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Digital Circuits & Computer Architecture

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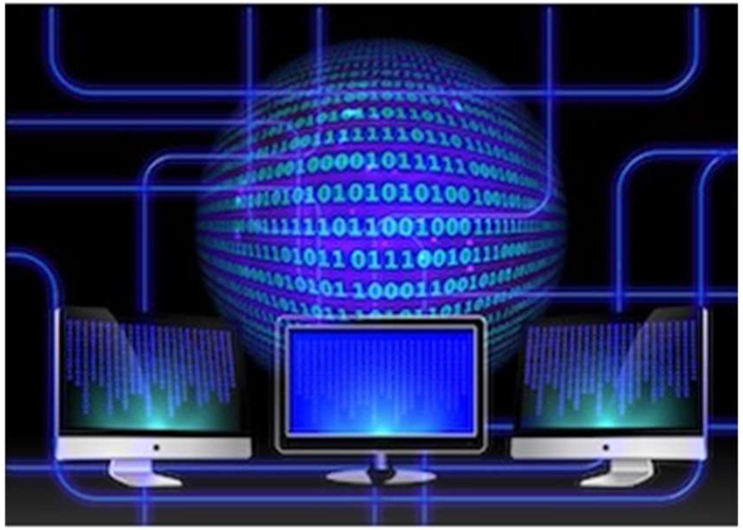
1. Introduction



- Machines interpret numbers differently from humans.
- Whenever we type something on a device, those letters convert into certain numbers which only the computer can understand.
- The number system is an essential part of computer technology enabling computers to perform all functions in just a few seconds,

2. Number systems

- There are several number systems which we normally use in modern computing and digital electronics, such as decimal (base 10), binary (base 2), octal (base 8) and hexadecimal (base 16),
- Amongst them we are most familiar with the decimal number system.
- These systems are classified according to the values of the base (B) of the number system.

2.1.Polynomial représentation

- In general, we can express any number in any base or radix “B.” Any number with base B, having n digits to the left and m digits to the right of the decimal point, can be expressed as:

$$a_n B^{n-1} + a_{n-1} B^{n-2} + a_{n-2} B^{n-3} + \cdots + a_2 B^1 + a_1 B^0 + b_1 B^{-1} + b_2 B^{-2} + .. + b_m B^{-m}$$

where a_n is the digit in the nth position. The coefficient $\boldsymbol{a_n}$ is defined as the **MSD (**Most Significant Digit) and $\boldsymbol{b_m}$ is defined as the **LSD** (Least Significant Digit).

2. Number systems

2.1. Decimal Number System (base 10)

- The most common system for number representation is the **decimal.** It's used in finances, engineering and biology, almost everywhere we see and use numbers.
- With a decimal system we have **10 different digits :**

DECIMAL SYMBOLS									
0	1	2	3	4	5	6	7	8	9

- As you can see there are 10 symbols from 0 to 9. With these symbols we can construct all the numbers in the decimal system, such as $(2023)_{10}$ $(208)_{10}$

2. Number systems

Examples:

$$(5462)_{10} = 5X10^3 + 4X10^2 + 6X10^1 + 2X10^0$$

$$(239.537)_{10} = 2X10^2 + 3X10^1 + 9X10^0 + 5X10^{-1} + 3X10^{-2} + 7X10^{-3}$$

We can keep in mind these characteristics of the decimal numbers system:

- it's using <u>10 symbols</u>
- can be decomposed in <u>factors containing powers of 10</u>
- it's the most common number representation system

2. Number systems

2.2. Binary Number System (base 2)

- A binary system has only 2 different digits: 0 and 1. So to deal with a binary number system is quite easier than a decimal system.

BINARY SYMBOLS	
0	1

- In a digital world, we can think in binary nature, e.g., a light can be either off or on. There is no state in between these two.
- So we generally use the binary system when we deal with the digital world : all the operations are done using two levels of voltage: high and low. Each level of voltage is assigned to a value/symbol: HIGH for 1 and LOW for 0. For a microcontroller which is supplied with +5V the 1 (high) will be represented by +5 V and the 0 (low) by 0 V.

2. Number systems

- All the decimal numbers we can think of can be represented into binary symbols i.e., 0 and 1.

Decimal (base 10)	Binary (base 2)
$(2510)_{10}$	$(11001)_2$

- Usually to distinguish between a decimal or binary number, we must specify the base to which we are referring to. The base is described as a subscript after the last character of the number
- By specifying the base of the number we eliminate the probability of confusion, because the same representation (e.g. 11) can mean different things for different bases.

$$(11)_{10} \neq (11)_2$$

- To specify a binary number we use the prefix 0b.
  Example: 0b1100

2. Number systems

01110011

binary digit

8 binary digits = 1 Byte

Example:
1000111100001111
16 binary digits = 2 Bytes

The characteristics of a binary system are:

- it’s using <u>2 symbols</u>
- can be decomposed in <u>factors containing powers of 2</u>
- it’s used in computers, microcontrollers

2. Number systems

2.3. Octal Number System (base 8)

- In an octal number system there are 8 digits:

OCTAL SYMBOLS							
0	1	2	3	4	5	6	7

- $(107)_8$ is an octal number and the subscript after the last character of the number clearly indicate the base 8

- Any octal number cannot have any digit greater than 7.
  Example :
  109 is it an octal number?
  109 isn't an octal number

2. Number systems

2.4. Hexadecimal Number System (base 16)

- Hexadecimal number system has 16 digits 0 to 9 and the rest of the six digits are specified by letter symbols as A,B, C, D, E, and F :

Hexadecimal symbols															
0	1	2	3	4	5	6	7	8	9	A (10)	B (11)	C (12)	D (13)	E (14)	F (15)

- Common practice is to use the prefix “0x” in order to distinguish from the decimal notation. Example: 0x105E9.

- The characteristics of a hexadecimal number representation system are:
  - It’s using 16 symbols
  - Can be decomposed in factors containing powers of 16
  - It’s used in computers, microcontrollers

2. Number systems

The table below summaries the characteristics of the above mentioned number representation systems:

System	Nomber of symbols	Symbols	Prefix	Example
Decimal	10	0,1,2,3,4,5,6,7,8,9	None	147
Binary	2	0,1	0b	11011001
Octal	8	0,1,2,3,4,5,6,7	None	41
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F	0X	7DE1

Decimal numbers	4 bits binary number	Hexadecimal number	3 bits binary number	Octal number
0	0000	0	000	0
1	0001	1	001	1
2	0010	2	010	2
3	0011	3	011	3
4	0100	4	100	4
5	0101	5	101	5
6	0110	6	110	6
7	0111	7	111	7
8	1000	8		10
9	1001	9		11
10	1010	A		12
11	1011	B		13
12	1100	C		14
13	1101	D		15
14	1110	E		16
15	1111	F		17

3. Conversion between number systems

- It is often required to convert a number in a particular number system to any other number system, e.g., it may be required to convert a decimal number to binary or octal or hexadecimal.
- The reverse is also true, i.e., a binary number may be converted into decimal and so on.

3.1. Binary, octal and hexadecimal to decimal conversion

- Converting a number N from any number system B to a decimal number is given by polynomial representation.
- Each of the binary, octal, or hexadecimal number system is a positional number system, i.e., each of the digits in the number systems has a positional weight as in the case of the decimal system.

3. Conversion between number systems

Examples:

Binary ⟶ Decimal

$(101)_2 = (\qquad)_{10}$

$(11010)_2 = (\qquad)_{10}$

$(1111)_2 = (\qquad)_{10}$

Octal ⟶ Decimal

$(12)_8 = (\qquad)_{10}$

$(107)_8 = (\qquad)_{10}$

$(200)_8 = (\qquad)_{10}$

Hexadecimal ⟶ Decimal

$(15)_{16} = (\qquad)_{10}$

$(FF)_{16} = (\qquad)_{10}$

$(2C0)_{16} = (\qquad)_{10}$

3. Conversion between number systems

3.2. Decimal to binary conversion

- To convert a **number in decimal to a number in binary** we have to **divide the decimal number** by **2** repeatedly, **until the quotient of zero is obtained**.
- This method of repeated division by 2 is called the ‘double-dabble’ method.
- The remainders are noted down for each of the division steps. Then the column of the remainder is read in reverse order i.e., from bottom to top order.
- We try to show the method with examples:

Decimal $\longrightarrow$ Binary

$(43)_{10} = (\qquad\qquad)_2$

$(9)_{10} = (\qquad\qquad)_2$

$(24)_{10} = (\qquad\qquad)_2$

$(33)_{10} = (\qquad\qquad)_2$

$(256)_{10} = (\qquad\qquad)_2$

3. Conversion between number systems

Conversion from a Binary to Octal Number and Vice Versa

- We know that the maximum digit in an octal number system is 7, which can be represented as $(111)_2$ in a binary system. Hence, **starting from the LSB**, we **group three digits** at a time and replace them by the decimal equivalent of those groups and we get the final octal number.
- Since at the time of grouping of three digits starting from the LSB, we find that a group cannot be completed, so we complete the group by **adding 0s to the MSB side**.

Binary ⟷ Octal

$(100101)_2 = (\qquad)_8$

$(10010110)_2 = (\qquad)_8$

3. Conversion between number systems

Conversion from a Binary to Hexadecimal Number and Vice Versa

- We know that the maximum digit in a hexadecimal system is 15, which can be represented by $(1111)_2$ in a binary system. Hence, **starting from the LSB**, we **group four digits** at a time and replace them with the hexadecimal equivalent of those groups and we get the final hexadecimal number

- Since at the time of grouping of four digits starting from the LSB, we find that a group cannot be completed, so we complete the group by **adding 0s to the MSB side**.

Binary ⟷ Hexadecimal

$(11010011)_2 = (\quad\quad)_{16}$

$(1111011011)_2 = (\quad\quad)_{16}$

3. Conversion between number systems

Conversion from an Octal to Hexadecimal Number and Vice Versa

- **To convert an octal number into a hexadecimal number** the following steps are to be followed:
  (*i*) First convert the octal number to its binary equivalent (as already discussed above).
  (*ii*) Then form groups of 4 bits, starting from the LSB.
  (*iii*) Then write the equivalent hexadecimal number for each group of 4 bits.

- For **converting a hexadecimal number into an octal number** the following steps are to be followed:
  (*i*) First convert the hexadecimal number to its binary equivalent.
  (*ii*) Then form groups of 3 bits, starting from the LSB.
  (*iii*) Then write the equivalent octal number for each group of 3 bits.

3. Conversion between number systems

Examples:

Octal ⟷ Hexadecimal

$(25)_8 = (\quad\quad\quad)_{16}$

$(6401)_8 = (\quad\quad\quad)_{16}$

3. Conversion between number systems

3.3. Binary-octal-hexadecimal and decimal-octal-hexadecimal conversion

Decimal to octal conversion

- To convert a number in decimal to a number in octal we have to **divide the decimal number by 8** repeatedly, until the quotient of zero is obtained.
- This method of repeated division by 8 is called ‘octal-dabble.’
- The remainders are noted down for each of the division steps. Then the column of the remainder is read from bottom to top order, just as in the case of the double-dabble method

Decimal to hexadecimal conversion

- The same steps are repeated to convert a number in decimal to a number in hexadecimal. Only here we have to **divide the decimal number by 16 repeatedly**, until the quotient of zero is obtained.
- This method of repeated division by 16 is called ‘hexdabble.’
- The remainders are noted down for each of the division steps. Then the column of the remainder is read from bottom to top

3. Conversion between number systems

Examples:

Decimal ⟷ Octal ⟷ Hexadecimal

$(68)_{10} = (\qquad)_8 = (\qquad)_{16}$

$(45)_{10} = (\qquad)_8 = (\qquad)_{16}$