> fail(m);
    fail[0] = 0; int j = 0;
    for(int i=1;i<m;i++){
        while(j>0&&p[i]!=p[j]) j = fail[j-1];
        if( p[i] == p[j] ){
            fail[i] = j+1; j++;
        }else{
            fail[i] = 0;
        }
    }
    return fail;
}

vector<int> kmp(string s, string p){
    auto fail = preprocess(p);
    vector<int> ans; int n = s.size(), m = p.size();
    int j = 0;
    for(int i=0;i<n;i++){
        while(j>0 && s[i]!=p[j]) j = fail[j-1];
        if( s[i] == p[j] ){
            if( j == m-1 ){
                ans.pb(i-m+1); j = fail[j];
            }else{
                j++;
            }
        }
    }
    return ans;
}

```

### 2.2 Aho Chorasick

```

struct AhoCorasick{
    struct Node{
        int fail;
        vector<int> output;
        int children[26];

        Node(){
            for(int i=0;i<26;i++) children[i] = -1;
            fail = -1;
        }
    };
};

```

```

    }
};

vector<Node> trie;
int new_node(){
    Node x;
    trie.push_back(x);
    return (int)trie.size()-1;
}

void add(int node, string &s, int idx, int string_num){
    //cout << node << " " << idx << endl;
    if( idx == s.size() ){
        trie[node].output.push_back(string_num);
        return;
    }
    int c = s[idx] - 'a';
    if( trie[node].children[c] == -1 ){
        int next = new_node();
        trie[node].children[c] = next;
    }

    add(trie[node].children[c], s, idx+1, string_num);
}

void build(vector<string> v){
    int root = new_node();
    for(int i=0;i<v.size();i++){
        add(root,v[i],0,i);
    }

    queue<int> q;
    q.push(root); trie[root].fail = root;
    while( !q.empty() ){
        int cur = q.front(); q.pop();
        for(int i=0;i<26;i++){
            int next = trie[cur].children[i];
            if( next == -1 ) continue;

            // build fail
            if( cur == root ){
                trie[next].fail = root;
            }
            else{
                int x = trie[cur].fail;
                while( x != root && trie[x].children[i] == -1 ) x = trie[x].fail;
                if( trie[x].children[i] != -1 ) x = trie[x].children[i];
                trie[next].fail = x;
            }
        }
        // build output
        int f = trie[next].fail;
        for(auto e : trie[f].output) trie[next].output.push_back(e);
        q.push(next);
    }
}

```

```

    }

    vector<Pi> find(string s){
        int n = (int) s.size();
        int cur = 0, root = 0;
        vector<Pi> ans;
        for(int i=0; i<n; i++){
            int c = s[i]-'a';
            while( cur != root && trie[cur].children[c] == -1 ) cur = trie[cur].fail;
            if( trie[cur].children[c] != -1 ) cur = trie[cur].children[c];

            for(auto e : trie[cur].output){
                ans.push_back({e,i});
            }
        }
        return ans;
    }
};

```

## 2.3 Suffix array

```

// Make sure to add !, #, $, %, & at the end of input string
class SuffixArray{
public:
    int n;
    string s;
    vector<int> rank, temprank, sa, tempsa, c;
    vector<int> lcp;
    SuffixArray(string _s){
        n = _s.size(); s = _s;
        rank.resize(n); temprank.resize(n); sa.resize(n); tempsa.resize(n);
        lcp.resize(n);
        constructSA();
        constructLCP();
    }

    void countingSort(int k){
        int sum = 0, maxi = max(270, n); //ASCII 256
        c.clear(); c.resize(maxi+10);
        for(auto& e : c) e = 0;
        for(int i=0; i<n; i++) c[ i+k<n ? rank[i+k] : 0 ] ++;
        for(int i=0; i<maxi; i++){
            int t = c[i]; c[i] = sum; sum += t;
        }
        for(int i=0; i<n; i++) tempsa[ c[ sa[i]+k < n ? rank[sa[i]+k] : 0 ] ++ ]
            = sa[i];
        for(int i=0; i<n; i++) sa[i] = tempsa[i];
    }

    void constructSA(){
        for(int i=0; i<n; i++) rank[i] = s[i];
        for(int i=0; i<n; i++) sa[i] = i;
    }
};

```

```

        for(int k=1; k<n; k<=1){
            countingSort(k);
            countingSort(0);
            int r = 0;
            temprank[sa[0]] = 0;
            for(int i=1; i<n; i++){
                temprank[sa[i]] = (rank[sa[i]] == rank[sa[i-1]] && rank[sa[i]+k]
                    == rank[sa[i-1]+k]) ? r : ++r;
            }
            for(int i=0; i<n; i++) rank[i] = temprank[i];
            if( rank[sa[n-1]] == n-1 ) break;
        }
    }

    // lcp Implementation from
    // http://m.blog.naver.com/dark__nebula/220419358547
    void constructLCP(){
        int h = 0;
        for(int i=0; i<n; i++){
            if( rank[i] ){
                int j = sa[rank[i]-1];
                while( s[i+h] == s[j+h] ) h++;
                lcp[rank[i]] = h;
            }
            if( h > 0 ) h--;
        }
    }
};

```

## 2.4 Manacher's algorithm

```

// finds radius of longest palindrome centered at s[i]
// If you also want to find even-length paidnromes, use dummy characters
// baab -> #b#a#a#b#
vector<int> ManacherAlgorithm(string s){
    int n = (int) s.size();
    int p = -1, r = -1;
    vector<int> A(n);
    for(int i=0; i<n; i++){
        if( r < i ){
            A[i] = 0;
            int j = 0;
            while( i + A[i] < n && i - A[i] >= 0 && s[ i+A[i] ] == s[ i-A[i] ] )
                A[i]++;
            A[i]--;
        }
        else{
            A[i] = min( A[2*p - i] , r-i );
            while( i + A[i] < n && i - A[i] >= 0 && s[ i+A[i] ] == s[ i-A[i] ] )
                A[i]++;
            A[i]--;
        }
    }
};

```

```

        // update r
        if( r < i + A[i] ){
            r = i + A[i];
            p = i;
        }
    }
    return A;
}

```

## 2.5 Z algorithm

```

// Calculates LCP[i] for all 0 <= i < n
vector<int> Zalgorithm(string s){
    int l=0, r=0;
    int n = (int) s.size();
    vector<int> Z(n);
    Z[0] = n;
    for(int i=1; i<n; i++){
        // reset and calculate again
        if( i > r ){
            l = r = i;
            while( r<n && s[r] == s[r-l] ) r++;
            r--;
            Z[i] = r-l+1;
        }

        // extend [l,r]
        else{
            int k = i-l;
            // not enough matching at position k
            if( Z[k] < r-i+1 ) Z[i] = Z[k];
            // enough matching. extend [l,r]
            else{
                l = i;
                while( r<n && s[r] == s[r-l] ) r++;
                r--;
                Z[i] = r-l+1;
            }
        }
    }

    return Z;
};

```

## 3 Graph & Flow

### 3.1 BCC

```

int N,M;
int timer = 0;
vector<int> E[300500];
int vis[300500], low[300500];

```

```

// dfs1 is to fill vis(discover time) and low array
int dfs1(int x, int pa){
    vis[x] = ++timer;
    low[x] = vis[x];
    for(auto e : E[x]){
        if( e!=pa ){
            if( vis[e] ){
                low[x] = min(low[x], vis[e]);
            }
            else{
                dfs1(e,x);
                low[x] = min(low[x], low[e]);
            }
        }
    }
    return low[x] ;
}

```

```

int color = 0;
vector<int> colors[300500], E2[300500];
int vis2[300500];

// dfs2 is to color every nodes
// Store node's colors into colors array
// Store new edges into E2
void dfs2(int x, int pa, int c){
    colors[x].pb(c);
    vis2[x] = 1;
    for(auto e : E[x]){
        if(!vis2[e]){
            // x-e is an articulation edge
            if( low[e] > vis[x] ){
                ++color;
                colors[x].pb(color);
                E2[c].pb(color); E2[color].pb(c);
                dfs2(e,x,color);
            }
            // x-e is not an articulation edge
            else dfs2(e,x,c);
        }
    }
}

```

```

int main(){
    geti(N,M);
    repp(i,M){
        int a, b; geti(a,b);
        E[a].pb(b); E[b].pb(a);
    }
    // fill vis & low
    dfs1(1,-1);
    // find out articulation edge and color of nodes
    color = 1;
    dfs2(1,-1,color);
}

```

### 3.2 Dinic

```

struct MaxFlowDinic{
    struct Edge{
        // next, inv, residual
        int to, inv; ll res;
    };

    int n;
    vector<vector<Edge>> graph;

    vector<int> lev,work;

    void init(int x){
        n = x+10;
        graph.resize(x+10);
        lev.resize(n); work.resize(n);
    }

    void make_edge(int s, int e, ll cap, ll caprev = 0){
        Edge forward = {e, (int)graph[e].size(), cap};
        Edge backward = {s, (int)graph[s].size(), caprev};
        graph[s].push_back(forward);
        graph[e].push_back(backward);
    }

    bool bfs(int source, int sink){
        queue<int> q;
        for(auto& e : lev) e = -1;
        lev[source] = 0; q.push(source);
        while(!q.empty()){
            int cur = q.front(); q.pop();
            for(auto e : graph[cur]){
                if(lev[e.to]==-1 && e.res > 0){
                    lev[e.to] = lev[cur]+1;
                    q.push(e.to);
                }
            }
        }
        return lev[sink] != -1;
    }

    ll dfs(int cur, int sink, ll flow){
        if( cur == sink ) return flow;
        for(int &i = work[cur]; i < (int)graph[cur].size(); i++){
            Edge &e = graph[cur][i];
            if( e.res == 0 || lev[e.to] != lev[cur]+1 ) continue;
            ll df = dfs(e.to, sink, min(flow, e.res) );
            if( df > 0 ){
                e.res -= df;
                graph[e.to][e.inv].res += df;
                return df;
            }
        }
        return 0;
    }
}

```

```

    ll solve( int source, int sink ){
        ll ans = 0;
        while( bfs(source, sink) ){
            for(auto& e : work) e = 0;
            while( true ){
                ll flow = dfs(source,sink,54321987654321LL);
                if( flow == 0 ) break;
                ans += flow;
            }
        }
        return ans;
    }
};

```

### 3.3 Bipartite matching (simple)

```

int yx[5000], xy[5000];
bool vis[5000];
vector<int> E[5000];
int dfs(int x){
    vis[x] = true;
    for(auto e : E[x]){
        if( yx[e] == -1 || (vis[yx[e]] == false && dfs(yx[e]) ) ){
            yx[e] = x;
            xy[e] = e;
            return 1;
        }
    }
    return 0;
}

int main(){
    memset(yx,-1,sizeof yx);
    int ans = 0;
    rep(i,N){
        memset(vis,0,sizeof vis);
        ans += dfs(i);
    }
    cout << ans;
}

```

### 3.4 MCMF

```

struct MCMF{
    struct edge{
        int to, inv, cap, flow, cost;
        int res(){
            return cap - flow;
        }
    };

    vector<vector<edge>> graph;

```

```

vector<int> pv, pe;
vector<int> dist, inq;

void init(int x){
    graph.resize(x+10);
    for(auto& e : graph) e.resize(x+10);
    pv.resize(x+10); pe.resize(x+10);
    dist.resize(x+10);
    inq.resize(x+10);
}

void make_edge(int from, int to, int cap, int cost){
    //printf("%d -> %d | cost = %d\n", from, to, cost);
    edge forward = {to, (int)graph[to].size(), cap, 0, cost};
    edge backward = {from, (int)graph[from].size(), 0, 0, -cost};
    graph[from].push_back(forward);
    graph[to].push_back(backward);
}

int solve(int source, int sink){
    int ans = 0;
    int totalflow = 0;
    while(true){
        for(auto& e : dist) e = INF;
        for(auto& e : inq) e = 0;
        queue<int> q;
        q.push(source); inq[source] = 1;
        dist[source] = 0;

        while(!q.empty()){
            int cur = q.front(); q.pop();
            inq[cur] = 0;
            for(int i=0; i<(int)graph[cur].size(); i++){
                auto& e = graph[cur][i];
                if( e.res() > 0 && dist[e.to] > dist[cur] + e.cost ){
                    dist[e.to] = dist[cur] + e.cost;
                    pv[e.to] = cur; pe[e.to] = i;
                    if( inq[e.to] == 0 ){
                        q.push(e.to); inq[e.to] = 1;
                    }
                }
            }
        }

        if( dist[sink] == INF ) break;

        // add this limit when we don't require maxflow
        //if( dist[sink] > 0 ) break;

        int mnflow = INF;
        for( int v = sink; v != source; v = pv[v] ){
            mnflow = min( mnflow, graph[pv[v]][pe[v]].res() );
        }

        for( int v = sink; v != source; v = pv[v] ){
            int tmp = graph[pv[v]][pe[v]].inv;

```

```

            graph[pv[v]][pe[v]].flow += mnflow;
            graph[v][tmp].flow -= mnflow;
        }
        totalflow += mnflow;
        ans += dist[sink] * mnflow;
    }
    return ans;
}

};

```

### 3.5 2SAT & answer recover

```

#define MAX_V 20010
int V,M;

vector<int> Edge[MAX_V];
vector<int> rEdge[MAX_V];
vector<int> vs;

bool vis[MAX_V];
int cmp[MAX_V];
set<int> printSet[MAX_V];

void addEdge(int from, int to){
    Edge[from].push_back(to);
    rEdge[to].push_back(from);
}

void dfs(int v){
    vis[v] = true;
    for( int i = 0; i < Edge[v].size(); i++){
        if (!vis[Edge[v][i]]) dfs(Edge[v][i]);
    }
    vs.push_back(v);
}

void rdfs(int v, int k){
    vis[v] = true;
    cmp[v] = k;
    printSet[k].insert(v);
    for( int i = 0; i < rEdge[v].size(); i++){
        if (!vis[rEdge[v][i]]) rdfs(rEdge[v][i], k);
    }
}

bool cmp1(set<int>& a, set<int>& b) {
    return *a.begin() < *b.begin();
}

int main()
{
    geti(V); geti(M);
    int cnt = 0;
    while (M--){

```

```

int a, b;
scanf("%d%d", &a, &b);
if (a > 0 && b > 0 ){
    addEdge(a + V, b);
    addEdge(b + V, a);
}
else if (a > 0 && b < 0){
    b = -b;
    addEdge(a + V, b + V);
    addEdge(b, a);
}
else if (a < 0 && b > 0){
    a = -a;
    addEdge(a, b);
    addEdge(b + V, a + V);
}
else{
    a = -a; b = -b;
    addEdge(a, b + V);
    addEdge(b, a + V);
}
}

memset(vis, false, sizeof(vis));
for (int i = 1; i <= 2*V; i++){
    if (!vis[i]) dfs(i);
}

memset(vis, false, sizeof(vis));
int k = 0;
for (int i = vs.size()-1; i >= 0 ; i--){
    if (!vis[vs[i]]) rdfs(vs[i],k++);
}

for (int i = 1; i <= V; i++){
    if (cmp[i] == cmp[V + i]){
        printf("0\n");
        return 0;
    }
}
printf("1\n");

for (int i = 1; i <= V; i++){
    if (cmp[i] > cmp[V + i]){
        printf("1 ");
    }
    else printf("0 ");
}
}

```

### 3.6 Stoer Wagner

```

// Stoer-Wagner algorithm
struct mincut {
    int n;
    vector<vector<int>> graph;

    void init(int nn) {
        n = nn;
        graph.resize(n, vector<int>(n, 0));
    }

    void addEdge(int u, int v, int w) {
        graph[u][v] += w;
        graph[v][u] += w;
    }

    pair<int, vector<int>> findMincut() {
        vector<vector<int>> weight = graph;
        vector<bool> used(n, 0);
        vector<int> best_cut;
        int best_weight = -1;

        vector<vector<int>> group(n);
        for(int i = 0; i < n; i++)
            group[i].push_back(i);

        for(int phase = n-1; phase >= 0; phase--) {
            int start = 0;
            vector<int> w = weight[start];
            vector<bool> inSet = used;
            inSet[start] = true;
            int prev, last = start;

            for(int i = 0; i < phase; i++) {
                prev = last;
                last = -1;
                for(int j = 0; j < n; j++)
                    if(!inSet[j] && (last == -1 || w[j] > w[last])) last = j;

                if(i < phase-1) {
                    inSet[last] = true;
                    for(int j = 0; j < n; j++)
                        w[j] += weight[last][j];
                } else { // last step - merge two nodes: prev & last
                    for(int j = 0; j < n; j++) {
                        weight[prev][j] += weight[last][j];
                        weight[j][prev] = weight[j][last];
                    }
                    used[last] = true;
                    group[prev].insert(group[prev].end(), group[last].begin(),
                                        group[last].end());
                    if(best_weight == -1 || w[last] < best_weight) {
                        best_weight = w[last];
                        best_cut = group[last];
                    }
                }
            }
        }
    }
}

```

```

    }
    }
    return make_pair(best_weight, best_cut);
}
};

```

### 3.7 LR-flow

G has a feasible (s,t)-flow iff G' has a saturating (s',t')-flow  
 in G' total capacity out of s' and into t' are both D (sum of demands)  
 saturating flow : flow with value exactly D.

1. Make new source, new sink (s', t')
2. for every v:  
 $c'(s' \rightarrow v) = \sum \{ d(u \rightarrow v) \}$  (give demands into v)  
 $c'(v \rightarrow t') = \sum \{ d(v \rightarrow w) \}$  (take demands out of v)
3. for every  $u \rightarrow v$ :  
 $c'(u \rightarrow v) = c(u \rightarrow v) - d(u \rightarrow v)$  (difference of cap, demand)
3. make  $t \rightarrow s$  cap: INF

## 4 Query

### 4.1 HLD

```

int N,K,M,tc,T;

struct segTree{ //range max query
    vector<int> v; int n;
    void init(int _n){
        _n+=3; v.resize(_n*2+10); n = _n;
    }
    void update(int x, int val){
        for(v[x+=n]=val; x>1; x>>=1) v[x/2] = max(v[x],v[x^1]);
    }
    int query(int l, int r){ // [l,r]
        r++; //to make range as [l,r+1)
        int res = 0;
        for(l+=n, r+=n; l<r; l>>=1, r>>=1){
            if( l&1 ) res = max(res, v[l++]);
            if( r&1 ) res = max(res, v[--r]);
        }
        return res;
    }
};

#define MAXV 100500
#define LOGV 18
// cNo: node# -> hld# mapping
int cNo[MAXV];
// other arrays are accesed using hld#

```

```

int cPos[MAXV], cSize[MAXV], cHead[MAXV], cN; int cLeaf[MAXV];
vector<Pi> E[MAXV]; int pa[LOGV][MAXV]; int sz[MAXV]; int val[MAXV]; int level[
MAXV];
bool vis[MAXV]; vector<segTree> tree; vector<Pi> edges;
int dfs_build(int x, int p, int v, int lev){
    pa[0][x] = p; sz[x] = 1; val[x] = v; level[x] = lev;
    for(auto e : E[x]) if(e.Fi!=p){
        sz[x] += dfs_build(e.Fi, x, e.Se, lev+1);
    }
    return sz[x];
}
void lca_build(){
    for(int k=1; k<LOGV; k++){
        repp(i, N){
            if( pa[k-1][i] != -1 ) pa[k][i] = pa[k-1][pa[k-1][i]];
            else pa[k][i] = -1;
        }
    }
}
int lca(int x, int y){
    if( level[x] < level[y] ) swap(x, y);
    int diff = level[x] - level[y];
    for(int k=0; k<LOGV; k++){
        if( diff & (1<<k) ) x = pa[k][x];

        if( x == y ) return x;
        for(int k=LOGV-1; k>=0; k--){
            if( pa[k][x] != pa[k][y] ){
                x = pa[k][x]; y = pa[k][y];
            }
        }
        return pa[0][x];
    }
}
void hld(int cur){
    vis[cur] = true;
    if( cHead[cN] == 0 ) cHead[cN] = cur;
    cLeaf[cN] = cur;
    cNo[cur] = cN;
    cPos[cur] = cSize[cN]; cSize[cN]++;
    int nxt = -1; int mx = -1;
    // get max subtree (special child)
    for(auto e : E[cur]) if(!vis[e.Fi]){
        if( sz[e.Fi] > mx ){
            nxt = e.Fi; mx = sz[e.Fi];
        }
    }

    if( mx >= 0 ) hld(nxt);
    for(auto e : E[cur]) if(!vis[e.Fi]){
        cN++; hld(e.Fi);
    }
}

void build_hld_segTree(){
    for(int n=1; n<=cN; n++){
        int cur = cLeaf[n];
    }
}

```



```

        tree[n].init(cSize[n]+5);
        while( cur!=-1 && cNo[cur]==n ){
            tree[n].update(cPos[cur],val[cur]);
            cur = pa[0][cur];
        }
    }
}

void update_query(int x, int val){
    tree[cNo[x]].update(cPos[x],val);
}

int query_up(int u, int v){
    int uc = cNo[u], vc = cNo[v]; int ret = 0;
    while(true){
        if( uc == vc ){
            ret = max(ret, tree[uc].query(cPos[v]+1,cPos[u]) );
            break;
        }
        ret = max(ret, tree[uc].query( cPos[cHead[uc]], cPos[u]) );
        u = cHead[uc]; u = pa[0][u]; uc = cNo[u];
    }
    return ret;
}

int query(int u, int v){
    int l = lca(u,v);
    return max(query_up(u,l), query_up(v,l));
}

int main(){
    geti(N);
    rep(i,N-1){
        int a,b,c; geti(a,b,c);
        E[a].push_back({b,c}); E[b].push_back({a,c});
        edges.push_back({a,b});
    }

    dfs_build(1,-1,0,0); lca_build();
    cN = 1;
    hld(1);
    tree.resize(cN+3);
    build_hld_segTree();
    geti(K);
    rep(i,K){
        int a,b,c; geti(a,b,c);
        if( a == 1 ){
            b--; int u = edges[b].Fi; int v = edges[b].Se;
            if( level[u] > level[v] ) swap(u,v);
            update_query(v,c);
        }else{
            printf("%d\n",query(b,c));
        }
    }
}

```

## 4.2 Centroid decomposition

```

int n;
set<int> adj[MAXN];
int sub[MAXN], dep[MAXN];

void dfsSubtree(int node, int pnode) {
    sub[node] = 1;
    for(auto cnode : adj[node]) {
        if(cnode != pnode) {
            dfsSubtree(cnode, node);
            sub[node] += sub[cnode];
        }
    }
}

int findCentroid(int node, int pnode, int size) {
    for(auto cnode : adj[node]) {
        if(cnode != pnode && sub[cnode] > size / 2)
            return findCentroid(cnode, node, size);
    }
    return node;
}

bool decompose(int node, int depth) {
    bool result = true;
    if(depth >= 26) {
        return false;
    }
    dfsSubtree(node, -1);
    int ctr = findCentroid(node, -1, sub[node]);
    dep[ctr] = depth;
    for(auto cnode : adj[ctr]) {
        adj[cnode].erase(ctr);
        result &= decompose(cnode, depth + 1);
    }
    adj[ctr].clear();
    return result;
}

int main() {
    geti(n);
    rep(i, n-1) {
        int u, v;
        geti(u, v);
        adj[u].insert(v);
        adj[v].insert(u);
    }
    if(decompose(1, 0)) {
        repp(i, n) printf("%c ", dep[i] + 'A');
    } else {
        cout << "Impossible!";
    }
}

```

## 4.3 Mo's algorithm

```

int N,M,K,tc;
ll c[1000005];
ll p[1000005]; int Bsize;
typedef struct query{
    int l,r,n; ll ans;
} query;
bool cmp(query& a, query& b){
    if( a.l/Bsize == b.l/Bsize ) return a.r < b.r;
    else return a.l/Bsize < b.l/Bsize;
}
bool cmp2(query&a, query& b ){ return a.n < b.n; }
int main(void)
{
    geti(N,M); rep(i,N) scanf("%lld",p+i);
    Bsize = (int) sqrt(1.0*N);
    vector<query> q;
    rep(i,M){
        int a,b; geti(a,b); a--;b--;
        q.push_back({a,b,i});
    }

    sort(all(q),cmp);
    int l=0, r=-1; ll sum = 0;

    for(int i=0;i<q.size();i++){
        query& e = q[i];
        int ql = e.l, qr = e.r;
        while( r < qr ){
            r++;
            sum += p[r]*(2*c[p[r]]+1); c[p[r]]++;
        }
        while( r > qr ){
            sum += p[r]*(1-2*c[p[r]]); c[p[r]]--;
            r--;
        }
        while( l < ql ){
            sum += p[l]*(1-2*c[p[l]]); c[p[l]]--;
            l++;
        }
        while( l > ql ){
            l--;
            sum += p[l]*(2*c[p[l]]+1); c[p[l]]++;
        }
        e.ans = sum;
    }

    sort(all(q),cmp2);
    for(auto e : q ){
        printf("%lld\n",e.ans);
    }
}

```

#### 4.4 Mo's algorithm on tree

```
int N;
```

```

int g[MAXN];
int f[MAXN];
int pa[LOGV][MAXV]; int level[MAXN];
int ST[MAXN], EN[MAXN], arr[MAXN*3];
int tt = 0;
vector<int> E[MAXN];

void dfs_build(int x, int p, int lev){
    pa[0][x] = p;
    level[x] = lev;
    ST[x] = ++tt; arr[tt] = x;
    for(auto e : E[x])if(e!=p){
        dfs_build(e,x,lev+1);
    }
    EN[x] = ++tt; arr[tt] = x;
}

void lca_build(){
    for(int k=1;k<LOGV;k++){
        repp(i,N){
            if( pa[k-1][i] != -1 )pa[k][i] = pa[k-1][pa[k-1][i]];
            else pa[k][i] = -1;
        }
    }
}

int lca(int x, int y){
    if( level[x] < level[y] ) swap(x,y);
    int diff = level[x] - level[y];
    for(int k=0;k<LOGV;k++){
        if( diff & (1<<k) ) x = pa[k][x];
    }
    if( x == y ) return x;
    for(int k=LOGV-1;k>=0;k--){
        if( pa[k][x] != pa[k][y] ){
            x = pa[k][x]; y = pa[k][y];
        }
    }
    return pa[0][x];
}

int Bsize;
struct query{
    int l,r,n;
};
bool cmp1(query& a, query& b){
    if( a.l/Bsize == b.l/Bsize ) return a.r < b.r;
    else return a.l/Bsize < b.l/Bsize;
};
bool cmp2(query&a, query& b ){ return a.n < b.n; }

ll ans[100500];
ll cnt[2][200500];
int vis[100500];
ll sum = 0;

void update(int x, int type){

```

```

// add node to range
if( type == 1 ){
    sum += cnt[g[x]^1][f[x]];
    cnt[g[x]][f[x]]++;
}
// remove node from range
if( type == 0 ){
    sum -= cnt[g[x]^1][f[x]];
    cnt[g[x]][f[x]]--;
}
}

int main(void){
    geti(N);
    repp(i,N) geti(g[i]);
    repp(i,N) geti(f[i]);

    set<int> flist;
    map<int,int> fmp;
    repp(i,N) flist.insert(f[i]);
    int tmp = 1;
    for(auto e: flist) fmp[e] = tmp++;
    repp(i,N) f[i] = fmp[f[i]];

    repp(i,N-1){
        int a,b; geti(a,b);
        E[a].pb(b); E[b].pb(a);
    }
    tt = 0;
    dfs_build(1,-1,0);
    lca_build();
    Bsize = (int) sqrt(1.0*tt);

    int Q; geti(Q);
    vector<query> v;
    repp(q,Q){
        int a,b; geti(a,b);
        if( ST[a] > ST[b] ) swap(a,b);
        int l = lca(a,b);
        if( a == l || b == l ){
            v.push_back({ST[a],ST[b],q});
        }
        else{
            v.push_back({EN[a],ST[b],q});
        }
    }

    sort(all(v),cmp1);
    int l=1, r=0;
    for(int i=0;i<v.size();i++){
        query& e = v[i];
        int ql = e.l, qr = e.r;
        while( r < qr ){
            r++;
            int node = arr[r];
            vis[node]++;

```

```

            if( vis[node] == 1 ) update(node,1);
            if( vis[node] == 2 ) update(node,0);
        }
        while( r > qr ){
            int node = arr[r];
            vis[node]--;
            if( vis[node] == 0 ) update(node,0);
            if( vis[node] == 1 ) update(node,1);
            r--;
        }
        while( l < ql ){
            int node = arr[l];
            vis[node]--;
            if( vis[node] == 0 ) update(node,0);
            if( vis[node] == 1 ) update(node,1);
            l++;
        }
        while( l > ql ){
            l--;
            int node = arr[l];
            vis[node]++;
            if( vis[node] == 1 ) update(node,1);
            if( vis[node] == 2 ) update(node,0);
        }

        int u = arr[ql]; int v = arr[qr];
        int l = lca(u,v);

        if( u != l && v != l ){
            int node = l;
            vis[node]++;
            if( vis[node] == 1 ) update(node,1);
            if( vis[node] == 2 ) update(node,0);
        }

        ans[e.n] += sum;

        if( u != l && v != l ){
            int node = l;
            vis[node]--;
            if( vis[node] == 0 ) update(node,0);
            if( vis[node] == 1 ) update(node,1);
        }
    }
    repp(i,Q) printf("%lld\n",ans[i]);
}

```

## 4.5 Mo Hilbert Order

```

inline int64_t gilbertOrder(int x, int y, int pow, int rotate) {
    if (pow == 0) {
        return 0;
    }
    int hpow = 1 << (pow-1);

```

```

    int seg = (x < hpow) ? (
        (y < hpow) ? 0 : 3
    ) : (
        (y < hpow) ? 1 : 2
    );
    seg = (seg + rotate) & 3;
    const int rotateDelta[4] = {3, 0, 0, 1};
    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
    int nrot = (rotate + rotateDelta[seg]) & 3;
    int64_t subSquareSize = int64_t(1) << (2*pow - 2);
    int64_t ans = seg * subSquareSize;
    int64_t add = gilbertOrder(nx, ny, pow-1, nrot);
    ans += (seg == 1 || seg == 2) ? add : (subSquareSize - add - 1);
    return ans;
}

struct Query {
    int l, r, idx;
    int64_t ord;

    inline void calcOrder() {
        ord = gilbertOrder(l, r, 21, 0);
    }
};

inline bool operator<(const Query &a, const Query &b) {
    return a.ord < b.ord;
}

```

## 4.6 Parallel binary search

```

int N,M,K,Q;

vector<Pi> edge[1000500];
int pa[MAXN]; int sz[MAXN];

// each query's answer
Pi ans[MAXN];
// each query's possible answer range for binary search
int low[MAXN], high[MAXN];
// focus[x] : list of query # where it's mid value is x
vector<int> focus[1000500];

int find(int x){
    if( x == pa[x] ) return x;
    return pa[x] = find(pa[x]);
}

int x[MAXN], y[MAXN];

void uni(int a, int b){
    a = find(a); b = find(b);
    if( a == b ) return;
    pa[a] = b;
    sz[b] += sz[a];
}

```

```

int main(void){
    //ios::sync_with_stdio(false);
    geti(N,M);
    int C = -1;
    repp(i,M){
        int a,b,c; geti(a,b,c);
        edge[c].push_back({a,b});
        C = max(C, c);
    }

    geti(Q);
    repp(i,Q){
        int a,b;
        geti(a,b); x[i] = a; y[i] = b;
        ans[i] = {INF,-1};
        // Initially, every query has answer in [0,C] range
        low[i] = 0; high[i] = C;
    }

    bool changed = true;
    while( changed ){
        changed = false;

        // Clear variables
        rep(i,C+1) focus[i].clear();
        repp(i,N) pa[i] = i, sz[i] = 1;

        // Put each query into corresponding focus group
        repp(i,Q){
            if( low[i] > high[i] ) continue;
            focus[ (low[i] + high[i])/2 ].push_back(i);
        }

        // for every time 0~C
        for(int k=0;k<=C;k++){
            // perform action of that time
            for(auto e : edge[k]) uni(e.Fi,e.Se);

            // for each focus group
            // determine it's answer & next position
            for(auto e : focus[k]){
                changed = true;
                int a = x[e]; int b = y[e];
                if( find(a) == find(b) ){
                    ans[e].Fi = min(ans[e].Fi, k);
                    ans[e].Se = sz[find(a)];
                    high[e] = k-1;
                }
                else{
                    low[e] = k+1;
                }
            }
        }
    }
}

```

```

    repp(i,Q){
        if( ans[i].Fi == INF ) printf("%d\n",-1);
        else printf("%d %d\n",ans[i].Fi, ans[i].Se);
    }
}

```

## 4.7 Lazy Propagation 1

```

int N,M,K;

struct segTree{
    struct Node{
        ll d, lazy;
    };
    vector<Node> data;
    int n;
    void init(int x){
        n = 1; while( n < x ) n *= 2;
        data.resize(n*2+10);
    }
    void propagate(int node, int nodeL, int nodeR){
        if( data[node].lazy == 0 ) return;
        ll len = nodeR - nodeL + 1;
        data[node].d += len*data[node].lazy;
        if( len > 1 ){
            data[node*2].lazy += data[node].lazy;
            data[node*2+1].lazy += data[node].lazy;
        }
        data[node].lazy = 0;
    }

    void update(int l, int r, ll val, int node, int nodeL, int nodeR){
        propagate(node, nodeL, nodeR);
        if( l > nodeR || r < nodeL ) return;
        if( l <= nodeL && nodeR <= r ){
            data[node].lazy += val;
            propagate(node,nodeL,nodeR);
            return;
        }
        update(l,r,val,node*2,nodeL,(nodeL+nodeR)/2);
        update(l,r,val,node*2+1,(nodeL+nodeR)/2+1,nodeR);
        data[node].d = data[node*2].d + data[node*2+1].d;
    }

    ll query(int l, int r, int node, int nodeL, int nodeR){
        propagate(node, nodeL, nodeR);
        if( l > nodeR || r < nodeL ) return 0;
        if( l <= nodeL && nodeR <= r ){
            return data[node].d;
        }
        ll sum = 0;
        sum += query(l,r,node*2,nodeL,(nodeL+nodeR)/2);
        sum += query(l,r,node*2+1,(nodeL+nodeR)/2+1,nodeR);
        return sum;
    }
}

```

```

    }
};

int main(){
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    segTree tree;
    cin >> N >> M >> K;
    tree.init(N);
    repp(i,N){
        ll x; cin >> x;
        tree.update(i,i,x,1,1,tree.n);
    }
    repp(i,M+K){
        int a; cin >> a;
        if( a == 1 ){
            int b,c; ll d;
            cin >> b >> c >> d;
            tree.update(b,c,d,1,1,tree.n);
        }
        else{
            int b,c; cin >> b >> c;
            printf("%lld\n",tree.query(b,c,1,1,tree.n));
        }
    }
}

```

## 5 Geometry

### 5.1 Closest pair

```

int N,M,T,K,V;

typedef struct Point{
    int x,y;
    bool operator<(const Point& l) const{
        if( y == l.y ) return x < l.x;
        return y < l.y;
    }
    bool operator==(const Point& l) const{
        return (x==l.x)&&(y==l.y);
    }
} Point;

bool cmp(const Point& l, const Point& r){
    if(l.x == r.x ) return l.y < r.y;
    return l.x < r.x;
}

int dist(Point& l, Point& r){
    return (l.x-r.x)*(l.x-r.x) + (l.y-r.y)*(l.y-r.y);
}

```

```

int main(void)
{
    geti(N); vector<Point> v(N);
    for(int i=0;i<N;i++){
        int x ,y; geti(x,y); v[i].x = x; v[i].y = y;
    }
    sort(all(v),cmp);
    int ans = dist(v[0],v[1]); int left = 0;
    set<Point> possible; possible.insert(v[0]); possible.insert(v[1]);

    for(int i=2;i<N;i++){
        while( (v[i].x - v[left].x)*(v[i].x - v[left].x ) > ans ){
            possible.erase(v[left]);
            left++;
        }
        int d = (int) sqrt(ans) + 1;
        auto bottom = possible.lower_bound({-100000,v[i].y-d});
        auto top = possible.upper_bound({100000,v[i].y+d});
        for(auto it = bottom; it != top; it++){
            Point cur = *it;
            if( dist(v[i],cur) < ans ) ans = dist(v[i],cur);
        }
        possible.insert(v[i]);
    }
    cout << ans;
}

```

## 5.2 Convex hull

```

typedef struct Point{
    ll x,y,n;
} Point;

ll ccw(Point a, Point b, Point c){
    b.x -= a.x, b.y -= a.y;
    c.x -= a.x, c.y -= a.y;
    return b.x*c.y - c.x*b.y;
}

vector<Point> convex_hull(vector<Point> ps){
    if (ps.size() < 3)return ps;
    vector<Point> upper, lower;
    sort(ps.begin(), ps.end(),[](const Point &a, const Point &b) {
        if (a.x == b.x) return a.y < b.y; return a.x < b.x;
    });
    for(const auto &p : ps){ // ccw without `=` when include every point in
        convex_hull
            while(upper.size() >= 2 && ccw(++upper.rbegin(), *upper.rbegin(), p) >=
                0)upper.pop_back();
            while(lower.size() >= 2 && ccw(++lower.rbegin(), *lower.rbegin(), p) <=
                0)lower.pop_back();
            upper.emplace_back(p);
            lower.emplace_back(p);
    }
}

```

```

        lower.insert(lower.end(), ++upper.rbegin(), --upper.rend());
        return lower;
    }

    vector<Point> convex_hull2(vector<Point> ps){ // sorting angle
        if (ps.size() < 3)return ps;
        vector<Point> convex;
        sort(ps.begin(), ps.end(), [](Point &a, Point &b){
            if(a.x == b.x)return a.y < b.y; return a.x<b.x;
        });
        Point d = ps[0];
        for(auto &p : ps){
            p.x -= d.x;p.y -= d.y;
        }
        sort(ps.begin(), ps.end(), [](Point &a, Point &b){
            if (ccw({0,0},a,b) == 0) return a.x*a.x + a.y*a.y < b.x*b.x + b.y*b.y;
            return ccw({0,0},a,b) > 0;
        });
        for(auto &p : ps){
            while(convex.size() >= 2 && ccw(++convex.rbegin(), *convex.rbegin(), p)
                <= 0)convex.pop_back();
            convex.emplace_back(p);
        }
        for(auto &p : convex){
            p.x += d.x;p.y += d.y;
        }
        return convex;
    }
}

```

## 5.3 Rotating Calipers

```

int main(){
    vector<Point> convex;
    int ans = 0;

    int mid = 0;
    // if you want iterate `only` antipodal pairs
    // while(ccw(convex.back(), convex[0], convex[mid], convex[mid+1]) > 0) mid
    ++;

    for(int i=0,j=mid; i < convex.size();){
        // do something with pair of i, j

        int nextj = (j+1) % convex.size();
        int nexti = (i+1) % convex.size();
        if (ccw(convex[i], convex[nexti], convex[j], convex[nextj]) > 0)j =
            nextj;
        else i++;
    }
}

```

## 6 Math

### 6.1 FFT

```
#include <cmath>
#include <complex>
using namespace std;
typedef pair<int,int> pii;
typedef complex<double> base;

void fft(vector<base> &a, bool invert){
    int n = a.size();
    for(int i=1,j=0;i<n;i++){
        int bit = n >> 1;
        for (;j>=bit;bit>>=1)j -= bit;
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }
    for(int len=2;len<=n;len<=1){
        double ang = 2*acos(-1)/len*(invert?-1:1);
        base wlen(cos(ang),sin(ang));
        for(int i=0;i<n;i+=len){
            base w(1);
            for(int j=0;j<len/2;j++){
                base u = a[i+j], v = a[i+j+len/2]*w;
                a[i+j] = u+v;
                a[i+j+len/2] = u-v;
                w *= wlen;
            }
        }
    }
    if (invert) {
        for(int i=0;i<n;i++) a[i] /= n;
    }
}

void multiply(const vector<int> &a, const vector<int> &b, vector<int> &res){
    vector<base> fa(a.begin(), a.end()), fb(b.begin(),b.end());
    int n = 1;
    while(n < max(a.size(), b.size())) n <<= 1;
    n <<= 1;
    fa.resize(n); fb.resize(n);
    fft(fa,false);fft(fb,false);
    for(int i=0;i<n;i++) fa[i] *= fb[i];
    fft(fa,true);
    res.resize(n);
    for(int i=0;i<n;i++) res[i] = int(fa[i].real() + (fa[i].real() > 0 ? 0.5 :
        -0.5));
}
```

### 6.2 Extended Euclidean

```
pair<int,int> ext_gcd(int a,int b){
    if(b){
```

```
        auto tmp = ext_gcd(b, a%b);
        return {tmp.second, tmp.first - (a/b) * tmp.second};
    } else return {1, 0};
}
```

```
int mod_inv(int a, int M){
    return (ext_gcd(a, M).first + M) % M;
}
```

### 6.3 Gaussian Elimination

```
#define MAX_N 300 // adjust this value as needed
struct AugmentedMatrix { double mat[MAX_N][MAX_N + MAX_N + 10]; };
struct ColumnVector { double vec[MAX_N]; };

// 0 indexed row and column
AugmentedMatrix GaussianElimination(int N, AugmentedMatrix Aug) {
    // input: N X 2N matrix [A I], output: [I invA]

    // forward eliminataion phase
    for(int i=0;i<N;i++){
        int l = i;
        // which row has largest column value
        for(int j=i+1;j<N;j++){
            if( fabs(Aug.mat[j][i]) > fabs(Aug.mat[l][i]) )
                l = j;
        }
        // swap this pivot row to minimize error
        for(int k=i;k<2*N;k++){
            swap(Aug.mat[i][k],Aug.mat[l][k]);
        }
        // calculate forward elimination
        for(int j=i+1;j<N;j++){
            for(int k=2*N-1;k>=i;k--){
                Aug.mat[j][k] -= Aug.mat[i][k] * Aug.mat[j][i] / Aug.mat[i][i];
            }
        }

        // normalize pivots
        for(int i=0;i<N;i++){
            for(int j=2*N-1;j>=i;j--){
                Aug.mat[i][j] /= Aug.mat[i][i];
            }
        }

        // backward elimination
        for(int i=N-1;i>0;i--){
            for(int j=i-1;j>=0;j--){
                for(int k=2*N-1;k>=i;k--){
                    Aug.mat[j][k] -= Aug.mat[i][k] * Aug.mat[j][i] / Aug.mat[i][i];
                }
            }
        }

        return Aug;
    }
}

int main() {

    AugmentedMatrix Aug;
    int N; geti(N);
    rep(i,N) rep(j,N) scanf("%lf",&Aug.mat[i][j]);
```

```

    for(int i=N;i<2*N;i++) Aug.mat[i-N][i] = 1;

    AugmentedMatrix res = GaussianElimination(N, Aug);

    // Print inversion of A
    for(int i=0;i<N;i++){
        for(int j=N;j<2*N;j++) printf("%f ",res.mat[i][j]);
        printf("\n");
    }

    return 0;
}

```

## 6.4 Prime Algorithms

```

typedef long long ll;
using namespace std;

ll gcd(ll a, ll b) {
    if (b == 0)
        return a;
    return gcd(b, a%b);
}

namespace miller_rabin {
    ll mul(ll x, ll y, ll mod) { return (__int128)x * y % mod; }
    //ll mul(ll x, ll y, ll mod) { return x * y % mod; }
    ll ipow(ll x, ll y, ll p) {
        ll ret = 1, piv = x % p;
        while (y) {
            if (y & 1) ret = mul(ret, piv, p);
            piv = mul(piv, piv, p);
            y >>= 1;
        }
        return ret;
    }
    bool miller_rabin(ll x, ll a) {
        if (x % a == 0) return 0;
        ll d = x - 1;
        while (1) {
            ll tmp = ipow(a, d, x);
            if (d & 1) return (tmp != 1 && tmp != x - 1);
            else if (tmp == x - 1) return 0;
            d >>= 1;
        }
    }
    bool isprime(ll x) {
        for (auto &i : { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 }) {
            if (x == i) return 1;
            if (x > 40 && miller_rabin(x, i)) return 0;
        }
        if (x <= 40) return 0;
        return 1;
    }
}

```

```

}
namespace pollard_rho {
    ll f(ll x, ll n, ll c) {
        return (c + miller_rabin::mul(x, x, n)) % n;
    }
    void rec(ll n, vector<ll> &v) {
        if (n == 1) return;
        if (n % 2 == 0) {
            v.push_back(2);
            rec(n / 2, v);
            return;
        }
        if (miller_rabin::isprime(n)) {
            v.push_back(n);
            return;
        }
        ll a, b, c;
        while (1) {
            a = rand() % (n - 2) + 2;
            b = a;
            c = rand() % 20 + 1;
            do {
                a = f(a, n, c);
                b = f(f(b, n, c), n, c);
            } while (gcd(abs(a - b), n) == 1);
            if (a != b) break;
        }
        ll x = gcd(abs(a - b), n);
        rec(x, v);
        rec(n / x, v);
    }
    vector<ll> factorize(ll n) {
        vector<ll> ret;
        rec(n, ret);
        sort(ret.begin(), ret.end());
        return ret;
    }
};

int main() {
    vector<ll> res;
    ll num;
    scanf("%lld", &num);
    res = pollard_rho::factorize(num);
    for (int i = 0; i < res.size(); ++i)
        printf("%lld\n", res[i]);
}

```

## 7 Miscellaneous

### 7.1 Grundy number

```

map<set<int>,int> grundy;
map<ll,set<int>> mp;

```



```

int get_grundy(set<int> x){
    // base case
    if( sz(x) == 0 ) return 0;
    if( grundy.find(x) != grundy.end() ) return grundy[x];

    set<int> S;
    int res = 0;

    auto iter = x.end(); iter--;
    int mx = *iter;

    // transition : which k to select
    for(int i=1;i<=mx;i++){
        set<int> nxt;
        for(auto e : x){
            if( e < i ) nxt.insert(e);
            else if( e == i ) continue;
            else nxt.insert(e-i);
        }
        S.insert(get_grundy(nxt));
    }

    // find mex and return
    while( S.find(res) != S.end() ) res++;
    grundy[x] = res;
    return res;
}

int main(void){
    int n; geti(n);

    // Simple prime factorization
    rep(i,n){
        ll x; scanf("%lld",&x);
        for(ll i=2;i*i<=x;i++){
            if( x>0 && x%i == 0 ){
                int cnt = 0;
                while( x>0 && x%i == 0 ){
                    cnt++; x/= i;
                }
                mp[i].insert(cnt);
            }
        }
        if( x > 1 ){
            mp[x].insert(1);
        }
    }

    int res = 0;
    for(auto e : mp){
        res ^= get_grundy(e.Se);
    }

    if( res == 0 ) printf("Arpa");
    else printf("Mojtaba");
}

```

```

}

```

## 7.2 Hungarian

```

// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs. In practice, it solves 1000x1000 problems in around 1
// second.
//
// cost[i][j] = cost for pairing left node i with right node j
// Lmate[i] = index of right node that left node i pairs with
// Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative. To perform
// maximization, simply negate the cost[][] matrix.

typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
    int n = int(cost.size());

    // construct dual feasible solution
    VD u(n);
    VD v(n);
    for (int i = 0; i < n; i++) {
        u[i] = cost[i][0];
        for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
    }
    for (int j = 0; j < n; j++) {
        v[j] = cost[0][j] - u[0];
        for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
    }

    // construct primal solution satisfying complementary slackness
    Lmate = VI(n, -1);
    Rmate = VI(n, -1);
    int mated = 0;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (Rmate[j] != -1) continue;
            if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
                Lmate[i] = j;
                Rmate[j] = i;
                mated++;
                break;
            }
        }
    }

    VD dist(n);
    VI dad(n);
}

```

```

VI seen(n);

// repeat until primal solution is feasible
while (mated < n) {

    // find an unmatched left node
    int s = 0;
    while (Lmate[s] != -1) s++;

    // initialize Dijkstra
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++)
        dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    while (true) {

        // find closest
        j = -1;
        for (int k = 0; k < n; k++) {
            if (seen[k]) continue;
            if (j == -1 || dist[k] < dist[j]) j = k;
        }
        seen[j] = 1;

        // termination condition
        if (Rmate[j] == -1) break;

        // relax neighbors
        const int i = Rmate[j];
        for (int k = 0; k < n; k++) {
            if (seen[k]) continue;
            const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
            if (dist[k] > new_dist) {
                dist[k] = new_dist;
                dad[k] = j;
            }
        }
    }

    // update dual variables
    for (int k = 0; k < n; k++) {
        if (k == j || !seen[k]) continue;
        const int i = Rmate[k];
        v[k] += dist[k] - dist[j];
        u[i] -= dist[k] - dist[j];
    }
    u[s] += dist[j];

    // augment along path
    while (dad[j] >= 0) {
        const int d = dad[j];
        Rmate[j] = Rmate[d];
        Lmate[Rmate[j]] = j;
        j = d;
    }
}

```

```

    }
    Rmate[j] = s;
    Lmate[s] = j;

    mated++;
}

double value = 0;
for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];

return value;
}

```

## 7.3 Hungarian2

```

/*
Tests
http://www.spoj.com/problems/GREED/
https://www.acmicpc.net/problem/8992
SRM 506 mid

```

Time complexity  $O(n^3)$

```

Usage
MinWeightBipartiteMatch matcher(n);
for (int i = 0; i < n; i++) for (int j = 0; j < n; j++) matcher.weights[i][j] =
    SOMETHING;
cost_t total = matcher.solve();

```

See `matcher.match(row -> col)` and `matcher.matched(col -> row)` for actual match  
\*/

```

struct MinWeightBipartiteMatch
{
    typedef long long cost_t;

    cost_t max_cost() const { return numeric_limits<cost_t>::max(); }

    // input
    int n;
    vector<vector<cost_t>> weights;
    // output
    vector<int> match, matched;

    MinWeightBipartiteMatch(int n) :
        n(n), match(n), matched(n), weights(n, vector<cost_t>(n))
    {

    }

    void resize(int n) {
        this->n = n;
        match.resize(n);
        matched.resize(n);
    }
}

```

```

weights.resize(n);
for (int i = 0; i < n; i++) {
    weights[i].resize(n);
}

/* for solve() */
vector<cost_t> slack;
vector<cost_t> potential_row, potential_col;
vector<int> reach_row, reach_col;
int rcnt;
vector<int> from;
void found_match(int r, int c) {
    do {
        int old_match = match[r];
        match[r] = c;
        matched[c] = r;
        tie(r, c) = make_pair(from[r], old_match);
    } while (r >= 0 && c >= 0);
}

void augment(int row_to_match) {
    slack.resize(n);
    for (int c = 0; c < n; c++) {
        slack[c] = weights[row_to_match][c] - potential_row[row_to_match] -
            potential_col[c];
    }
    ++rcnt;
    vector<int> q; q.reserve(n);
    int h = 0;
    q.push_back(row_to_match);
    reach_row[row_to_match] = rcnt;
    from[row_to_match] = -1;
    for (;;) {
        while (h < q.size()) {
            int r = q[h++];
            for (int c = 0; c < n; c++) {
                cost_t gap = weights[r][c] - potential_row[r] - potential_col[c];
                slack[c] = min(slack[c], gap);
                if (gap != cost_t()) continue;
                int next = matched[c];
                if (next < 0) {
                    found_match(r, c);
                    return;
                }
                reach_col[c] = rcnt;
                if (reach_row[next] == rcnt) continue;
                q.push_back(next);
                reach_row[next] = rcnt;
                from[next] = r;
            }
        }
        cost_t delta = max_cost();
        for (int c = 0; c < n; c++) {
            if (reach_col[c] == rcnt) continue; // non-covered -> continue
            delta = min(delta, slack[c]);

```

```

        }
        for (int r = 0; r < n; r++) {
            if (reach_row[r] == rcnt) continue;
            potential_row[r] -= delta;
        }
        for (int c = 0; c < n; c++) {
            if (reach_col[c] == rcnt) continue;
            potential_col[c] += delta;
            slack[c] -= delta;
        }
        int lastsize = q.size();
        for (int c = 0; c < n; c++) {
            if (reach_col[c] == rcnt) continue;
            if (slack[c] != cost_t()) continue;
            int next = matched[c];
            if (next >= 0 && reach_row[next] == rcnt) continue;
            for (int qi = 0; qi < lastsize; qi++) {
                int r = q[qi];
                cost_t gap = weights[r][c] - potential_row[r] - potential_col[c];
                if (gap != cost_t()) continue;
                if (next < 0) {
                    found_match(r, c);
                    return;
                }
                reach_col[c] = rcnt;
                q.push_back(next);
                reach_row[next] = rcnt;
                from[next] = r;
                break;
            }
        }
    }
}

void initialize() {
    potential_row.assign(n, cost_t());
    potential_col.assign(n, cost_t());
    match.assign(n, -1);
    matched.assign(n, -1);
    reach_row.assign(n, 0);
    reach_col.assign(n, 0);
    from.resize(n);
    rcnt = 1;
    for (int i = 0; i < n; i++) {
        cost_t row_min_weight = *min_element(weights[i].begin(), weights[i].end());
        ;
        potential_row[i] = row_min_weight;
    }
    for (int i = 0; i < n; i++) {
        cost_t col_min_weight = weights[0][i] - potential_row[0];
        for (int j = 1; j < n; j++) col_min_weight = min(col_min_weight, weights[j][i] - potential_row[j]);
        potential_col[i] = col_min_weight;
    }
}

```

```

cost_t solve() {
    initialize();
    for (int row_to_match = 0; row_to_match < n; row_to_match++) {
        augment(row_to_match);
    }
    cost_t ans = cost_t();
    for (auto v : potential_row) ans += v;
    for (auto v : potential_col) ans += v;
    return ans;
}
};

```

## 7.4 LiChao Tree

```

// LiChaoTree for dynamic CHT trick
// This example maintains CHT for finding MAXIMUM of corresponding x
// op=1 : add ax + b into CHT
// op=2 : find max value of position x
// https://cp-algorithms.com/geometry/convex_hull_trick.html
ll f(Pll line, ll x){
    return line.Fi*x + line.Se;
}

vector<ll> xlist;
struct LiChaoTree{
    int n; vector<Pll> d;
    void init(int x){
        n = 1; while (n < x) n *= 2;
        d.resize(n*2+10);
        for(auto& e : d){
            e = {0, -3*(1e18)};
        }
    }

    void insert(int node, int nL, int nR, Pll newline){
        if( nL == nR ){
            if( f(d[node], xlist[nL]) < f(newline, xlist[nL]) ) d[node] =
                newline;
            return;
        }
        bool left = f(d[node], xlist[nL]) < f(newline, xlist[nL]);
        bool right = f(d[node], xlist[nR]) < f(newline, xlist[nR]);

        // take upper, lower line based on leftmost point of the segment
        Pll upper = d[node], lower = newline;
        if( left ) swap(upper, lower);

        // one line totally cover another line
        if( left == right ){
            d[node] = upper; return;
        }

        int m = (nL+nR)/2;
        // intersection in left half segment
        if( f(upper, xlist[m]) <= f(lower, xlist[m]) ){

```

```

            d[node] = lower;
            insert(node*2, nL, m, upper);
        }
        // intersection in right half segment
        else{
            d[node] = upper;
            insert(node*2+1, m+1, nR, lower);
        }
    }

    ll query(int node, int nL, int nR, int pos){
        if( nL == nR ) return f(d[node], xlist[pos]);

        int m = (nL+nR)/2;
        ll nval = -3*(1e18);
        if( pos <= m ) nval = query(node*2, nL, m, pos);
        else nval = query(node*2+1, m+1, nR, pos);

        return max(nval, f(d[node], xlist[pos]) );
    }
};

int main(){
    int Q; scanf("%d",&Q);
    vector<pair<int,Pll>> qlist;
    repp(q,Q){
        int op; scanf("%d",&op);
        if( op == 1 ){
            ll a,b; scanf("%lld%lld",&a,&b);
            qlist.push_back({1,{a,b}});
        }
        else{
            ll x; scanf("%lld",&x);
            xlist.push_back(x);
            qlist.push_back({2,{x,x}});
        }
    }

    xlist.push_back(-2*(1e12) - 10);
    sort(all(xlist));
    xlist.erase(unique(all(xlist)), xlist.end());
    LiChaoTree tree;
    tree.init( sz(xlist)+1 );

    // careful to put padding into xlist
    // so that it fits to tree size
    while( sz(xlist) < tree.n+5 ) xlist.push_back(2*(1e12));

    for(auto q : qlist){
        if( q.Fi == 1 ){
            tree.insert(1,1,tree.n,q.Se);
        }
        if( q.Fi == 2 ){
            int pos = lower_bound(all(xlist), q.Se.Fi) - xlist.begin();
            printf("%lld\n",tree.query(1,1,tree.n,pos));
        }
    }
}

```

```

    }
}
}

```

## 7.5 Persistence Segment Tree

```

int n, cnt;
int root[MAXN];

struct node {
    int sum, left, right;
} tree[3 * MAXN * LOGN];

int build(int l = 0, int r = n) {
    int idx = ++cnt;
    if(r - l <= 1) {
        tree[idx] = {0, 0, 0};
        return idx;
    }
    int mid = (l + r) >> 1;
    tree[idx] = {0, build(l, mid), build(mid, r)};
    return idx;
}

int update(int x, int prev, int l = 0, int r = n) {
    if(x < l || r <= x) return prev;
    int idx = ++cnt;
    if(r - l <= 1) {
        tree[idx] = {1, 0, 0};
        return idx;
    }

    int mid = (l + r) >> 1;
    int L = update(x, tree[prev].left, l, mid);
    int R = update(x, tree[prev].right, mid, r);
    tree[idx] = {tree[L].sum + tree[R].sum, L, R};
    return idx;
}

int query(int x, int y, int k, int l = 0, int r = n) {
    if(r - l <= 1) return l;
    int mid = (l + r) >> 1;
    int leftSum = tree[tree[y].left].sum - tree[tree[x].left].sum;
    if(leftSum >= k)
        return query(tree[x].left, tree[y].left, k, l, mid);
    else
        return query(tree[x].right, tree[y].right, k - leftSum, mid, r);
}

int a[MAXN], rev[MAXN];
map<int, int> M;

```

```

int main() {
    int q;
    geti(n, q);
    for(int i = 1; i <= n; i++) {
        geti(a[i]);
        rev[i-1] = a[i];
    }
    sort(rev, rev + n);
    for(int i = 0; i < n; i++)
        M[rev[i]] = i;
    for(int i = 1; i <= n; i++)
        a[i] = M[a[i]];

    root[0] = build();
    for(int i = 1; i <= n; i++)
        root[i] = update(a[i], root[i-1]);

    while(q--) {
        int i, j, k;
        geti(i, j, k);
        printf("%d\n", rev[query(root[i-1], root[j], k)]);
    }
}

```

## 7.6 XOR FFT

```

#include <cstdio>
#include <complex>

const int SZ = 20, N = 1 << SZ;

using namespace std;

int Rev(int x) {
    int i, r = 0;
    for (i = 0; i < SZ; i++) {
        r = r << 1 | x & 1;
        x >>= 1;
    }
    return r;
}

void FFT(int *a, bool f) {
    int i, j, k, z;
    for (i = 0; i < N; i++) {
        j = Rev(i);
        if (i < j) {
            z = a[i];
            a[i] = a[j];
            a[j] = z;
        }
    }
    for (i = 1; i < N; i <= 1) for (j = 0; j < N; j += i << 1) for (k = 0; k <
        i; k++) {
        z = a[i + j + k];

```

```
const double EPS = 0.00001;

typedef complex<double> base;

void fft(vector<base> &a, bool invert){
    int n = a.size();
    for(int i=1,j=0;i<n;i++){
        int bit = n >> 1;
        for (;j>=bit;bit>>=1)j -= bit;
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }
    for(int len=2;len<=n;len<=<=1){
        double ang = 2*acos(-1)/len*(invert?-1:1);
        base wlen(cos(ang),sin(ang));
        for(int i=0;i<n;i+=len){
            base w(1);
            for(int j=0;j<len/2;j++){
                base u = a[i+j], v = a[i+j+len/2]*w;
                a[i+j] = u+v;
                a[i+j+len/2] = u-v;
                w *= wlen;
            }
        }
    }
    if (invert) {
        for(int i=0;i<n;i++) a[i] /= n;
    }
}
```

```

void multiply(const vector<int> &a, const vector<int> &b, vector<int> &res){
    vector<base> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while(n < max(a.size(), b.size())) n <= 1;
    n <= 1;
    fa.resize(n); fb.resize(n);
    fft(fa, false); fft(fb, false);
    for(int i=0; i<n; i++) fa[i] *= fb[i];
    fft(fa, true);
    res.resize(n);
    for(int i=0; i<n; i++) res[i] = int(fa[i].real() + (fa[i].real() > 0 ? 0.5 :
        -0.5));
}

void multiply_complex(const vector<base> &a, const vector<base> &b, vector<base>
    &res){
    vector<base> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while(n < max(a.size(), b.size())) n <= 1;
    n <= 1;
    fa.resize(n); fb.resize(n);
    fft(fa, false); fft(fb, false);
    for(int i=0; i<n; i++) fa[i] *= fb[i];
    fft(fa, true);
    res.resize(n);
    for(int i=0; i<n; i++) res[i] = fa[i];
}

const int MAXN = 405;
const int LOGN = 19;

string S[MAXN], T[MAXN];

int main() {
    int n, m;
    geti(n, m);
    for(int i = 0; i < n; i++)
        cin >> S[i];
    int r, c;
    geti(r, c);
    for(int i = 0; i < r; i++)
        cin >> T[i];

    int p = 1, q = 1;
    while(q < m+c) q <= 1;
    while(p < n+r) p <= 1;

    vector<vector<base>> a(p, vector<base>(q)), b(p, vector<base>(q));
    for(int i = 0; i < p; i++) {
        for(int j = 0; j < q; j++) {
            int t = S[i%n][j%m] - 'a';
            double ang = 2*acos(-1)*t/26;
            a[i][j] = base(cos(ang), sin(ang));
        }
    }
}

```

```

    }
    int cnt = 0;
    for(int i = 0; i < r; i++) {
        for(int j = 0; j < c; j++) {
            if(T[i][j] != '?') {
                cnt++;
                int t = T[i][j] - 'a';
                double ang = 2*acos(-1)*t/26;
                b[(r-1)-i][(c-1)-j] = base(cos(-ang), sin(-ang));
            }
        }
    }

    vector<vector<base>> fa, fb, res;
    for(int i = 0; i < p; i++) {
        vector<base> ta(a[i].begin(), a[i].end()), tb(b[i].begin(), b[i].end());
        fft(ta, false);
        fft(tb, false);
        fa.push_back(ta);
        fb.push_back(tb);
    }

    for(int j = 0; j < q; j++) {
        vector<base> ta(p), tb(p), tmp;
        for(int i = 0; i < p; i++) {
            ta[i] = fa[i][j];
            tb[i] = fb[i][j];
        }
        multiply_complex(ta, tb, tmp);
        if(j == 0)
            res.resize(tmp.size(), vector<base>(q));

        for(int i = 0; i < res.size(); i++)
            res[i][j] = tmp[i];
    }

    for(int i = 0; i < res.size(); i++)
        fft(res[i], true);

    for(int i = 0; i < n; i++) {
        for(int j = 0; j < m; j++) {
            if(abs(res[i+r-1][j+c-1].real() - cnt) < EPS && abs(res[i+r-1][j+c-1].imag()) < EPS) printf("1");
            else printf("0");
        }
        printf("\n");
    }
}

```

## 7.9 Divide and Conquer DP optimization

```

void dfs(int s, int e, int p, int q)
{
    if (s > e) return;
    int m = (s+e)>>1, opt;
}

```

```

D[m] = 1e18;
for (int k=p;k<=q&&k<m;k++){
    lld v = E[k] + (lld)(m-k-1)*(S[m]-S[k]);
    if (D[m] > v)
        D[m] = v, opt = k;
}
dfs(s, m-1, p, opt);
dfs(m+1, e, opt, q);
}

```

## 7.10 Order Statistic Tree

```

#include <ext/pb_ds/assoc_container.hpp> // Common file
#include <ext/pb_ds/tree_policy.hpp> // Including
    tree_order_statistics_node_update

```

```

// Need this
// We can run this code on codeforces
// http://codeforces.com/blog/entry/11080
using namespace __gnu_pbds;

```

```

typedef tree<
int,
null_type,
less<int>,
rb_tree_tag,
tree_order_statistics_node_update>
ordered_set;

```

```

int main(){
    ordered_set X;
    X.insert(1);
    X.insert(2);
    X.insert(4);
    X.insert(8);
    X.insert(16);

    cout<<*X.find_by_order(1)<<endl; // 2
    cout<<*X.find_by_order(2)<<endl; // 4
    cout<<*X.find_by_order(4)<<endl; // 16
    cout<<(end(X)==X.find_by_order(6))<<endl; // true

    cout<<X.order_of_key(-5)<<endl; // 0
    cout<<X.order_of_key(1)<<endl; // 0
    cout<<X.order_of_key(3)<<endl; // 2
    cout<<X.order_of_key(4)<<endl; // 2
    cout<<X.order_of_key(400)<<endl; // 5
}

```

## 7.11 BITSET

```

#define M 32
int main()

```

```

{
    // default constructor initializes with all bits 0
    bitset<M> bset1;

    // bset2 is initialized with bits of 20
    bitset<M> bset2(20);

    // bset3 is initialized with bits of specified binary string
    bitset<M> bset3(string("1100"));

    // cout prints exact bits representation of bitset
    cout << bset1 << endl; // 00000000000000000000000000000000
    cout << bset2 << endl; // 0000000000000000000000000000000010100
    cout << bset3 << endl; // 000000000000000000000000000000001100
    cout << endl;

    // declaring set8 with capacity of 8 bits

    bitset<8> set8; // 00000000

    // setting first bit (or 6th index)
    set8[1] = 1; // 00000010
    set8[4] = set8[1]; // 00010010
    cout << set8 << endl;

    // count function returns number of set bits in bitset
    int numberof1 = set8.count();

    // size function returns total number of bits in bitset
    // so there difference will give us number of unset(0)
    // bits in bitset
    int numberof0 = set8.size() - numberof1;
    cout << set8 << " has " << numberof1 << " ones and "
        << numberof0 << " zeros\n";

    // test function return 1 if bit is set else returns 0
    cout << "bool representation of " << set8 << " : ";
    for (int i = 0; i < set8.size(); i++)
        cout << set8.test(i) << " ";

    cout << endl;

    // any function returns true, if atleast 1 bit
    // is set
    if (!set8.any())
        cout << "set8 has no bit set.\n";

    if (!bset1.any())
        cout << "bset1 has no bit set.\n";

    // none function returns true, if none of the bit
    // is set
    if (!bset1.none())
        cout << "bset1 has all bit set\n";

    // bset.set() sets all bits
}

```



```
    cout << set8.set() << endl;

    // bset.set(pos, b) makes bset[pos] = b
    cout << set8.set(4, 0) << endl;

    // bset.set(pos) makes bset[pos] = 1 i.e. default
    // is 1
    cout << set8.set(4) << endl;

    // reset function makes all bits 0
    cout << set8.reset(2) << endl;
    cout << set8.reset() << endl;

    // flip function flips all bits i.e. 1 <-> 0
    // and 0 <-> 1
    cout << set8.flip(2) << endl;
    cout << set8.flip() << endl;

    // Converting decimal number to binary by using bitset
    int num = 100;
    cout << "\nDecimal number: " << num
         << "   Binary equivalent: " << bitset<8>(num);

    return 0;
}

int main()
{
    bitset<4> bset1(9);      // bset1 contains 1001
    bitset<4> bset2(3);      // bset2 contains 0011

    // comparison operator
    cout << (bset1 == bset2) << endl; // false 0
    cout << (bset1 != bset2) << endl; // true 1

    // bitwise operation and assignment
    cout << (bset1 ^= bset2) << endl; // 1010
    cout << (bset1 &= bset2) << endl; // 0010
    cout << (bset1 |= bset2) << endl; // 0011

    // left and right shifting
    cout << (bset1 <<= 2) << endl;    // 1100
    cout << (bset1 >>= 1) << endl;    // 0110

    // not operator
    cout << (~bset2) << endl;        // 1100

    // bitwise operator
    cout << (bset1 & bset2) << endl;  // 0010
    cout << (bset1 | bset2) << endl;  // 0111
    cout << (bset1 ^ bset2) << endl;  // 0101
}
```









