Remote State Preparation: Arbitrary remote control of photon polarizations for quantum communication

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ABSTRACT

By using a partial polarizer to apply a generalized polarization measurement to one photon of a polarization-entangled pair, we remotely prepare single photons in arbitrary polarization qubits. Specifically, we are able to produce a range of states of any desired degree of mixedness or purity, over (and within) the entire Poincaré sphere, with a typical fidelity exceeding 99.5%. Moreover, by using non-degenerate entangled pairs as a resource, we can prepare states in multiple wavelengths. Finally, we discuss the states remotely preparable given a particular two-qubit resource state.

Keywords: SPDC, entanglement, remote state preparation, quantum communication

1. INTRODUCTION

The transmission of quantum information may be achieved by one of two approaches. One can either send the physical quantum system directly or one can use shared entanglement and a limited amount of classical communication to transmit quantum information as in teleportation[1] and remote state preparation (RSP).[2–4] The latter approach is remarkable, as quantum systems are generally described by n real variables, which contain many more than n classical bits of information. For example, the two-level system (or qubit) is characterized by three real parameters, where each parameter may in principal encode an infinite amount of information[*]. Thus, using only classical communication, the *precise* transmission of a qubit would require *many* classical bits. This is not the case when an entangled resource is shared between a sender (Alice) and a receiver (Bob), as they may use either teleportation or remote state preparation and only a modicum of classical communication.

In quantum teleportation, Alice sends an *unknown* qubit's state to Bob using a shared entangled qubit pair and two bits of classical communication. Alice does a Bell-state measurement on the unknown qubit and her half of the entangled pair. Then, based on her measurement result, she tells Bob to perform one of four operations on his qubit to obtain the unknown state. The goal of remote state preparation is similar, except that Alice *knows* the state she wants to send to Bob, so she need only perform a local measurement, not a Bell-state measurement, currently very difficult with photons.

RSP has been demonstrated with control of some of the parameters of a qubit in liquid-state NMR,[5] vacuum and single-photon superposition states,[6] and photon degree of polarization.[7,8] Until recently,[9,10] however, no RSP implementation has achieved control over the three parameters required to prepare *arbitrary* single-qubit states.

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[*]It is important to note that even though a qubit may in principle only be specified by an infinite amount of classical information, *measurement* of a single quantum bit yields only one classical bit.

1.1. Qubit representation

Our qubits are encoded in photon polarization states, where the horizontal and vertical polarization states form a basis:

$$|0\rangle \equiv |H\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |1\rangle \equiv |V\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{1}$$

or in density matrix notation,

$$\rho_H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } \rho_V = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2}$$

The density matrix of an arbitrary single qubit can be represented by three independent real parameters (θ, ϕ, and λ):

$$\rho_{rp}((\theta,\phi,\lambda)) = (1-\lambda)|\psi(\theta,\phi)\rangle\langle\psi(\theta,\phi)| + \frac{\lambda}{2}\mathbf{1}, \tag{3}$$

where $|\psi(\theta,\phi)\rangle \equiv \cos\theta|D\rangle + \sin\theta e^{i\phi}|A\rangle$, and $|D\rangle \equiv (|H\rangle + |V\rangle)/\sqrt{2}$ and $|A\rangle \equiv (|H\rangle - |V\rangle)/\sqrt{2}$ label diagonally and anti-diagonally polarized states, respectively. (Below we will explain why we employ this particular representation.)

1.2. Fidelity

To quantify how well states agree, we calculate the fidelity between them. In the simple case of two pure states $|\eta_m\rangle$, and $|\eta_e\rangle$, the fidelity is $|\langle\eta_e|\eta_m\rangle|^2$, i.e., simply the state overlap. For the more general states, a measured state ρ_m and an expected state ρ_e, where either are allowed to be mixed, the fidelity is given by[11]

$$F(\rho_e, \rho_m) \equiv \left| \text{Tr} \left(\sqrt{\sqrt{\rho_e}\rho_m\sqrt{\rho_e}} \right) \right|^2. \tag{4}$$

The fidelity is zero for orthogonal states and one for identical states.

2. RSP

In this section, we describe RSP using photon polarization qubits, although the methods are applicable to any physical qubit implementation. Consider an initial maximally entangled state resource: $|\phi^+\rangle \equiv (|H_t H_{rp}\rangle + |V_t V_{rp}\rangle)/\sqrt{2} \equiv (|D_t D_{rp}\rangle + |A_t A_{rp}\rangle)/\sqrt{2}$. Here the subscripts label the trigger and remotely prepared photons, and $|H\rangle$ and $|V\rangle$ label horizontal and vertical polarization states, respectively. Here is a simple illustration of RSP: if Alice measures the trigger photon in the state $|D_t\rangle$ (i.e., she detects the photon after it passes through a diagonal polarizer) then, using a classical communication channel†, she tells Bob to accept his photon, thereby conditionally remotely preparing the state $|D_{rp}\rangle$. Remote preparation of a general *pure* state proceeds in a similar way, except the trigger photon is detected after projection into an arbitrary pure polarization state using a quarter-wave plate (QWP), a half-wave plate (HWP) and a polarizer. Thus, to remotely prepare $|\psi_{rp}(\theta,\phi)\rangle$, Alice projects the trigger photon so that the two-photon state is $|\phi^+\rangle \rightarrow (|\zeta_t(\theta,\phi)D_{rp}\rangle + |\zeta_t^\perp(\theta,\phi)A_{rp}\rangle)/\sqrt{2} \equiv (|D_t\psi_{rp}(\theta,\phi)\rangle + |A_t\psi_{rp}^\perp(\theta,\phi)\rangle)/\sqrt{2}$, where $|\zeta_t(\theta,\phi)\rangle \equiv \cos\theta|D\rangle - e^{-i\phi}\sin\theta|A\rangle$, and $\langle\zeta^\perp|\zeta\rangle = 0$. Projection of the trigger into $\langle D|$ ($\langle A|$), remotely prepares the state $|\psi(\theta,\phi)\rangle$ ($|\psi^\perp(\theta,\phi)\rangle$), the state orthogonal to $|\psi(\theta,\phi)\rangle$). Note that this process is only 50% efficient. It is possible to improve the efficiency to 100% if Alice constrains herself to sending states from a single great circle on the Poincaré sphere. In this case, Bob transforms his photon $|\psi_b^\perp\rangle \rightarrow |\psi_b\rangle$ whenever Alice measures her target photon in $|A\rangle$ instead of $|D\rangle$. However, because a universal single-qubit NOT operation is forbidden,[12] the efficiency is limited to 50% in the general case.

The previous procedure suffices to set the relative amplitude and phase (θ and ϕ, respectively, from Eq. 3) between the two levels of the qubit, allowing one to remotely prepare any *pure* state. However, one

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†The classical communication requirement prevents any possibility of superluminal communication.

must additionally control the *strength* of the projection to set the remaining parameter (λ, which determines the mixedness), and ultimately prepare an arbitrary qubit. As an extreme example, consider the case where the polarizer strength is zero, that is, the polarizer is removed from the trigger measurement. In this case, the polarization state of the trigger is traced over and a completely mixed (unpolarized) state is remotely prepared:

$$\rho_{rp} = \langle D_t|\phi^+\rangle\langle\phi^+|D_t\rangle + \langle A_t|\phi^+\rangle\langle\phi^+|A_t\rangle = \tfrac{1}{2}(|D_{rp}\rangle\langle D_{rp}| + |A_{rp}\rangle\langle A_{rp}|) = \tfrac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{5}$$

By using a partial polarizer in her measurement, Alice can control the *strength* of her polarization projection (between the polarizer and no-polarizer cases) and remotely prepare states of any mixedness. With wave plates preceeding this partial polarizer, she can remotely prepare the arbitrary states of Eq. (3).

2.1. Partial polarizer

The partial polarizer[7,9] consists of two birefringent beam displacers (BD shown in Fig. 1), oriented so that after the first element, $|D\rangle$-polarized light is undeviated, while $|A\rangle$-polarized light undergoes a 4-mm displacement[‡]. The second element recombines these polarizations into the original mode (defined by the two-crystal source and an iris before the detector). This configuration will trace over the polarization of Alice's photon, i.e., no polarization components are filtered out. To enable Alice to change the strength of the partial polarizer, a multi-pixel liquid crystal (LC)[§] with an optic axis in the H-V basis is placed between the BDs. The liquid crystal can independently change the polarizations in the $|D\rangle$ and $|A\rangle$ modes, thereby controlling the diagonal (T_D) and antidiagonal (T_A) transmissions in the partial polarizer's output mode. The achievable transmissions of the partial polarizer are less than unity.

For our purposes, the overall transmission of the partial polarizer is much less important than the relative transmission between the orthogonal polarization components, so the components are normalized with respect to each other using $N \equiv 1/(T_D + T_A)$. If the transmitted components have equal amplitude, then the partial polarizer behaves as if there is no polarization analysis. In contrast, if one component has zero transmission, then any transmission of the orthogonal component will cause the partial polarizer to behave as a perfect polarizer for the transmitted component. Using only the partial polarizer remotely prepares the states[7,9]

$$\begin{aligned}
\rho_{rp}(T_D, T_A) &= N(T_D\langle D_t|\phi^+\rangle\langle\phi^+|D_t\rangle + T_A\langle A_t|\phi^+\rangle\langle\phi^+|A_t\rangle) \\
&= \tfrac{N}{2}(T_D|D_{rp}\rangle\langle D_{rp}| + T_A|A_{rp}\rangle\langle A_{rp}|) \\
&= \tfrac{1}{2}\begin{pmatrix} 1 & N(T_D - T_A) \\ N(T_D - T_A) & 1 \end{pmatrix},
\end{aligned} \tag{6}$$

where the result is written in the $|H\rangle$, $|V\rangle$ basis.

2.2. Implementation

The experiment divides into three steps: entangled state creation, remote state preparation, and tomography of the remotely prepared state. Photons pairs are created by spontaneous parametric downconversion (SPDC, see FIG. 1). Two type-I phasematched β-Barium Borate (BBO) nonlinear crystals are pumped with a cw Ar-ion 351-nm pump, generating photon pairs at 702-nm, emitted at a 3° angle from the pump. The

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[‡]If instead they were oriented to displace horizontal and vertical components or some other set of orthogonal linear polarization states, the liquid crystal optic axis would need to be mounted so that it is at 45° with respect to the polarizations of the displaced states. In our experiment they were mounted diagonally for technical reasons. The partial polarizer will act in whatever basis the BDs are set.

[§]As the liquid crystal is a custom element, one could instead use half-wave plates. One must take care that such wave plates have very little wedge so that they do not deflect the beams to different locations with each setting. We did not use HWPs in our experiment because the 4-mm separation of the beams affords little room for wave plate mounting hardware.
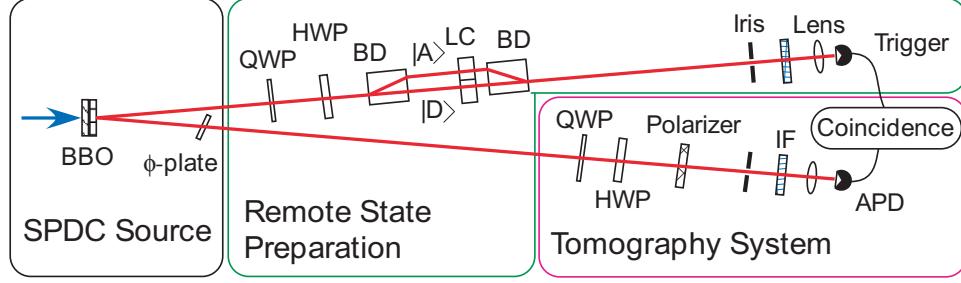
Figure 1. Setup for remote state preparation. The entangled two-qubit state $(|DD\rangle + |AA\rangle)/\sqrt{2}$ is created using a two crystal technique[13] where the relative phase between $|DD\rangle$ and $|AA\rangle$ is set by tipping a quarter wave plate about its vertically oriented optic axis (ϕ-plate). A quarter-wave plate (QWP), half-wave plate (HWP) and partial polarizer (see text) partially project the trigger photon into an arbitrary polarization state. When the trigger photon is detected, the sister photon is conditionally prepared in the expected state.

two crystals' optic axes are oriented in perpendicular planes to give the maximally entangled superposition $(|HH\rangle + |VV\rangle)/\sqrt{2}$.[13] The polarization state of the entangled resource is completely characterized by measuring 36 polarization correlations (by making two-photon polarization projections $\langle VV|$, $\langle DV|$, $\langle AA|$,..., etc.) from which a maximum likelihood technique is used to find the density matrix.[14]

To an excellent approximation, the measurement of one photon prepares its sister in a single-photon Fock state.[15] The trigger photon is projected into an arbitrary polarization state with two wave plates and a partial polarizer, remotely preparing a qubit of the form of Eq. (3). The RSP classical communication requirement is fulfilled by counting the trigger and remotely prepared photons in coincidence (within a 4.5 ns window). For a given remotely prepared state, much like for direct state preparation,[16] the wave plate angles may be calculated assuming the birefringent retardance is 90° for a QWP and 180° for a HWP. If the wave plate phases differ greatly from the ideal (for example, as we soon discuss, using 702-nm wave plates for 737-nm photons), wave plate settings are numerically determined. For the numeric case, the fidelity between the desired remotely prepared state and the expected state (calculated from the measured entangled resource state and the known wave plate retardances) is maximized.

To determine the state of the remotely prepared qubits to a high degree of certainty, a tomography is preformed on a large ensemble of identically prepared states. The ensemble is projected into $\langle H|$, $\langle V|$, $\langle D|$ and $\langle A|$, $\langle L| \equiv \langle H| - i\langle V|$ and $\langle R| \equiv \langle H| + i\langle V|$ for 702-nm photons, where the last two are the left- and right-circular polarization states, respectively.

2.2.1. Degenerate results

Remotely prepared states of 702-nm photons (determined by 2-nm bandwidth interference filters centered at 702 nm, placed before each detector) are shown plotted in the Poincaré sphere in Fig. 2(a). The color
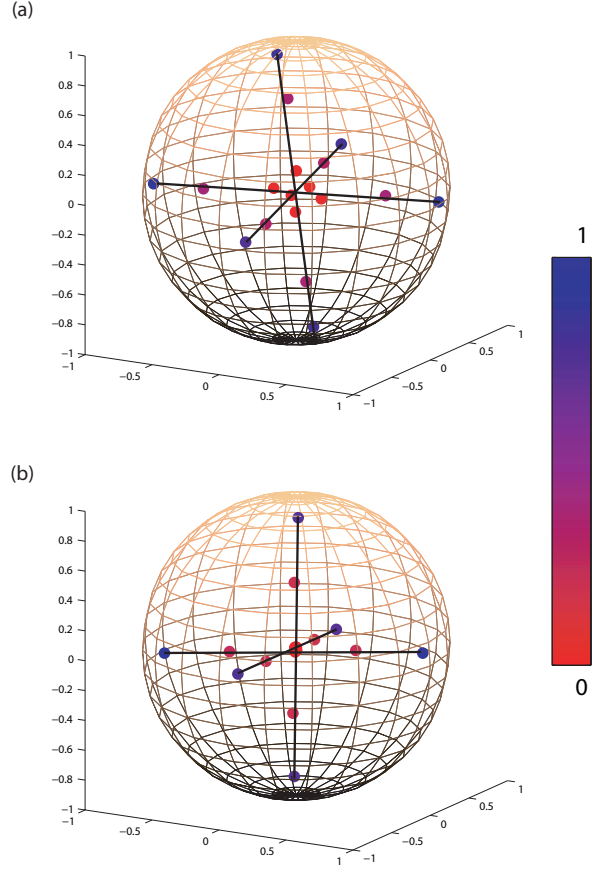
Figure 2. Plots of remotely prepared states over (and within) the entire Poincaré sphere. (a) Remotely prepared states using degenerate entangled resource, i.e., both the trigger and remotely prepared photons have 702-nm wavelengths. (b) Remotely prepared states using non-degenerate entanglement (where the trigger and remotely prepared photon wavelengths are 737 nm and 670 nm, respectively). In both cases, the color of the states plotted transitions from red (0) for mixed (unpolarized) states to blue (1) for pure states. Lines are drawn to guide the eye. Reprinted figure with permission from N. A. Peters *et al.*, *Phys. Rev. Lett.* **94**, 150502 (2005) http://link.aps.org/abstract/PRL/v94/e150502 © 2004 the American Physical Society. Readers may view, browse, and/or download material for temporary copying purposes only, provided these uses are for noncommercial personal purposes. Except as provided by law, this material may not be further reproduced, distributed, transmitted, modified, adapted, performed, displayed, published, or sold in whole or part, without prior written permission from the publisher.

corresponds to state purity, from red (for unpolarized mixed states) to blue (for strongly polarized pure states). We created states in all regions of the Poincaré sphere as indicated by remote preparation in the cardinal directions of the Poincaré sphere at several entropies. Given the parameters of our system, the states we expected matched well with the states we remotely prepared[9]: the average fidelity was 0.996, with all 18 states better than 0.99.

2.2.2. Nondegenerate results

We have thus far only considered the case of a degenerate-wavelength entangled state resource. The ability to remotely prepare states at wavelengths away from the design wavelengths of our utilized optical components may be useful for tailoring remotely prepared photons as inputs for other experiments. One can envision selecting the wavelength to maximize fiber or atmospheric transmission for fiber and free-space cryptography experiments. Wavelength control may also be useful for coupling to other quantum systems, e.g., atoms in a cavity or quantum dots, or optimizing detector efficiency.

To obtain polarization entangled pairs at the conjugate wavelengths of 737 nm and 670 nm, the 702-nm interference filters were replaced with a 670-nm and a 737-nm interference filter and the collection angles were slightly adjusted. The remaining physical resources were used at the different wavelengths, e.g., the wave plates (by optimizing wave plate settings for use away from design wavelengths as previously discussed), crystals, and the partial polarizer. Qubits remotely prepared at 670 nm, conditional on the detection of a 737-nm trigger photon are plotted in Fig. 2(b)[¶]. The average fidelity of states we remotely prepared[9] with the expected states was 0.996, with 17 of the 18 measured states above 0.99. In order to characterize the repeatability of our RSP techniques in Fig. 3, we show 10 repeated measurements of five 670-nm remotely prepared qubits of varying entropies. Here the average fidelity is 0.995, with all 50 states at or above 0.99.

2.3. Theoretical discussions

In this section we discuss the qubit states that can be remotely prepared given shared two-qubit states between Alice and Bob. The most general operation Alice can perform on her qubit can be described by at most four local filters[17]:

$$\rho_A \to \sum_{i=1}^{4} p_i \, \mathcal{M}_i \rho_A \mathcal{M}_i^\dagger, \tag{7}$$

where $\sum_i p_i \, \mathcal{M}_i^\dagger \mathcal{M}_i \leq \mathbf{1}$, and each local filter $\mathcal{M}_i$ can be expressed in the singular-value decomposition

$$\mathcal{M} = V^\dagger \mathcal{D} U. \tag{8}$$

Here $\mathcal{D}$ is a non-negative, no-greater-than-unity diagonal matrix, representing Procrustean filtering,[18–21] and U and V are unitary matrices, not necessarily adjoint to each other.

Under the above (generally) non-trace preserving operation, the joint two-qubit state ρ_{AB} becomes,[9] neglecting normalization,

$$\rho'_{AB} = \sum_{i=1}^{4} p_i \, \mathcal{M}_i \otimes \mathbf{1} \, \rho_{AB} \, \mathcal{M}_i^\dagger \otimes \mathbf{1}, \tag{9}$$

and Bob's qubit becomes $\rho'_B = \mathrm{Tr}_A \rho'_{AB}$. Thus, the most general states Alice can remotely prepare are mixtures of states which she can prepare from a single local filter. Thus we need only analyze the optimal capability of a general local filter applied to RSP, namely, the extremal states of $\rho'_B \equiv \mathrm{Tr}_A(\rho'_{AB})$, where

$$\rho'_{AB} = \mathcal{M} \otimes \mathbf{1} \, \rho_{AB} \, \mathcal{M}^\dagger \otimes \mathbf{1}. \tag{10}$$

[¶]Since we know the precise wave plate retardances at 702 nm and the optic axes' orientations, we use the 702-nm settings for the 670-nm measurements discussed below. Thus we take the 670-nm tomography by projecting into different states than those of the 702-nm tomographies.
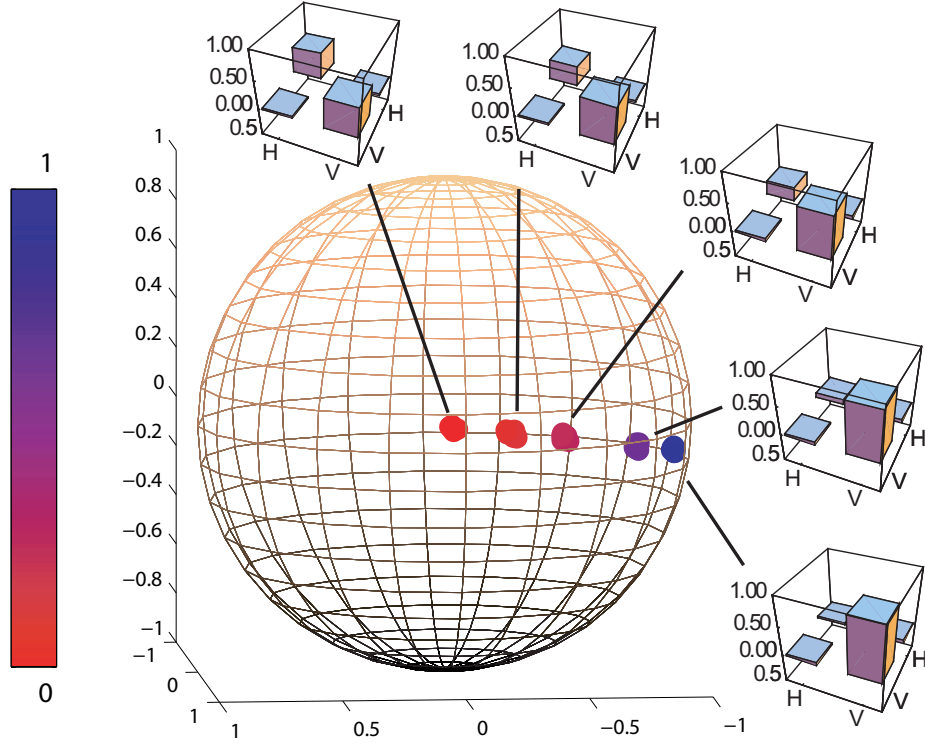
Figure 3. Additional states remotely prepared with nondegenerate entanglement. Shown are five different entropy states (nominally $[(1 - \lambda)|V\rangle\langle V| + \frac{\lambda}{2}\mathbf{1}]$) which were measured 10 times each, showing that counting fluctuations change the plotted states negligibly. The color of the states plotted transitions from red (0) for mixed (unpolarized) states to blue (1) for pure states. Also shown are one of the ten remotely prepared density matrices for each state. The average fidelity of produced states with expected states (given our wave plate, partial polarizer, and entangled resource parameters) is 99.5% for these 50 states, with no fidelities less than 99%.
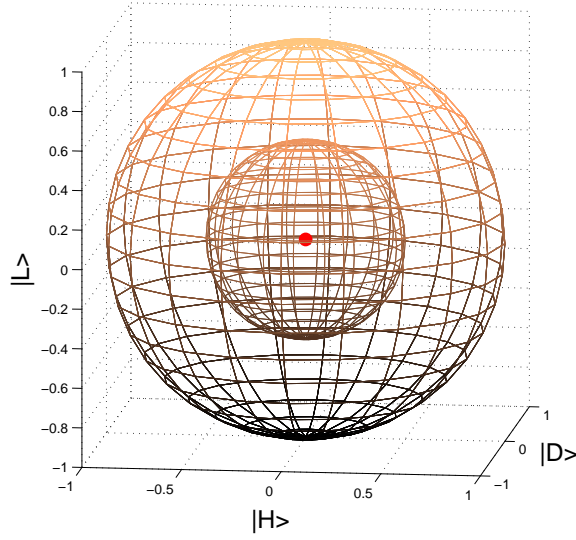
Figure 4. States remotely preparable via a Werner state $\rho_W(r = 1/2)$. The unit sphere is Bob's Poincaré sphere, the origin being Bob's initial state, i.e., his state is completely mixed if Alice performs no polarization filtering. The smaller sphere (and its interior) represents states that Alice can remotely prepare.

The interpretation of this filter is clear: first, apply a local unitary transformation U, followed by a "Procrustean" operation[18–21]

$$\mathcal{D} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, \tag{11}$$

with $0 \leq (a, b) \leq 1$, and lastly apply another unitary transformation $V^\dagger$. This final transformation has no effect on Bob's state, so can be ignored in the analysis of RSP.

With a suitable parameterization of U, e.g., $U = \cos\theta\mathbf{1} + i\hat{n} \cdot \vec{\sigma}\sin\theta$, where $\hat{n}$ is a unit vector and the components of $\vec{\sigma}$ are the Pauli spin matrices, it is straightforward to analyze the states that can be remotely prepared by the filter. To illustrate the results, we consider the case where ρ_{AB} is a Werner state:

$$\rho_W(r) \equiv r|\psi^-\rangle\langle\psi^-| + \frac{1-r}{4}\mathbf{1}. \tag{12}$$

The initial state of Bob $\rho_B = \mathbf{1}/2$ is a completely mixed state. What are the possible states Alice can remotely prepare? Going through the above procedure, one finds that Alice can remotely prepare states that lie on and inside the sphere of radius r centered at the origin[∥]; for an example using the state $\rho_W(r = 1/2)$ see Fig. 4. We note that the Werner state has concurrence $C = \max\{(3r - 1)/2, 0\}$, which indicates that the larger the two-qubit entanglement, the larger capability of the remote state preparation. But this, in general, cannot be used to compare the entanglement quantitatively. For example, even a pure nonmaximally entangled state can be utilized to remotely prepare arbitrary one-qubit states: a Procrustean distillation on one side yields a maximally entangled state, which can then be used to remotely prepare arbitrary one-qubit states. Perhaps, knowing the range of remotely preparable states, and the efficiency of preparation, one may be able to make a quantitative statement of the resource state's two-qubit entanglement. However, this requires further investigation.

[∥] A different approach in the context of quantum steering also yields the same results.[22, 23]

3. CONCLUSIONS

Provided two parties share an entangled resource, they may quantum teleport the state of an unknown qubit using two bits of classical communication, despite that each qubit "encodes" an infinite amount of classical information. If instead one wishes to send the state of a *known* qubit, it may be "remotely prepared" using only a single bit of classical communication, and without the experimentally difficult need to perform Bell-state analysis. By using a partial polarizer to apply a generalized polarization measurement to one photon of a polarization-entangled pair we have remotely prepared arbitrary single-photon polarization qubits. Specifically, we are able to produce a range of states of any desired degree of mixing or pureness, with high fidelity. The states produced range over (and within) the entire Poincaré sphere. These methods may be useful in linear optics feed-forward quantum computation[24, 25] and in understanding the influence of the environment on quantum communications protocols. Additionally, by using non-degenerate entangled pairs as a resource, we can prepare states in multiple wavelengths. Such wavelength control may be useful in preparing states for other quantum communication protocols.

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