

6.5. The Gram-Schmidt Process.

- orthonormal basis or orthogonal basis \Leftrightarrow orth.

Thm 293. V : imp/F , $V = \text{sp}\{u_1, \dots, u_n\}$.

$$\hat{u}_1 = u_1$$

$$\hat{u}_2 = u_2 - \frac{\langle u_2, \hat{u}_1 \rangle}{\langle \hat{u}_1, \hat{u}_1 \rangle} \hat{u}_1$$

$$\hat{u}_3 = u_3 - \left(\frac{\langle u_3, \hat{u}_1 \rangle}{\langle \hat{u}_1, \hat{u}_1 \rangle} \hat{u}_1 + \frac{\langle u_3, \hat{u}_2 \rangle}{\langle \hat{u}_2, \hat{u}_2 \rangle} \hat{u}_2 \right)$$

$$\vdots$$

$$\hat{u}_n = \sim$$

$$\text{proj}_{\hat{u}_1} u_2 = \alpha \hat{u}_1$$

$$\langle u_2 - \alpha \hat{u}_1, \hat{u}_1 \rangle = 0.$$

$$\Rightarrow \alpha = \frac{\langle u_2, \hat{u}_1 \rangle}{\langle \hat{u}_1, \hat{u}_1 \rangle}$$

G-S process.

$\{\hat{u}_1, \dots, \hat{u}_n\}$ orthogonal basis.

HW: example 294, ~~296~~, 297.