CE 474 - LOGIC OF COMPUTER SCIENCE

Lecture Note 2

Learning Objectives

By the end of this lecture, students should be able to:

- Build and evaluate truth tables.
- Identify tautologies, contradictions, and contingencies.

1. Syntax of Propositional Logic

Propositional logic is a branch of formal logic that deals with propositions and their relationships through logical connectives. A **proposition** is a statement that is either **true (T)** or **false (F)**.

Atomic Propositions and Connectives

Propositions are typically represented by letters like p, q, and r. Connectives include:

- $\neg$ (NOT)
- $\wedge$ (AND)
- $\vee$ (OR)
- $\rightarrow$ (IMPLIES)
- $\leftrightarrow$ (IF AND ONLY IF)

A **well-formed formula (WFF)** is any syntactically correct combination of these symbols.

2. Semantics and Interpretation

Semantics assigns truth-values to formulas based on a **truth-value assignment** (or **interpretation**). Each atomic proposition is mapped to either **T** or **F**.

Example:

Given an interpretation where p is T and q is F:

- $p \wedge q$ is **F**
- $p \vee q$ is **T**
- $p \rightarrow q$ is **F**

3. Truth Table Construction

A **truth table** lists all possible truth-value combinations for the atomic variables and determines the truth-value of the formula under each case.

Example:

Evaluate $(p \wedge q) \to \neg r$

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \to \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	T
F	T	F	F	T	T
F	F	T	F	F	T
F	F	F	F	T	T

4. Logical Equivalences and Laws

Logical equivalences allow us to transform and simplify formulas. Key identities include: 2

- **De Morgan's Laws:**
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- **Distributive Laws:**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- **Associative Laws:**
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Identity Laws:**
  - $p \wedge T \equiv p$
  - $p \vee F \equiv p$
- **Domination Laws:**
  - $p \wedge F \equiv F$
  - $p \vee T \equiv T$
- **Double Negation:**
  - $\neg(\neg p) \equiv p$

5. Tautologies, Contradictions, and Contingencies

- A **tautology** is always true under all possible interpretations, e.g., $p \lor \neg p$.
- A **contradiction** is always false, e.g., $p \land \neg p$.
- A **contingency** is true under some interpretations and false under others, e.g., $p \to q$.

6. In-Class Exercise

Construct a truth table for the following formula:

$$(p \to q) \leftrightarrow (\neg q \to \neg p)$$

- Determine whether the formula is a tautology, contradiction, or contingency.
- Discuss and verify the equivalence (contrapositive) in small groups.
- Present your findings.

Summary

This week we examined the foundations of propositional logic, including:

- Building formulas using logical syntax.
- Evaluating their meanings through truth-value assignments.
- Using truth tables for formal analysis.
- Understanding logical identities and formula classification.

Students should now be able to:

- Analyze formulas using truth tables.
- Recognize tautologies, contradictions, and contingencies.
- Apply equivalences to simplify formulas.

References

- Huth, M., & Ryan, M. (2004). *Logic in Computer Science: Modelling and Reasoning about Systems*. Cambridge University Press.
- Ben-Ari, M. (2012). *Mathematical Logic for Computer Science*. Springer.
- Sipser, M. (2012). *Introduction to the Theory of Computation*. Cengage Learning.