Quantum State Tomography
Paul Kwiat's Quantum Information
Group

Website Tomography Tutorial
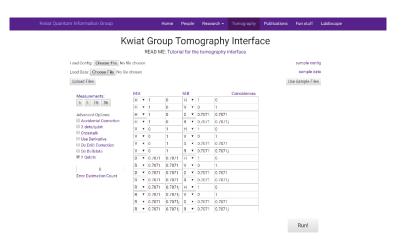
Scott Turro

1   Introduction

Quantum state tomography is the process by which a quantum state is reconstructed using measurements on an ensemble of identical quantum states. Because measurement of a quantum state (in general) changes the state being measured, getting a complete picture of that state requires measurements on many state copies.

The website is organized into two major sections. Tomography Theory contains the theoretical foundations necessary for tomography, information about the specific systems that the Paul Kwiat Quantum Information Group use, and the modifications, measurements, and data processing that are necessary to perform a real, experimental tomography. (Most of this information is contained in a large book chapter available for download.) The second part discusses Tomography Code which contains all of the MATLAB code used to analyze data, while the web interface is a web-based implementation of this code.

The web interface for the state tomography has all the features of the code in an easy to access and intuitive setting.
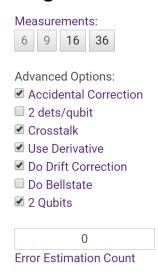
2   Walk-through of running the program

The program is relatively versatile when it comes to inputting data. It can calculate state tomography using one detector per photon and 16 or 36 measurements, or using two detectors per photon and 9, 16, or 36 measurements. It can also perform drift correction, accidental correction, and beamsplitter crosstalk correction. The user also has the option of inputting data and configurations manually, or importing configurations using a data file, or importing both a configuration and data file on which to calculate a state tomography. Figures 1 through 9 take a user step-by-step through running the web-interface with a prepared configuration and data file.

Step 1: The tomography webpage interface.

For manually entered data, continue to the next subsection, and skip the subsection on loading configuration files. Otherwise, for loading files, skip the manual section and continue to subsection 2.2.

2.1 Manually Entering Configuration and Data

Measurements:

6 | 9 | 16 | 36

Advanced Options:
- ☑ Accidental Correction
- ☐ 2 dets/qubit
- ☑ Crosstalk
- ☑ Use Derivative
- ☑ Do Drift Correction
- ☐ Do Bellstate
- ☑ 2 Qubits

0

Error Estimation Count

Step 2: Select the type of configuration needed for analysis of the dataset. In this example, a 1 detector per qubit, 16-measurement analysis with accidental correction, drift correction, and crosstalk will be performed. A 9-measurement tomography is only possible with 2 detectors per qubit. It is also advisable to always select 'Use Derivative when performing the calculation to speed up the calculation.

Time	M:A			M:B			Coincidences	Singles		Intensity
1	H ▾	1	0	H ▾	1	0		0	0	1
1	H ▾	1	0	V ▾	0	1		0	0	1
1	H ▾	1	0	D ▾	0.7071	0.7071		0	0	1
1	H ▾	1	0	R ▾	0.7071	0.7071j		0	0	1
1	V ▾	0	1	H ▾	1	0		0	0	1
1	V ▾	0	1	V ▾	0	1		0	0	1
1	V ▾	0	1	D ▾	0.7071	0.7071		0	0	1
1	V ▾	0	1	R ▾	0.7071	0.7071j		0	0	1
1	D ▾	0.7071	0.7071	H ▾	1	0		0	0	1
1	D ▾	0.7071	0.7071	V ▾	0	1		0	0	1
1	D ▾	0.7071	0.7071	D ▾	0.7071	0.7071		0	0	1
1	D ▾	0.7071	0.7071	R ▾	0.7071	0.7071j		0	0	1
1	R ▾	0.7071	0.7071j	H ▾	1	0		0	0	1
1	R ▾	0.7071	0.7071j	V ▾	0	1		0	0	1
1	R ▾	0.7071	0.7071j	D ▾	0.7071	0.7071		0	0	1
1	R ▾	0.7071	0.7071j	R ▾	0.7071	0.7071j		0	0	1

Step 3: This represents the core part of the program for data entry. M:A and M:B represent the basis for those measurements. There are drop-down menus for H, V, D, A, R, L and which will load the proper values into the rows next to the measurement columns; there is also a custom option, allowing the user to enter an arbitrary basis (the vectors entered need not be normalized). The Coincidence and Singles columns should contain the data measured by the detectors for each measurement setting. There will be 4 coincidence columns and 4 singles columns for a 2-detector case. The intensity column is used for drift correction.
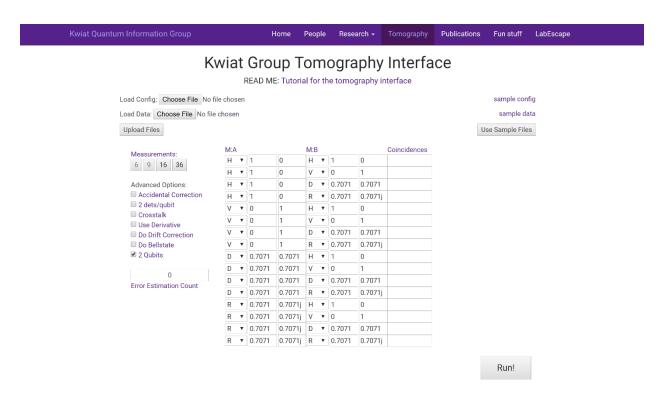
Window	0		

Crosstalk

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Step 4: This part of the interface allows the user to set the coincidence window in nanoseconds for accidental correction and the crosstalk matrix for the polarizing beam splitter. The two fields shown, W*indow* and *Crosstalk,* will only be seen if accidental correction and crosstalk are enabled, respectively. Once all the values are set, press *Run!* and continue to subsection 2.3.
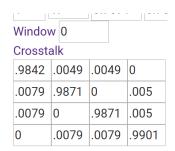
2.2   For Loading Files

Load Config: [Choose File] No file chosen   sample config

Load Data: [Choose File] No file chosen   sample data

[Upload Files]   [Use Sample Files]

Step 2: Load each file using this menu and then press the 'Upload Files' button.

Step 3: The webpage interface after loading example configuration and data files.

Crosstalk

.9842	.0049	.0049	0
.0079	.9871	0	.005
.0079	0	.9871	.005
0	.0079	.0079	.9901

Step 4: Click the *Run!* button to run the tomography.
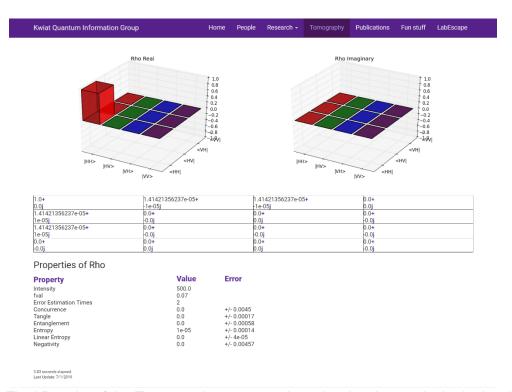
2.3 After pressing *Run!*

Download data or scripts for this tomography:

- eval file
- matlab file
- data file
- conf file

Or just

Calculate tomography

Step 5: The next screen has links to the configuration and data files generated from the inputs on the previous screen. These are .txt files that contain the MATLAB commands that are parsed by the web interface or can be used with the downloadable MATLAB code as input. When ready to run the tomography code, press *Calculate tomography.*

Rho Real

Rho Imaginary

1.0+ 0.0j	1.41421356237e-05+ -1e-05j	1.41421356237e-05+ -1e-05j	0.0+ 0.0j
1.41421356237e-05+ 1e-05j	0.0+ -0.0j	0.0+ 0.0j	0.0+ -0.0j
1.41421356237e-05+ 1e-05j	0.0+ -0.0j	0.0+ -0.0j	0.0+ -0.0j
0.0+ -0.0j	0.0+ 0.0j	0.0+ 0.0j	0.0+ -0.0j

Properties of Rho

Property	Value	Error
Intensity	500.0	
fval	0.07	
Error Estimation Times	2	
Concurrence	0.0	+/- 0.0045
Tangle	0.0	+/- 0.00017
Entanglement	0.0	+/- 0.00058
Entropy	1e-05	+/- 0.00014
Linear Entropy	0.0	+/- 4e-05
Negativity	0.0	+/- 0.00457

3.83 seconds elapsed.
Last Update: 7/1/2019

Step 6: Final Results of the Tomography program show the density matrix, both visually and numerically, plus values for the Tangle, Linear Entropy, Entropy, and Intensity, as well as numerically estimated error bars, if that option was selected. Information on these quantities can be found in the definition section.

3   Load Config and Data

The web interface has the ability to import configurations and datasets to run tomography. This lets a user easily repeat measurements with the same configuration. The files themselves are simple text files that contain the appropriate commands in MATLAB. A user can import a configuration file, or both a configuration file and a data file.

- All configuration files contain the same fields no matter what options are needed.

- **conf. NDectectors** determines if the measurement was done with 2 detectors per qubit or 1. The only valid argument is 1 or 2.
- **conf. Crosstalk** is a 4 x 4 matrix for the crosstalk in the polarizing beam-splitter. If you would like not to correct for this, or want to model an ideal PBS, insert the identity matrix.
- **conf. UseDerivative** is generally set to one because it speeds up the calculation.
- **conf. DoDriftCorrection** determines if drift correction is performed. Accepted values are "yes" or "no."
- **conf.Window** is the coincidence window used to calculate accidentals from the single counts supplied in the data.
- **conf. Efficiency** is a vector that lists the relative coincidence efficiencies of detector-pairs when using 2 detectors per qubit. The order is detector 1-2, 1-4, 3-2, 3-4.

For more information on the specific functions, please see the Definitions section.

3.1   Config Example - 1 Detector

1 detector configuration file with no drift correction but crosstalk correction.

conf.NQubits=2;
conf.NDetectors=1;
conf.Crosstalk=
[[0.9842,0.0049,0.0049,0],[0.0079,0.9871,0,0.0050],
[0.0079,0,0.9871,0.0050],[0.001,0.0079,0.0079,0.9901]];
conf.UseDerivative=1;
conf.DoDriftCorrection='no';
conf.Window=0;
conf.Efficiency=[1,1,1,];

3.2   Config Example - 2 Detector

2 detector configuration file with relative detector efficiencies listed, crosstalk correction, and drift correction enabled.

```
conf.NQubits=2;
conf.NDetectors=2;
conf.Crosstalk=
[[0.9842,0.0049,0.0049,0],[0.0079,0.9871,0,0.0050],
[0.0079,0,0.9871,0.0050], [0.001,0.0079,0.0079,0.9901]];
conf.UseDerivative=1;
conf.DoDriftCorrection='yes';
conf.Window=[0,0,0,0];
conf.Efficiency=[0.9998,1.0146,0.9195,0.9265];
```

3.3   Data

The data file is a text file containing a matrix in MATLAB syntax of either 9, 16, or 36 rows and 8 or 13 columns. Columns correspond to the number of detectors per qubit, 8 for 1, or 13 for 2 due to the 3 extra coincidence slots and 2 extra singles counts. The rows represent the number of measurements. 9 measurements are only possible with 2 detectors. The columns for the 1 detector case are:

- 1 - Time
- 2,3 - Singles Counts
- 4 - Coincidence Counts
- 5,6 - Measurement setting for A
- 7,8 - Measurement setting for B

and the following for the 2 detector case:

- 1 - Time
- 2-5 - Singles Counts
- 6-9 - Coincidence Counts
- 10,11 - Measurement setting for A
- 12,13 - Measurement setting for B

For more information on the specific functions, please see the definitions section.

4   Definitions

Accidental Correction

Perform accidental correction based on provided singles rates and coincidence window duration. For systems that have total efficiencies much less than 1 (i.e., there is a very good chance that any given signal or idler photon will not be detected), there is a high probability that one half of a correlated photon pair will be detected while the other half will not. These lead to singles counts that do not contribute to a coincidence count. Sometimes, however, orphaned

singles counts from two different entangled pairs happen to be detected together as a coincidence this is referred to as an accidental coincidence count. This code can automatically estimate -and then subtract - the number of accidental counts that contributed to a given coincidence measurement, allowing the tomography algorithm to correctly reconstruct the created quantum state. The accidental counts are calculated by multiplying the idler singles counts by the signal singles counts by the coincidence window duration, and dividing by the total measurement time.

Note: This calculation uses two assumptions. The source is CW. PULSED SOURCES WILL NOT BE RECONSTRUCTED ACCURATELY. The source is not near saturation, i.e., there is only a small chance that there will be counts found in any given coincidence window.

Coincidences

Coincidences are the heart of the tomography program. They represent the strength of correlations in the given basis. When using 4 detectors to measure the two-qubit states, as this program does, we number the detectors 1-4, with odd numbers on qubit A and even numbers for qubit B. Detectors 1 and 2 project onto the state specified, with detectors 3 and 4 projecting onto their orthogonal states. Given this numbering scheme, the four singles counts should be entered in the order: 1,2,3,4 (number corresponds to detector number). The four coincidence counts should be entered in the order of the pairs 1-2, 1-4, 3-2, 3-4, where X-Y corresponds to a coincidence measurement between detector X and detector Y.

Crosstalk

Imperfect projectors (polarizing beam splitters) can lead to errors m tomography results. However, if four detectors are used to measure the output, and the behavior of the projectors is well characterized, accurate tomography results can be reconstructed even with faulty beam splitters. The most common error is crosstalk - a polarizing beam splitter (PBS) transmits some fraction of the light it should have reflected or vice versa. The crosstalk of a pair of PBS's can be input into a 4x4 matrix of the following form: Element (I,J) of the crosstalk matrix represents the probability that incoming state I will be measured as state J. Here I and J range from O to 3, and represent O=HH, l=HV, 2=VH, 3=VV. Example: Consider a pair of PBS's which are each faulty in the same way. They each transmit 90% and absorb 10% of incoming horizontal light. They reflect 80% and transmit 10% of incoming vertical light. The crosstalk matrix corresponding to a pair of these PBS's is the following:

[[. 81   0   0   0],

[.09   .72   0   0],

[.09   0   .72   0],

[.01   .08   .08   . 64]]

Derivative

The use of calculated derivatives in the Matlab optimization allows for faster computing by taking advantage of the convex nature of the search space. In general, this option should always be on, but is not selected by default.

Drift Correction

It is possible that the pair-production rate varies in time as successive measurements are taken. If this production rate is measured (e.g., by measuring the pump beam intensity) then the reconstruction can correct for this. Input normalized pair-production rate for each measurement in the Intensity column.

Efficiency

Relative efficiency of each coincidence detector pair. The four efficiencies should be entered in the order of the detector pairs 1-2, 1-4, 3-2, 3-4, where A-B corresponds to a coincidence measurement between detector A and detector B.

Entropy

Von Neumann entropy, mixture measurement ranging from (0,2), defined as
$S = -Tr(\rho ln\rho)$

Linear Entropy

Mixture measurement ranging from (1/2,1), where 1 corresponds to a pure state and 1/2 corresponds to a completely mixed state. Linear entropy is defined as $S_L = 1 - Tr(\rho^2)$

fval

fval is the final value of the internal optimization function. Values greater than the number of measurements indicate poor agreement with a quantum state.

Intensity

Relative pump power (arb. units) during measurement; used for drift correction.

Measurements

For a standard tomography, a 16-parameter density matrix requires 16 measurements for a full specification. However, when using 2 detectors, it takes less total time to achieve the same precision if more measurements are taken. For almost all purposes, taking 36 measurements in the canonical basis {H, V, D, A, R, L} $\otimes$ {H, V, D, A, R, L} will give the most precise tomography for a given number of total input pairs. (For more details, see: J. B. Altepeter, et al. "Experimental Methods for Detecting Entanglement", Phys. Rev. Lett. 95, 033601 (2005) found

For a four-detector tomography (i.e., 2 detectors/qubit), four of these measurements are in effect taken simultaneously, allowing 9 measurements to collect as much data as 36 2-detector measurements. It is often advantageous, however, to take a full set of 36 measurements in order to detect systematic errors.

Measurement setting for the first qubit

Measurement setting for qubit A; measurements are defined by the projectors on each photon. Each projection can be defined as the state $\alpha_1 <H| + \alpha_2 <V|$, where in general α_1 and α_2 are complex. Projections can be selected from the canonical list or custom projections can be entered;

non-normalized values of α_1 and α_2 will be normalized by the program, e.g., (1, 1) will correctly represent analysis along D (diagonal).

Measurement setting for the second qubit

Measurement setting for qubit B; measurements are defined by the projectors on each photon. Each projection can be defined as the state $\beta_1 <H| + \beta_2 <V|$, where in general β_1 and β_2 are complex. Projections can be selected from the canonical list or custom projections can be entered; non-normalized values of β_1 and β_2 will be normalized by the program, e.g., (1, 1) will correctly represent analysis along D (diagonal).

Singles

Singles counts per second for each detector, used to calculate accidentals.

Tangle

Entanglement measure ranging from (0,1). The tangle is the square of the concurrence.

Two Detectors per Qubit

If you are using 2 detectors per measured qubit, then you are able to project a state into the full measurement basis. For two polarization-entangled qubits, that means using four detectors to measure not just HH, but HH, HV, VH, and VV - all at the same time. Any input pair will be projected into one of these four outcomes. In practice this greatly reduces the measurement time necessary to give a precise tomography.

When using this type of measurement for two-qubit states, as this program does, the detectors should be numbered 1-4, with odd numbers on qubit A and even numbers for qubit B. Detectors 1 and 2 project onto the state specified, with detectors 3 and 4 projecting onto their orthogonal states. Given this numbering scheme, the four singles counts should be entered in the order: 1,2,3,4 (number corresponds to detector number). The four coincidence counts should be entered in the order of the pairs 1-2, 1-4, 3-2, 3-4, where X-Y corresponds to a coincidence measurement between detector X and detector Y. When using 4 detectors, it is very possible that the relative efficiencies of each detector pair can be different. In practice, these must be measured. Those efficiencies should be input in the four fields marked Efficiency in the same order as coincidences.

Window

Coincidence window durations (in nanoseconds) to calculate the accidental rates. The four windows should be entered in the order of the detector pairs 1-2, 1-4, 3-2, 3-4, where A-B corresponds to a coincidence measurement between detector A and detector B.