Logic of Computer Science

Lecture 2: Propositional Logic – Syntax & Semantics

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Knowledge | Truth | Excellence

Learning Objectives

By the end of this session, students should be able to:

- Build and evaluate truth tables.
  - List all possible truth-value assignments.
  - Evaluate complex formulas row-by-row.
- Identify tautologies, contradictions, and contingencies.
  - Tautology: always true.
  - Contradiction: always false.
  - Contingency: sometimes true, sometimes false.

Syntax of Propositional Logic

1. Atomic Propositions

- Variables: $p, q, r, p_1, p_2, \ldots$
- Represent statements with True (T) or False (F)

2. Logical Connectives

- $\neg\phi$: Negation (not)
- $\phi \wedge \psi$: Conjunction (and)
- $\phi \vee \psi$: Disjunction (or)
- $\phi \rightarrow \psi$: Implication (implies)
- $\phi \leftrightarrow \psi$: Biconditional (iff)

Formation Rules (Well-Formed Formulas)

Recursive Rules

1. Every propositional variable is a formula.
2. If α is a formula, then $\neg\alpha$ is a formula.
3. If α and β are formulas, so are: $\alpha \land \beta$, $\alpha \lor \beta$, $\alpha \to \beta$, $\alpha \leftrightarrow \beta$

Semantics & Interpretation

Truth-Value Assignments

- $v : \{\text{propositional variables}\} \to \{T, F\}$
- Extend v recursively to complex formulas

Satisfaction

- $v \models \phi$ means $v(\phi) = T$
- ϕ is:
  - Satisfiable if $v(\phi) = T$ for some v
  - Valid (tautology) if $v(\phi) = T$ for all v
  - Unsatisfiable (contradiction) if $v(\phi) = F$ for all v

Truth Table Example

Evaluate $(p \wedge q) \rightarrow \neg r$

p	q	r	$p \wedge q$	$\neg r$	Result
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	T
F	T	F	F	T	T
F	F	T	F	F	T
F	F	F	F	T	T

Logical Equivalences

Selected Laws

- $\phi \to \psi \equiv \neg\phi \lor \psi$
- $\neg(\phi \land \psi) \equiv \neg\phi \lor \neg\psi$ (De Morgan)
- $\neg(\phi \lor \psi) \equiv \neg\phi \land \neg\psi$
- $\neg(\neg\phi) \equiv \phi$ (Double Negation)

Tautology vs. Contradiction vs. Contingency

Definitions

- Tautology: always true.
- Contradiction: always false.
- Contingency: true under some, false under others.

Examples

- Tautology: $p \lor \neg p$
- Contradiction: $p \land \neg p$
- Contingency: $p \leftrightarrow q$

In-Class Exercise

Build a truth table for:

$$(p \to q) \leftrightarrow (\neg q \to \neg p)$$

- Prove this is a tautology.
- Interpret each row and explain the contrapositive law.

Wrap-Up & Homework

Summary

- Defined syntax and semantics of propositional logic.
- Evaluated truth tables and formula equivalence.
- Identified tautologies, contradictions, contingencies.

Homework (Due Week 3)

- Build truth tables for two complex formulas.
- Determine logical status (tautology/contradiction/contingency).
- Convert a formula to CNF/DNF.

References I

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Thank You!