CE 474 - LOGIC OF COMPUTER SCIENCE

Problem Set 1

Instructions

- Submit a typed or clearly handwritten and scanned PDF on the LMS before class begins.

- You may work in groups, but your submission must be written in your own words.

- Justify your reasoning fully in each problem.

1. Union–Intersection Distributivity

Prove:
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Hint: Show both $\subseteq$ and $\supseteq$ separately by selecting an arbitrary element x and considering cases based on membership.

2. Transitivity of Inclusion

Let A, B, C be sets such that $A \subseteq B$ and $B \subseteq C$. Prove that $A \subseteq C$.
   Hint: Begin by assuming $x \in A$, and use the inclusions to conclude $x \in C$.

3. Relational Properties

Let $R \subseteq S \times S$ be a relation on a set S.
   Given:

- **Symmetric:** If $(a, b) \in R$, then $(b, a) \in R$.

- **Transitive:** If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.

Prove: For all $a, b \in S$, if $(a, b) \in R$, then both $(a, a) \in R$ and $(b, b) \in R$.

4. Asymmetry of Set Difference

Give an explicit example of two sets A and B such that:

$$A - B \neq B - A$$

Also: Briefly explain in 1–2 sentences why they differ.
   Recall: $A - B = \{x \in A \mid x \notin B\}$