Logic of Computer Science
Lecture 1: Foundations & Set Theory Review

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Learning Objectives

By the end of this session, you will be able to:

1. **Recall basic set operations.**
   - Form unions, intersections, set–differences, complements.
   - Interpret notation $A \subseteq B$, $x \in A$, $A \cup B$, etc.
2. **Reinforce proof techniques on set identities.**
   - Practise direct proofs and contrapositives.
   - Canonical identity example: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

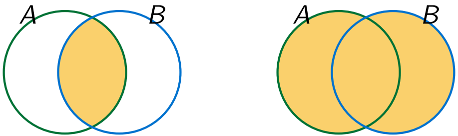
Key Definitions

1. Set, Element, Subset

- **Set** A: collection of distinct objects.
- **Element** $x \in A$: x belongs to A.
- **Subset** $A \subseteq B$: $\forall x (x \in A \Rightarrow x \in B)$.
- Empty set $\varnothing$.

2. Operations

- Union $A \cup B = \{x : x \in A \vee x \in B\}$.
- Intersection $A \cap B = \{x : x \in A \wedge x \in B\}$.
- Difference $A - B = \{x : x \in A \wedge x \notin B\}$.
- Complement $\overline{A} = U - A$.



Intersection vs. Union visual

Relations & Functions

Relation $R \subseteq A \times B$

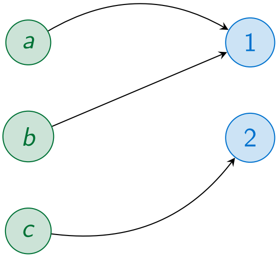
$(a, b) \in R \;\Leftrightarrow\; a\,R\,b$

Domain $(R) = \{a \in A \mid \exists b\,(a, b) \in R\}$

Function $f : A \to B$

Each $a \in A$ has exactly one $b \in B$ with $f(a) = b$.

- **Injective**: distinct inputs $\Rightarrow$ distinct outputs.
- **Surjective**: every $y \in B$ hit by some x.
- **Bijective**: both injective and surjective.



Example:

surjective but not injective.

Proof Techniques Refresher

Set Equality $X = Y$

Show $X \subseteq Y$ *and* $Y \subseteq X$.

Direct Proof

Assume P; derive Q.

Contrapositive

Prove $\neg Q \Rightarrow \neg P$ instead of $P \Rightarrow Q$.

Tip: identify P and Q clearly before choosing a method.

Example Proof — Distributive Law

Proposition

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\subseteq$ Let $x \in A \cap (B \cup C)$. Then $x \in A$ and $(x \in B \vee x \in C)$.

- If $x \in B \Rightarrow x \in A \cap B$.
- If $x \in C \Rightarrow x \in A \cap C$.

Hence $x \in (A \cap B) \cup (A \cap C)$.

$\supseteq$ Let $y \in (A \cap B) \cup (A \cap C)$.

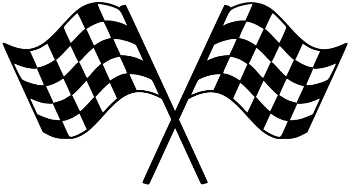
- If $y \in A \cap B \Rightarrow y \in A$ and $y \in B \Rightarrow y \in B \cup C$.
- Similar for $A \cap C$.

Thus $y \in A \cap (B \cup C)$.

$\blacksquare$

In-Class Activity — Set-Proof Race

1. Split into **small groups**
2. Each group receives one set identity to prove, e.g.
   - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   - $\overline{A \cup B} = \overline{A} \cap \overline{B}$
   - $A \triangle B = (A \cup B) - (A \cap B)$
3. 12 minutes to craft proofs & choose a presenter.
4. Rapid-fire presentations (2 minutes each).



Summary & Takeaways

Key Points

- Set theory is foundational for logic and programming semantics.
- Precise **definitions and notation** underpin logical reasoning.
- Proof techniques build rigor and form the basis for formal logic systems.
- These concepts set the stage for propositional and predicate logic.

Up Next — Week 2

Truth tables, logical connectives, more complex reasoning.

Home Practice: Prove $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

References I

- M. Huth and M. Ryan, *Logic in Computer Science: Modelling and Reasoning about Systems*, Cambridge University Press, 2004.
- D. Gries and F. B. Schneider, *A Logical Approach to Discrete Math*, Springer-Verlag, 1993.
- K. H. Rosen, *Discrete Mathematics and Its Applications*, 7th ed., McGraw-Hill, 2011.
- P. Halmos, *Naive Set Theory*, Van Nostrand, 1960.

Thank You!