Logic of Computer Science Lecture 2: Propositional Logic – Syntax & Semantics

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Knowledge | Truth | Excellence

Learning Objectives

By the end of this session, students should be able to:

- Build and evaluate truth tables.
 - List all possible truth-value assignments.
 - Evaluate complex formulas row-by-row.
- Identify tautologies, contradictions, and contingencies.
 - Tautology: always true.
 - Contradiction: always false.
 - Contingency: sometimes true, sometimes false.

Syntax of Propositional Logic

1. Atomic Propositions

- Variables: p, q, r, p_1, p_2, \dots
- Represent statements with True (T) or False (F)

2. Logical Connectives

- $\neg \phi$: Negation (not)
- $\phi \wedge \psi$: Conjunction (and)
- $\phi \lor \psi$: Disjunction (or)
- $\phi \rightarrow \psi$: Implication (implies)
- $\phi \leftrightarrow \psi$: Biconditional (iff)



Formation Rules (Well-Formed Formulas)

Recursive Rules

- Every propositional variable is a formula.
- **2** If α is a formula, then $\neg \alpha$ is a formula.
- **3** If α and β are formulas, so are: $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \to \beta$, $\alpha \leftrightarrow \beta$

Semantics & Interpretation

Truth-Value Assignments

- $v : \{\text{propositional variables}\} \rightarrow \{T, F\}$
- Extend v recursively to complex formulas

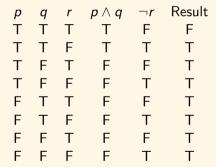
Satisfaction

- $v \models \phi$ means $v(\phi) = T$
- ullet ϕ is:
 - Satisfiable if $v(\phi) = T$ for some v
 - Valid (tautology) if $v(\phi) = T$ for all v
 - Unsatisfiable (contradiction) if $v(\phi) = F$ for all v

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Truth Table Example

Evaluate $(p \wedge q) ightarrow eg r$



Logical Equivalences

Selected Laws

- $\bullet \ \phi \to \psi \equiv \neg \phi \lor \psi$
- $\neg(\phi \land \psi) \equiv \neg\phi \lor \neg\psi$ (De Morgan)
- $\neg(\phi \lor \psi) \equiv \neg\phi \land \neg\psi$
- $\neg(\neg\phi) \equiv \phi$ (Double Negation)



Tautology vs. Contradiction vs. Contingency

Definitions

- Tautology: always true.
- Contradiction: always false.
- Contingency: true under some, false under others.

Examples

- Tautology: $p \lor \neg p$
- Contradiction: $p \land \neg p$
- Contingency: $p \leftrightarrow q$



In-Class Exercise

Build a truth table for:

$$(p
ightarrow q) \leftrightarrow (\neg q
ightarrow \neg p)$$

- Prove this is a tautology.
- Interpret each row and explain the contrapositive law.



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|Wrap-Up & Homework

Summary

- Defined syntax and semantics of propositional logic.
- Evaluated truth tables and formula equivalence.
- Identified tautologies, contradictions, contingencies.

Homework (Due Week 3)

- Build truth tables for two complex formulas.
- Determine logical status (tautology/contradiction/contingency).
- Convert a formula to CNF/DNF.



References I



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- P. Halmos, Naive Set Theory, Van Nostrand, 1960.

Thank You!