강의 09

Conditional PDF

$$p_X(x) = P(X=x)$$

$$p_{X|A}(x) = P(X=x|A) \qquad f_{X|A}(x)\cdot\delta \approx P(x \le X \le x+\delta \,|\, A)$$

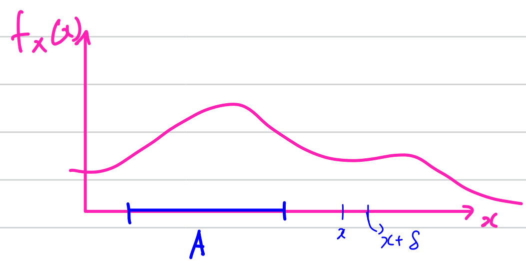
$$P(X\in B|A) = \int_B f_{X|A}(x)\,dx$$

Contional PDF of X, given that $X \in A$

Case 1

$$P(x \le X \le x+\delta \,|\, X\in A) \approx f_{X|X\in A}(x)\cdot\delta$$

$$= \frac{P(x\le X \le x+\delta,\ X\in A)}{P(A)}$$

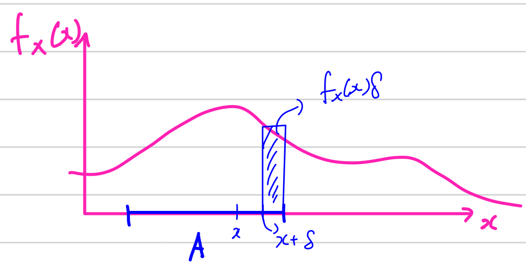


$= 0$ cause the x isn't inside the range A

Case 2

$$P(x \le X \le x+\delta \,|\, X\in A) \approx f_{X|X\in A}(x)\cdot\delta$$

$$= \frac{P(x\le X \le x+\delta,\ X\in A)}{P(A)}$$

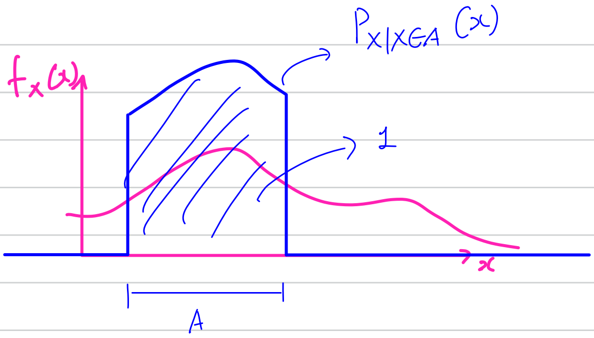


$$\approx \frac{f_X(x)\,\delta}{P(A)} = \frac{f_X(x)}{P(A)}$$

$f_X(x)\,\delta$ → 기둥의 넓이

δ는 아주 작은 숫자이기에 $f_X(x)\,\delta = f_X(x)$로 쓸수 있다

$$\therefore \quad f_{X|X\in A}(x) = \begin{cases} 0, & \text{if } x \notin A \\ \dfrac{f_X(x)}{P(A)} & \text{if } x \in A \end{cases}$$



Conditional E

$$E[X|A] = \int x \, f_{X|A}(x)\,dx$$

$$E[g(X)|A] = \int g(x)\, f_{X|A}(x)\,dx$$

Memorylessness of exponential PDF

Total Probability & expectation

$$F_X(x) = P(X \le x) = P(A_1) P(X \le x | A_1) + \cdots$$

$$= P(A_1) F_{X|A_1}(x) + \cdots$$

$$\rightsquigarrow f_X(x) = P(A_1) f_{X|A_1}(x) + \cdots P(A_n) f_{X|A_n}(x)$$

$$\rightsquigarrow E[X] = P(A_1) E[X|A_1] \cdots P(A_n) E[X|A_n]$$

ex) Bill은 $\frac{1}{3}$ 확률로 오늘 슈퍼마켓에 갈 예정 (0~2시에)
$\frac{2}{3}$ " 내일 " (6~8)

A_1 : 오늘 감. A_2 : 내일 감. X : 시간

$P(A_1) = \frac{1}{3}$

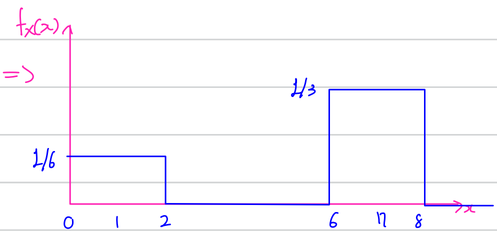
$f_{X|A_1} \sim uniform[0, 2]$ -> 오늘 supermarket 에 가는데, 0~2시에 가는 경우

$P(A_2) = \frac{2}{3}$

$f_{X|A_2} \sim$ " $[6, 8]$

$$f_X(x) = P(A_1) f_{X|A_1}(x) + P(A_2) f_{X|A_2}(x)$$

=>



Mixed distributions

$$X = \begin{cases} \text{uniform on } [0, 2] & \text{w.p. } 1/2 \\ 1 & \text{w.p. } 1/2 \end{cases}$$

→ Is X discrete → ✗

Is X continuous → ✗

X is mixed

Y: discrete

Z: continuous

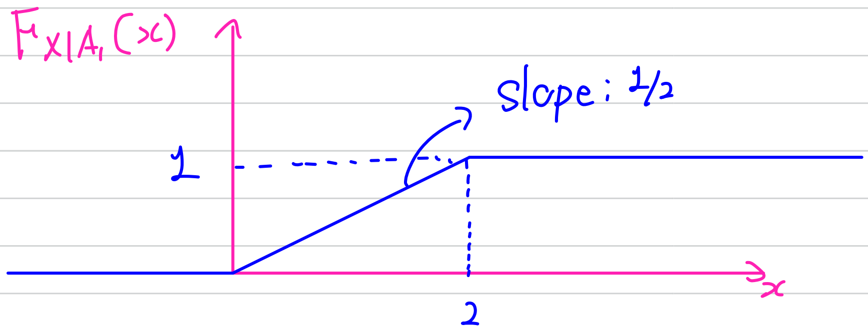
$$X = \begin{cases} Y & , \quad p \\ Z & , \quad 1-p \end{cases}$$

$$F_X(x) = p\,P(Y \leq x) + (1-p)\,P(Z \leq x)$$

$$= p F_Y(x) + (1-p) F_Z(x)$$

$$X = \begin{cases} \text{uni}[0,2] & \text{w.p. } 1/2 \cdots A_1 \\ 1 & \text{w.p. } 1/2 \cdots A_2 \end{cases}$$

CDFs



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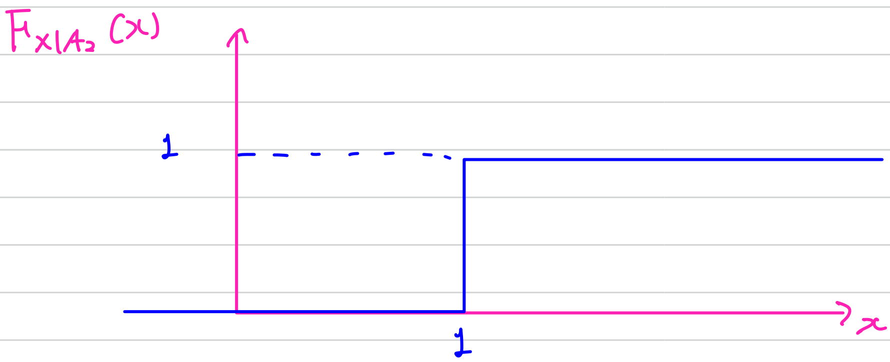

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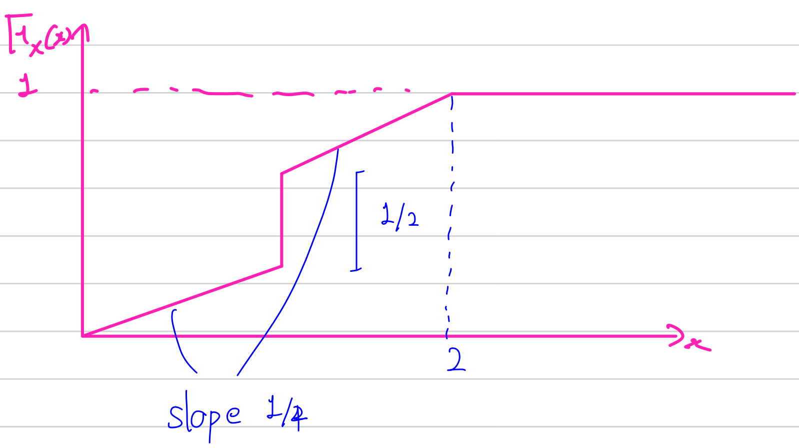
$$F_X(x) = P(A_1) F_{X|A_1}(x) + P(A_2) F_{X|A_2}(x)$$



Jointly continuous r.v.'s and joint PDFs

$p_{X,Y}(x,y) \quad f_{X,Y}(x,y)$

$$P((X,Y)\in B) = \iint_{(x,y)\in B} f_{X,Y}(x,y)\,dx\,dy$$

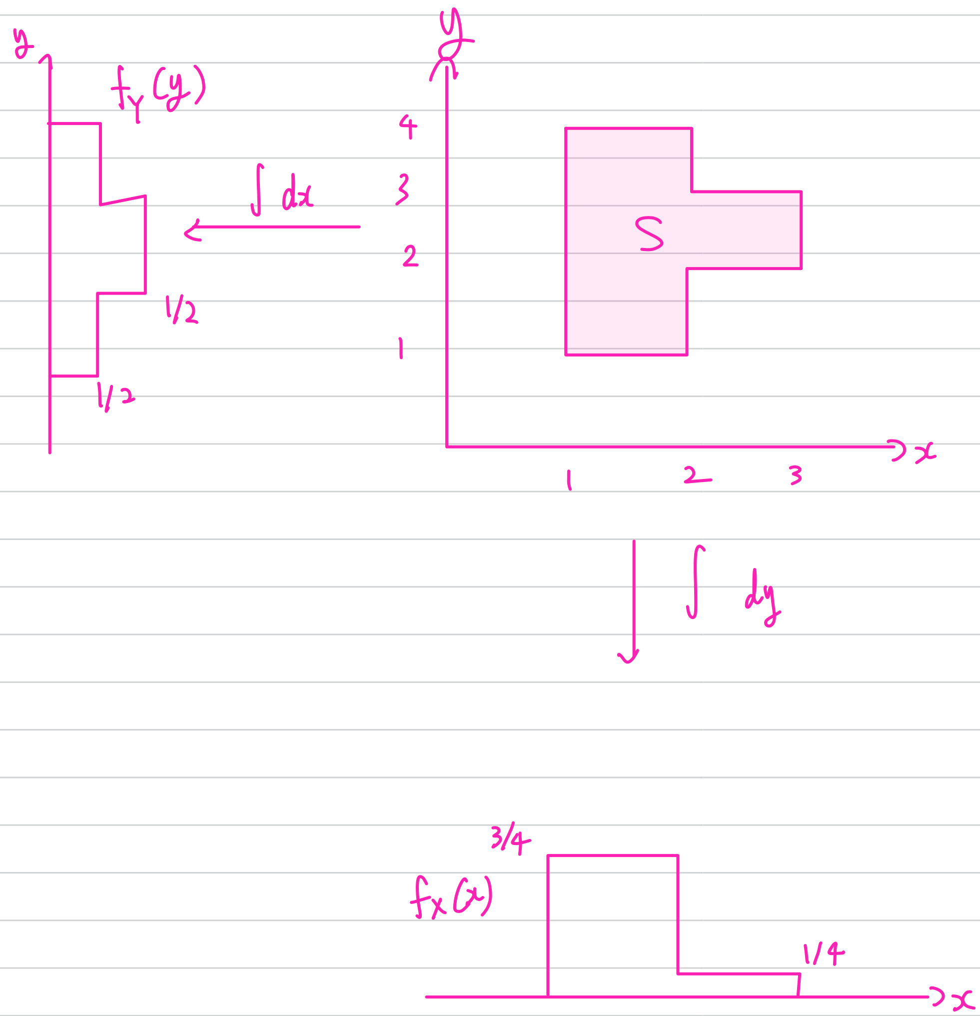
From joint to marginals

$$f_X(x) = \int f_{X,Y}(x, y)\, dy$$

$$f_Y(y) = \int f_{X,Y}(x, y)\, dx$$

marginal	joint

ex)

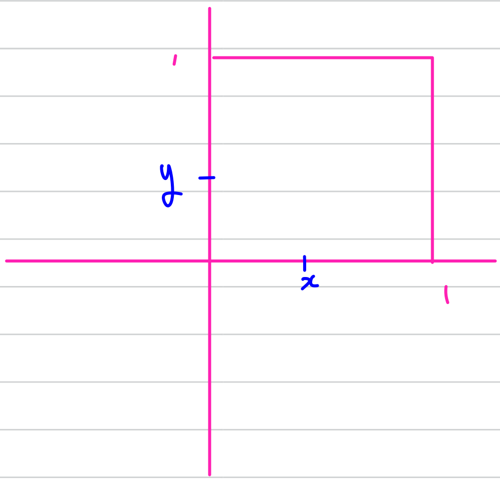


Joint CDF

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$$

$$F_{X,Y}(x,y) = P(X \le x, Y \le y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f_{X,Y}(s,t)\,ds\,dt$$

$$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}}{\partial x\,\partial y}(x,y)$$



$$F_{X,Y}(x,y) = xy$$