강의 7

## Conditional PMFs

$$p_{X|A}(x|A) = P(X=x|A)$$

$$p_{X|Y}(x|y) = P(X=x|Y=y)$$

$$= \frac{P(X=x, Y=y)}{P(Y=y)}$$

$$= \frac{p_{X,Y}(x,y)}{p_Y(y)}$$

$$\Downarrow \text{ 확장}$$

1. $p_{X|Y,Z}(x|y,z) = \dfrac{p_{X,Y,Z}(x,y,z)}{p_{Y,Z}(y,z)}$

2. $p_{X,Y|Z}(x,y|z) = \dfrac{p_{X,Y,Z}(x,y,z)}{p_Z(z)}$

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$P(A\cap B\cap C) = P(A)P(B|A)P(C|A\cap B)$ 에서

$A = \{X=x\}$, $B = \{Y=y\}$, $C = \{Z=z\}$ 라고 하면

$$p_{X,Y,Z}(x,y,z) = p_X(x)\, p_{Y|X}(y|x)\, p_{Z|X,Y}(z|x,y)$$

Conditional expectation

$$E[X|A] = \sum_x x P_{X|A}(x)$$

$$A = \{Y=y\} \qquad E[X|Y=y] = \sum_x x P_{X|Y}(x|y)$$

Total probability of E

$A_1, A_2 \cdots A_n$ : partition of $\Omega$

$$P_X(x) = P(A_1) P_{X|A_1}(x) + \cdots$$

$$P(A_n) P_{X|A_n}(x)$$

$$A_i = \{Y_i = y\}$$

$$P_X(x) = \sum_y P_Y(y) P_{X|Y}(x|y)$$

$$E[X] = \sum_y P_Y(y) E[X|Y=y]$$

Independent

$\rightarrow P(A \cap B) = P(A)P(B), \quad P(A|B) = P(A)$

of r.v. $\quad P(X=x \text{ and } A) = P(X=x)P(A)$ for all $x$

$p_{X|A}(x) = p_X(x)$

$P(A|X=x) = P(A)$

of two r.v. $\quad P(X=x \,\&\, Y=y) = P(X=x)P(Y=y)$

$p_{X,Y}(x,y) = p_X(x)p_Y(y)$

$p_{X|Y}(x|y) = p_X(x)$

$p_{Y|X}(y|x) = p_Y(y)$

Independence E

If X, Y are independent

$E[XY] = E[X]E[Y]$

Proof $\quad E[g(x,y)] \quad g(x,y) = xy$

$$= \sum_x \sum_y xy\, p_{X,Y}(x,y) = \sum_x \sum_y xy\, p_X(x)p_Y(y)$$

$$= \sum_x x\, p_X(x) \sum_y y\, p_Y(y)$$

$$= E[X]E[Y]$$

# Independence & variance

If $X, Y$ are independent : $V(X+Y) = V(X) + V(Y)$ assume $E(X) = E(Y) = 0$

Proof $V(X+Y) = E[(X+Y)^2] = E[X^2+2XY+Y^2]$

$= E(X^2) + 2E[XY] + E[Y^2]$

assume $E[X] = E[Y] = 0$
$\rightarrow \mu_X = \mu_Y = 0$

$= E[(X-\mu_X)^2] + 2E[XY] + E[(Y-\mu_Y)^2]$

$= V(X) + 2\underbrace{E[X]E[Y]}_{2 \times 0 \times 0} + V(Y)$

$= V(X) + V(Y)$

# Variance of binomial

$X = X_1 + \cdots X_n$ → 각각의 시도는 independent

$V(X) = V(X_1) \cdots V(X_n)$

$= np(1-q)$