Logic of Computer Science
Lecture 5: Proof Techniques and Strategy

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Why Proof Techniques Matter

Real-World Importance of Proofs

In programming, systems design, and AI, proofs are how we **guarantee correctness**.

- **Sorting Algorithm**: Prove it truly sorts any input.
- **Authentication Protocol**: Show it's secure under attack assumptions.
- **Compiler Optimization**: Prove the transformation preserves semantics.

Key Takeaway

Proofs are not just theoretical — they are your guarantee of **trust in systems**.

2. Learning Objectives

By the end of this lecture, you should be able to:

- Distinguish and apply direct proof, proof by contrapositive, and proof by contradiction.
- Plan proofs effectively by breaking complex statements into sub-goals.
- Identify which proof technique is most suitable for a given theorem.

3. Direct Proof vs. Contrapositive

3.1 Direct Proof

Goal: Prove an implication $P \rightarrow Q$

Method

- **Assume** P is true.
- **Use** definitions, algebra, and earlier results to derive Q.

Example

Prove: If n is even, then $n + 1$ is odd.

- Assume $n = 2k$ for some integer k.
- Then $n + 1 = 2k + 1$, which is odd by definition.

3.2 Proof by Contrapositive

Key Idea

An implication $P \to Q$ is logically equivalent to its contrapositive: $\neg Q \to \neg P$.

When to Use

Use this method when assuming $\neg Q$ makes the argument easier or more natural.

Method

- Assume $\neg Q$
- Show that $\neg P$ logically follows

3.3 Proof by Contradiction

Goal

Prove P by assuming $\neg P$ and reaching a contradiction.

Example: $\sqrt{2}$ is irrational

- Assume $\sqrt{2} = a/b$ in lowest terms
- $2 = a^2/b^2 \Rightarrow a^2 = 2b^2 \Rightarrow$a$even Let a$ $= 2$k $\Rightarrow b$ also even
- Contradicts lowest terms assumption

4. Proof Planning Strategy

Steps to Success

- Understand what to prove
- Choose suitable proof technique
- Break down complex statements
- Work backwards if needed
- Outline steps before full proof

5. Example Walkthrough

Theorem

If n^2 is even, then n is even.

Proof (Contrapositive)

- Assume n is odd: $n = 2k + 1$
- Then $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$
- $\Rightarrow n^2$ is odd
- $\therefore$ If n^2 is even, n is even

6. In-Class Strategy Workshop

Activity

Work on one of the following:

- The sum of two odd integers is even.
- Every factor of a prime is either 1 or itself.
- If $a \mid bc$ and $\gcd(a, b) = 1$, then $a \mid c$.

Tasks

- Choose a proof technique
- Outline steps (5–7 bullets)
- Present to the class

7. Summary Key Takeaways

Summary

- Direct: forward logic
- Contrapositive: flip and simplify
- Contradiction: assume negation and find inconsistency
- Plan before proof

References I

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Thank You!