

4 (a)

Return: $G_t = \sum_{k=0}^{T-t-1} \gamma^k R_{t+k+1}$, R is reward $\begin{cases} -1 & \text{failure} \\ 0 & \text{success} \end{cases}$

The end of the episode is when failure occurs. That means

$$G_t = 0 + 0 + \dots + 0 + \gamma^{T-t-1} (-1) = -\gamma^{T-t-1}$$

So the return is $-\gamma^{T-t-1}$ if there is discounting factor

Others is same where we have return as $-\gamma^k$, k is the time step before failure.

(b) Let $G_5 = 0$ terminal

$$\hookrightarrow G_4 = R_5 = 2$$

$$\hookrightarrow G_3 = \gamma G_4 + R_4 = 4$$

$$\hookrightarrow G_2 = \gamma G_3 + R_3 = 8$$

$$\hookrightarrow G_1 = \gamma G_2 + R_2 = 6$$

$$\hookrightarrow G_0 = \gamma G_1 + R_1 = 2$$

(c) $G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$, $G_0 = R_1 + \frac{\gamma \sum_{k=0}^{\infty} \gamma^k R_{k+2}}{\gamma G_1}$

$$R_{2 \sim \infty} = 1, \gamma = 0.9 \rightarrow G_0 = 2 + \frac{0.9 \times 1}{1 - 0.9} = 6.5$$

\hookrightarrow 등비공식 이용