

CE 474-LOGIC OF COMPUTER SCIENCE

Problem Set 3

Instructions

- Read each question carefully and attempt all parts.
- Show all your work clearly. Partial credit may be awarded for correct reasoning.
- Submission Format: Typed (preferred) or neatly handwritten and scanned as PDF.
- This assignment is to be completed individually.

1. Translating English to Predicate Logic (6 marks)

Translate each of the following into a well-formed predicate-logic formula. Be sure to:

- State your domain and predicate definitions clearly.
 - Use \forall for “for all” and \exists for “there exists.”
- a) “Every student in this class has submitted at least one assignment.”
 - b) “There is a student who has never missed a quiz.”
 - c) “All algorithms that terminate do so in finite time.”

2. Negating Quantified Statements (4 marks)

For each of the formulas below, write its negation in *prenex form* (i.e., with all quantifiers in front), then simplify so no negation appears immediately before a predicate:

- a) $\neg(\forall x P(x) \rightarrow Q(x))$
- b) $\neg(\exists y R(y) \wedge S(y))$

Hint: Use the equivalences:

- $\neg(\forall x \phi) \equiv \exists x \neg\phi$
- $\neg(\exists x \phi) \equiv \forall x \neg\phi$
- $\neg(\phi \rightarrow \psi) \equiv \phi \wedge \neg\psi$

3. Counterexamples & Validity (6 marks)

Determine whether each of the following universally quantified statements is true or false. If false, provide a counterexample; if true, give a brief justification.

- a) $\forall n \in N, (n > 1 \rightarrow n \text{ is prime})$
- b) $\forall x \in R, (x^2 \geq x)$
- c) $\forall s \in \text{Students}, (\text{Passed}(s, \text{Logic}) \rightarrow \text{Assigned}(s, \text{Project}))$

4. Domains & Scope (4 marks)

Consider the formula $\exists x \forall y \text{ Loves}(x, y)$

- a) Describe in English what this formula asserts about the domain of discourse. (1 mark)
- b) Swap the quantifiers to write $\forall y \exists x \text{ Loves}(x, y)$, and explain in English how the meaning changes. (2 marks)
- c) If our domain is “all people,” which of the two statements above is more plausible in real life? Why? (1 mark)

Tips for Success

- Define your domain (e.g., Students, N , R) before writing formulas.
- Keep quantifier scope clear: Parentheses help, e.g. $\forall x(P(x) \rightarrow Q(x))$.
- Push negations all the way to predicates in Question 2.
- Counterexamples must pick a specific element in the domain and show the statement fails.

Good luck! Let me know if you need hints or worked examples.