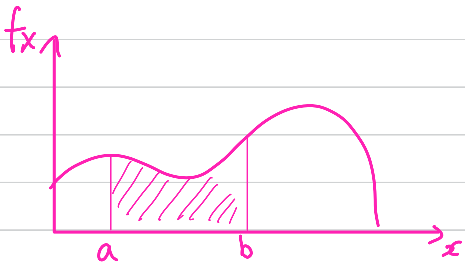
강의 08 Continuous r.v.

PDFs Probability density functions

→ 

$P(a \le X \le b)$

$= \int_a^b f_X(x)\,dx$

Properties

$f_X(x) \ge 0$

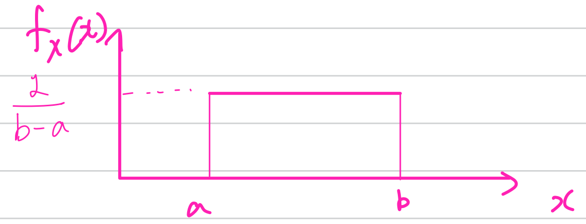
$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

※ PDF or continuous r.v. 에서
특정 지점, x, 의 확률 $P(X=x)$는 0 이다.

ex) $P(a \le X \le b) = P(X=a) + P(X=b) + P(a < X < b)$
$= P(a < X < b)$

=) 개구간 & 폐구간의 확률은 동일

Cotinuous uniform PDF



E of c.r.v

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx \qquad E[X] = \sum_x x P_X(x)$$

x) PMF와 동일한 형태

$$\rightarrow E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx \qquad E[g(X)] = \sum_x g(x) P_X(x)$$

$$\text{ex)}\ E[x^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

Variance $V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f_X(x)\, dx$

S.d $\sigma_X = \sqrt{V(X)}$

Continuous uniform variance

$$E[X^2] = \int_a^b x^2 \frac{1}{b-a}\,dx$$

$$E[X] = \int_a^b x \frac{1}{b-a}\,dx$$

$$V(X) = E[X^2] - E^2[X] \Rightarrow \frac{(b-a)^2}{12}$$

Exponential random variable $\lambda > 0$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

→ similiar to geometric PMF

$$E[X] = \int_0^\infty x f_X(x)\,dx$$

$$= \int_0^\infty x \lambda e^{-\lambda x}\,dx = 1/\lambda$$

$$V(X) = E[X^2] - E^2[X]$$

$$= 1/\lambda^2$$

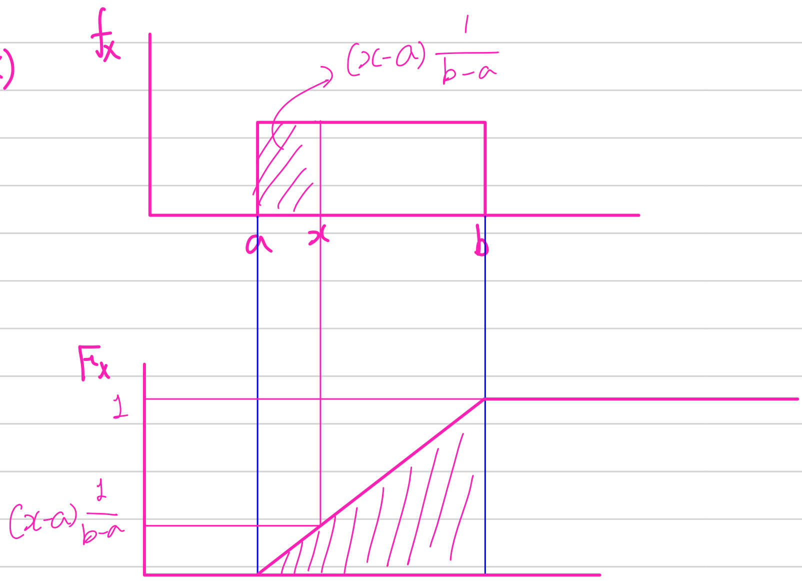
$$E[X^2] = \int_0^\infty x^2 \lambda e^{-\lambda x}\,dx = 2/\lambda^2$$

Cumulative Distribution Function (CDF)

definition $F_X(x) = P(X \le x)$

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(t)\,dt$$

ex)



Properties

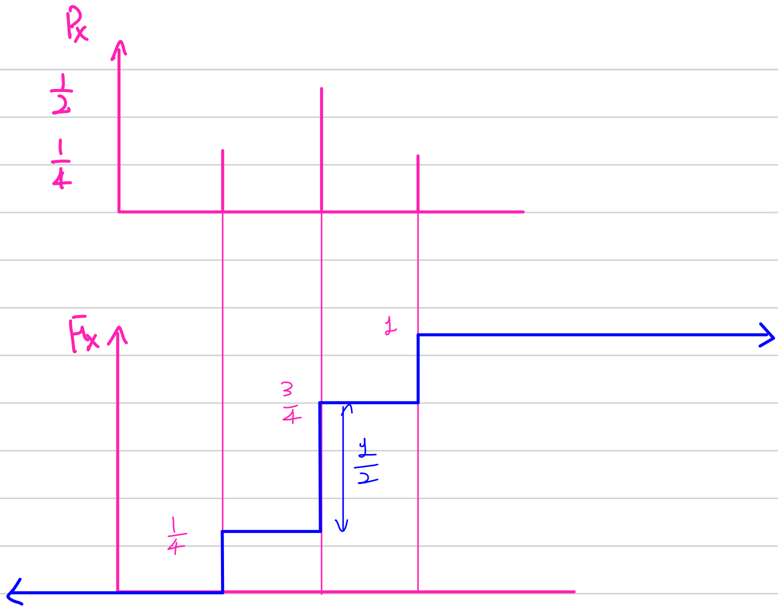
1 $\quad \text{CDF} \underset{\text{Integral}}{\overset{\text{Defferential}}{\rightleftarrows}} \text{PDF}$

2 $\quad y \ge x \longrightarrow F_X(y) \ge F_X(x)$

3 $\quad \lim_{x \to -\infty} F_X(x) = 0$

4 $\quad \lim_{x \to \infty} F_X(x) = 1$

ex)



Normal (Gaussian) r.v.

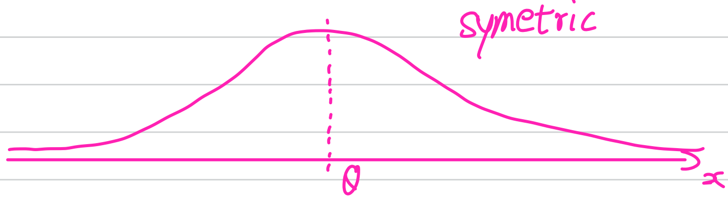
- Central limit theorem

- Prevalent in applications

  ex) noise

Standard Normal

$$N(0,1): f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \qquad x \in \mathbb{R}$$

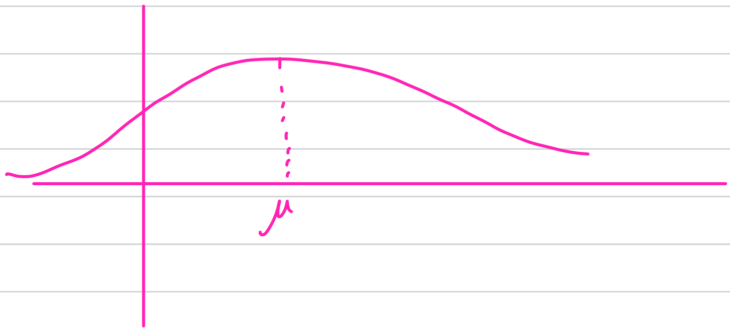


$E[X] = 0$

$V(X) = 1$

General normal r.v.

$$N(\mu, \sigma^2) : f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$



$E[X] = \mu$

$V(X) = \sigma^2$

Linear functions of n.r.v

Let $Y = aX + b$ $\quad X \sim N(\mu, \sigma^2)$

$E[Y] = a\mu + b$

$V(Y) = a^2\sigma^2$

Special case : $a = 0$?

$Y = b$ (discrete)

$N(b, 0)$

Standard normal tables

CDF가 함수로 존재하지 않음 but we use lookup table