3. Proof

Proving that T^{π} is a contraction is proving $d(T^{\pi}(V_1(s)), T^{\pi}(V_2(s))) \leq \gamma d(V_1(s), V_2(s))$

Let $d(x_1, x_2) = \|x_1 - x_2\|_{\infty} = \max|x_1 - x_2|$

For V_1, V_2, $\|T^{\pi}(V_1) - T^{\pi}(V_2)\|_{\infty} = \max|T^{\pi}(V_1) - T^{\pi}(V_2)|$

$$\|T^{\pi}(V_1) - T^{\pi}(V_2)\|_{\infty} = \max_{s \in S}\left|R^{\pi}(s) + \gamma \sum_{s'} P^{\pi}_{ss'} V^{\pi}_1(s') - R(s) - \gamma \sum_{s'} P^{\pi}_{ss'} V^{\pi}_2(s')\right|$$

$$= \gamma \max_{s \in S}\left|\sum_{s'} P^{\pi}_{ss'}(V^{\pi}_1(s') - V^{\pi}_2(s'))\right|$$

$$= \gamma \max_{s \in S} \sum_{s'} P^{\pi}_{ss'} |V^{\pi}_1(s') - V^{\pi}_2(s')|$$

$$\leq \gamma \sum_{s'} P^{\pi}_{ss'} \max_{s \in S} |V^{\pi}_1(s') - V^{\pi}_2(s')|$$

$$\leq \gamma \max_{s \in S} |V^{\pi}_1(s') - V^{\pi}_2(s')|$$

$$= \gamma \|V^{\pi}_1 - V^{\pi}_2\|_{\infty}$$