PRODUCTION AND USES OF HYPER-ENTANGLED STATES

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Entangled states are central to the new field of quantum information, including quantum dense coding, teleportation, and computation. However, only a relatively small class of entangled states has been discussed extensively, much less investigated experimentally. In particular, efforts to date have focussed on two particles entangled in a single degree of freedom, for example polarization, or energy, or momentum direction. Novel phase-matching arrangements in spontaneous parametric down-conversion allow the preparation of pairs of photons that are simultaneously entangled in polarization, momentum-direction, and energy. We shall call such a multiply-entangled state "hyper-entangled". In addition, an even more general state – a non-maximally entangled state – should be realizable, in which the amplitudes of the contributing terms are not equal.

Key words: entangled states, quantum states, Bell inequalities, nonlocality, parametric down-conversion.

1. INTRODUCTION

Entangled states are arguably the most "quantum-esque" aspect of quantum mechanics. In fact, Schrödinger described entanglement as "*the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought." [1] Entangled states are inextricably linked to the measurement problem, and are central to the demonstration of nonlocality, e.g., in tests of Bell's inequalities. They form the basis, figuratively and literally, of experiments in quantum information, such as quantum dense coding [2] and quantum teleportation [3]. And in terms of applications, they

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have been proposed for use in quantum cryptography [4], and are crucial in all implementations of quantum computers [5]. Nevertheless, even at the two-particle level, only a small class of entangled states has been discussed extensively, much less investigated experimentally. In particular, efforts to date have focussed on two particles entangled in a single degree of freedom, for example polarization [6], or energy [7], or momentum direction [8].[1]

A novel type-II phase-matching arrangement in spontaneous parametric down-conversion has recently permitted the demonstration of the first of these with very encouraging results [6]. All four of the Bell states were produced, and strong violations of Bell's inequalities were observed in all cases. The desired polarization-entangled states are produced directly out of the nonlinear crystal, and the source was seen to have unprecedented brightness and stability. Actually, due to the very nature of the down-conversion process, the photons are automatically also produced into an energy-entangled state. Moreover, using a modification of the phase-matching arrangement, one should be able to prepare momentum direction-entangled photon pairs. The final result will be pairs of photons that are simultaneously entangled in polarization, momentum-direction, and energy. We shall call such a multiply-entangled state "hyper-entangled". Finally, it should be possible to prepare photon pairs in a non-maximally entangled state, in which the two contributing terms do not have equal amplitudes. Such a state would be very important for enabling a loophole-free test of Bell's inequalities [11,12].

2. ENTANGLEMENT VIA PARAMETRIC DOWN-CONVERSION

An entangled state is a nonfactorizable sum of product states of two (or more) quantum systems [13]. More particularly, it is a state which cannot be factorized in *any* basis. The general form of such a state for two particles is:

$$\Psi_{1,2} = \sum_i c_i|\alpha_i\rangle \otimes |\beta_i\rangle, \tag{1}$$

where α and β are the basis vectors of particles 1 and 2, respectively. As we will see below, the sum sometimes is extended into an integral, when the relevant Hilbert space is continuous. The most familiar example of an entangled state is the Bohm singlet state, $\Psi_{1,2} = (|\uparrow_1\rangle|\downarrow_2\rangle - |\downarrow_1\rangle|\uparrow_2\rangle)/\sqrt{2}$, which represents the state of two spin-1/2 particles decaying from a spin-zero parent. The two particles are correlated — their spins are always anti-parallel — and they remain that way no matter the separation between them. In this sense entangled systems can demonstrate *nonlocal* quantum effects.

Albeit identical in principle, it is much easier in practice to work with

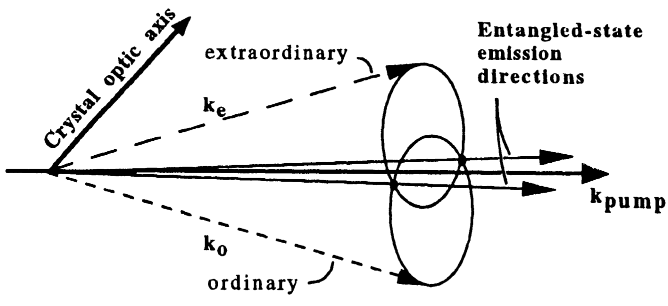

Fig. 1. Spontaneous down-conversion cones from type-II phase-matching.

photons that are correlated, because they are (now) easier to produce and their intrinsic correlation is not readily destroyed by decoherence through interaction with the environment. While initial experiments used photon pairs produced in an atomic cascade, now the source of choice is spontaneous parametric down-conversion [14]. In this process, an ultraviolet "pump" photon (typically produced in a laser) spontaneously decays inside a crystal with a $\chi^{(2)}$ nonlinearity into two highly correlated red photons of nearly equal energy. Energy and momentum are conserved in this process: $\hbar\omega_p = \hbar\omega_s + \hbar\omega_i$, $\hbar\mathbf{k}_p \approx \hbar\mathbf{k}_s + \hbar\mathbf{k}_i$, where $\hbar\omega_i$ and $\hbar\mathbf{k}_i$, $(i = p, s, i)$ are the respective energies and momenta of the parent [p] and two daughter photons (conventionally called the "signal" [s] and the "idler" [i]).

The conservation of momentum inside the crystal, which is also known as "phase-matching," is enabled by using the birefringent properties of the crystal itself to compensate for the dispersion of the material. The result is that the down-conversion photons are produced in a rainbow of colored cones, with conjugate photons (i.e., the members of a given pair) lying on opposite sides of the pump beam (see Fig. 1). For the case of "type-II" phase-matching, one of each pair is ordinary-polarized and its conjugate photon is extraordinary-polarized. For a negative uniaxial crystal such as BBO, the axis of the extraordinary cones is between the pump beam direction and the crystal optic axis, while the axis of the ordinary cones is further away from the optic axis than the pump beam (in both cases, the pump beam, crystal optic axis, and cone axis lie in a plane).

In Fig. 2 we see that there is a rich structure to the emissions from a down-conversion crystal. Various types of entangled states are produced for photons emitted in particular directions. For example, a pair of photons emitted along the directions 3 and 3′ are automatically in a polarization-entangled state. Table 1 lists the various entanglements present for the

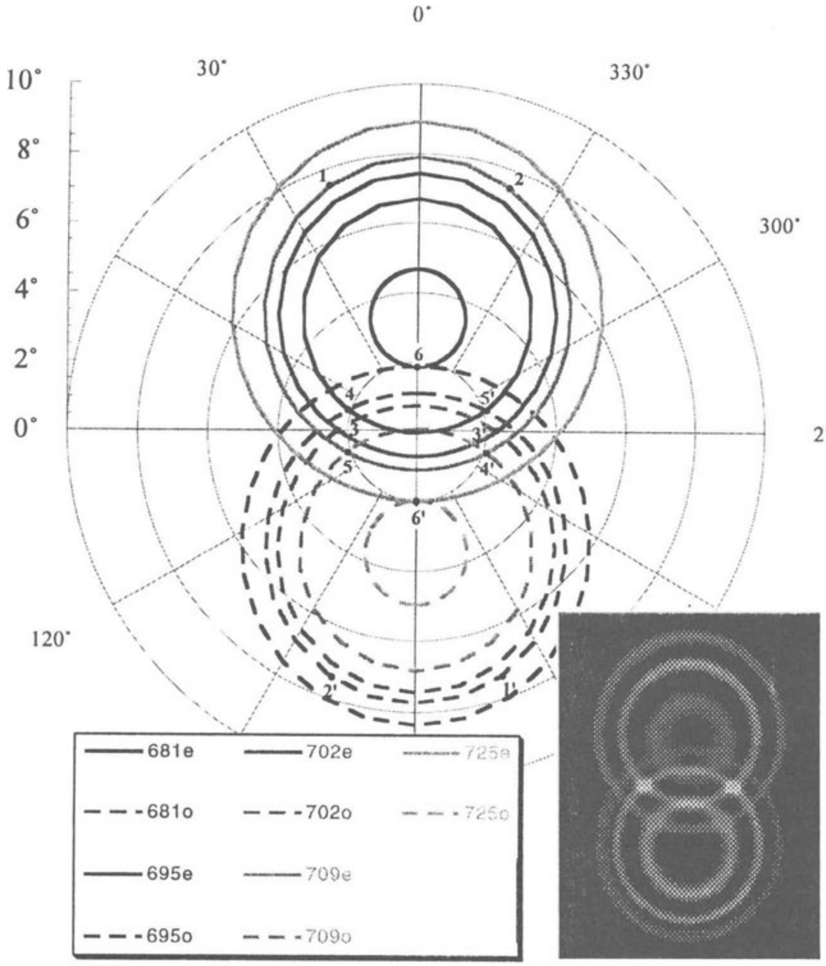

Fig. 2. Calculated type-II phase-matching curves in BBO (for a crystal cut at $\phi = 49.2°$, with a pump wavelength of 351 nm), showing various cones of down-conversion light (the pump beam would be coming directly out of the paper at the origin). The wavelengths of the five extraordinary-polarized cones (upper half, solid lines) are, from innermost to outermost: 681nm, 695 nm, 702 nm, 709 nm, and 725 nm. The ordinary-polarized conjugate cones to these (lower half, dashed lines) have their wavelengths reversed (e.g., smallest bottom cone is 725 nm). The numbered points represent sampling areas to extract various entangled states, as listed in Table 1. Inset: Photograph (courtesy of Michael Reck, Innsbruck) of three sets of conjugate cones emerging from a BBO crystal.

Table 1. The various entanglements available at the numbered points indicated in Fig. 2.

Conjugate points from Fig. 1	Energy-Time entangled	Momentum-entangled	Polarization-entangled	Non-maximally entangled
1 - 1'	✓			
1 - 1' -- 2 - 2'	✓	✓		
3 - 3'	✓		✓	
4 - 4' -- 5 - 5'	✓	✓	✓	
6 - 6'	✓		✓	✓

emission directions indicated in Fig. 2.

3. SINGLY ENTANGLED STATES

3.1. Time-Energy Entangled States

The energy-entangled states from down-conversion photons are in some sense the most universal, because they are present for *any* pair of photons. Due to the fact that there are many ways to partition the energy of the parent photon, each daughter photon has a broad spectrum, and hence a narrow wave packet in time. However, the *sum* of the two daughter photons' energies is extremely well-defined, since they must add up to the energy of the extremely monochromatic parent laser photon.[2] This correlation is represented by the following energy-entangled state:

$$|\Psi\rangle = \int_0^{E_p} dEA(E)|E\rangle_s|E_p - E\rangle_i, \tag{2}$$

where each ket describes the energy of one of the photons, s and i stand for "signal" and "idler", respectively, and $A(E)$ is essentially the spectral distribution of the collected down-conversion light.

That the photons can display nonlocal correlations arising from this entanglement was demonstrated by a number of groups [7], who implemented the experiment first proposed by Franson [15]. Each of the photons is directed into its own unbalanced Mach-Zehnder interferometer (see Fig. 3), giving it a long path (L) and a short path (S) to the detectors. Because the path length difference is much longer than the coherence length of the photons, no interference is observed in the singles rates at any of the detectors when the phase in, say, one of the long paths is changed. However, there *is* interference in the coincidence rate between detectors. The reason is that there are two processes that could lead to such a coincidence

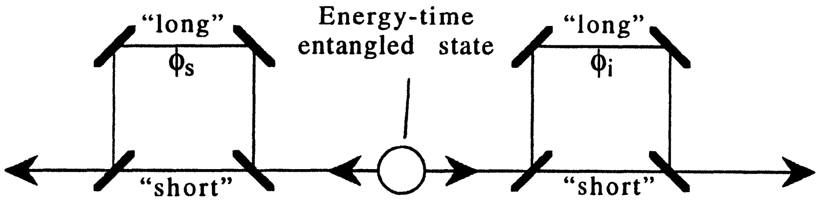

Fig. 3. Basic setup for the Franson experiment

count – both photons could have taken their respective long paths or both could have taken their respective short paths – these processes are indistinguishable, and so interfere. The relevant state is the first two terms of the following:

$$|\psi\rangle = \frac{1}{2}\left(|S_s, S_i\rangle - e^{i(\phi_s+\phi_i)}|L_s, L_i\rangle + e^{i\phi_i}|S_s, L_i\rangle + e^{i\phi_s}|L_s, S_i\rangle\right). \quad (3)$$

[Note, however, that it is first necessary to use a large path imbalance and very fast detectors to be able discard the second two terms, arising from the distinguishable (and hence *non*-interfering) events in which one photon takes its short path and the other takes the long path. We will see below how use of a doubly-entangled state can remove this requirement.] The rate of coincidences is thus dependent only on the *sum* of phases in the two interferometers. This nonlocal effect allowed experimenters to test a suitable version of Bell's inequalities [7].

3.2. Momentum Entangled States

The next most "common" entangled state from the down-conversion process is one in which the *momentum directions* of the photons are entangled (though of course the photons will still automatically possess the above-mentioned energy entanglement[2]). For simplicity we consider photons that have nearly the same energy (though this is not necessary), but which are emitted into two sets of directions (c.f. points 1-1′ – 2-2′ from Fig. 2). Concerning ourselves now only with the spatial modes, the emitted state can then be written as

$$|\psi\rangle = \left(|1_s, 1'_i\rangle + e^{i\phi}|2_s, 2'_i\rangle\right)/\sqrt{2}\ , \quad (4)$$

where the 1, 1′, 2, and 2′ indicate the directions of the photons. Such a state was employed by Rarity and Tapster to demonstrate a violation of

Bell's inequalities based on momentum entanglement [8], also depending nonlocally on separated phase elements.

3.3. Polarization Entangled States

Perhaps the simplest examples of entangled states of two photons are the polarization-entangled "Bell states" [16]:

$$\begin{aligned} |\psi^{\pm}\rangle &= \left(|H_s, V_i\rangle \pm |V_s, H_i\rangle\right)/\sqrt{2}, \\ |\phi^{\pm}\rangle &= \left(|H_s, H_i\rangle \pm |V_s, V_i\rangle\right)/\sqrt{2}, \end{aligned} \tag{5}$$

where H and V denote horizontal and vertical polarization, respectively. It is with these states that the advantage of type-II phase-matching over the more familiar type-I (in which the down-conversion photons both have the same [ordinary] polarization, and there is only a single set of cones, centered on the pump beam) becomes clear: Such states cannot be produced simply using the latter. To see how they arise in type-II phase-matching, consider points 3 and 3′ from Fig. 2. These points both lie on the degenerate set of cones (i.e., exactly half the pump frequency goes to each daughter photon). Moreover, at these points – the intersections of the two cones – a given photon belongs to *both* the extraordinary and the ordinary cones. Hence it does not have a definite polarization. Nevertheless, due to the nature of the emission process, if one of the photons is measured to have ordinary polarization (call it H), then the other is certain to have extraordinary polarization (call it V). In this way, merely by selecting out the correct directions from the down-conversion crystal we prepare the state $|\psi^+\rangle$.

By using only standard optical elements in one of the two output beams, it is possible to transform any one of the Bell states into any of the others. For example, using a polarization rotator to exchange H and V for one photon immediately changes $|\psi^+\rangle \leftrightarrow |\phi^+\rangle$ and $|\psi^-\rangle \leftrightarrow |\phi^-\rangle$. A birefringent phase-shifter in one of the beams similarly transforms $|\psi^+\rangle \leftrightarrow |\psi^-\rangle$ and $|\phi^+\rangle \leftrightarrow |\phi^-\rangle$. We were therefore able to test Bell's inequalities with all four states [6], and observe large violations in all cases (see Fig. 4), modulo the usual auxilliary assumptions to account for low detection efficiency and no rapid analyzer setting. In fact, due to an unexpected robustness in the source to larger collection irises, we were able to obtain the necessary statistics for a 102 σ violation in less than 5 minutes.

The four Bell states were also employed in a recent experiment [17] to demonstrate the possibility of "quantum dense coding", in which up to 2 bits of information may be encoded in the polarization state of a single photon [2]. A transmitting party and a receiving party each receive one member of a correlated pair of photons, initially in the state $|\psi^+\rangle$. By making one of four operations on his particle alone, the transmitter can change the entire

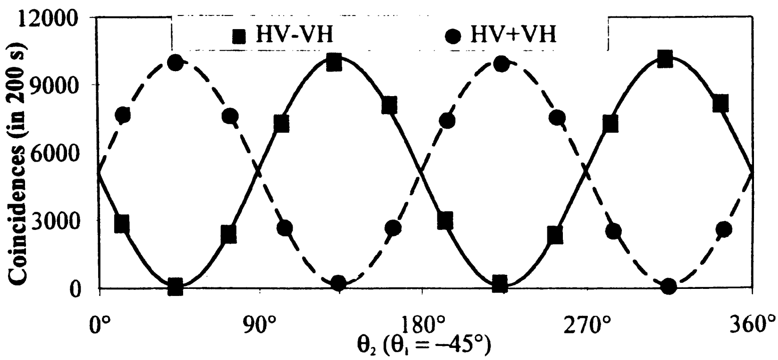

Fig. 4. High-visibility polarization correlations allowing one to observe strong violations of Bell's inequality.

joint two-particle system into one of the four Bell states. He then sends his particle to the receiver, who must make a joint measurement on the two particles to determine which of the four Bell states (which of four messages) was prepared by the transmitter. In fact, this Bell-state measurement is non-trivial, and to date it has only been possible to distinguish three of the four possibilities [17]. Below we will see how the use of a multiply-entangled state can remedy this.

4. MULTIPLY-ENTANGLED STATES

4.1. Improved Franson Experiment

As mentioned earlier, the energy-time correlations are present for all pairs of down-conversion photons, so that, in particular, they are present when the photons are also in a polarization-entangled state. The extra degrees of freedom allow one to improve some of the previous experiments. For example, one can perform a Franson-type experiment in which there is no need at all to discard any counts. Let us start out with the photons jointly in an energy-entangled state and in the polarization-entangled state $|\phi^+\rangle$, so that the photons have the same polarization. Consider now the original Franson setup (Fig. 3), but with *polarizing* beam splitters, which transmit horizontal and reflect vertical polarized light. Under this case it is clear that either the photons will both follow the short paths (term 1 of $|\phi^+\rangle$) or they will both follow the long paths (term 2 of $|\phi^+\rangle$) of their respective interferometers. In this case, *there is no non-interfering background* of "long-short" processes, for they simply do not occur. At the end we use a polarizer at 45° to "erase" the polarization information.[3] This system has a tremendous

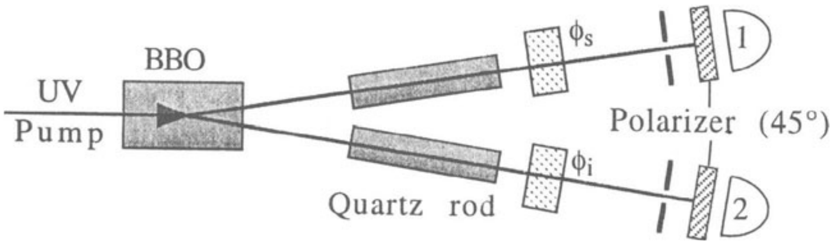

Fig. 5. Setup demonstrating the use of multiply-entangled photons (in energy-time and in polarization) to dramatically improve the performance of the Franson-type experiment.

advantage over the previous implementations in that it can be much smaller and therefore more robust.

A recent implementation [18] of this idea went one step farther, and used a birefringent quartz crystal as the entire interferometer (see Fig. 5). The analog of the short and long paths become the fast and slow modes in the quartz — the quartz is aligned with the fast axis along horizontal and the slow axis along vertical. The high-visibility correlations (again observed only in the coincident rate) implied a 17 σ violation of Bell's inequality. Moreover, the system was remarkably stable, and thus might find use in a realizable quantum cryptography setup.

4.2. Improved Bell-State Analysis

As a second example of the application of multiply-entangled photons, we consider the problem of Bell-state analysis. We wish to make a joint measurement on two photons and determine which of the four polarization Bell states (5) they are in. As alluded to in Sec. 3.3, at present there is no way to distinguish more than three of the Bell states (unless one has a non-linear interaction that is significant at the single-photon level). This problem can be solved by embedding the polarization entangled states into a larger Hilbert space, in whose degrees of freedom the photons are also entangled. Below we briefly describe one example based on the extra time correlations (connected with the energy entanglement) of the down-converted photons. A similar scheme based instead on an additional momentum entanglement is also possible.

Consider Fig. 6, in which the two photons are directed to opposite sides of a 50-50 beam splitter. It can be shown that only for the state ψ^- will the photons exit the beam splitter in different directions – for the other three states the photons will travel off together [17] On this basis alone we can distinguish the first of the four states. Next, we pass the photons

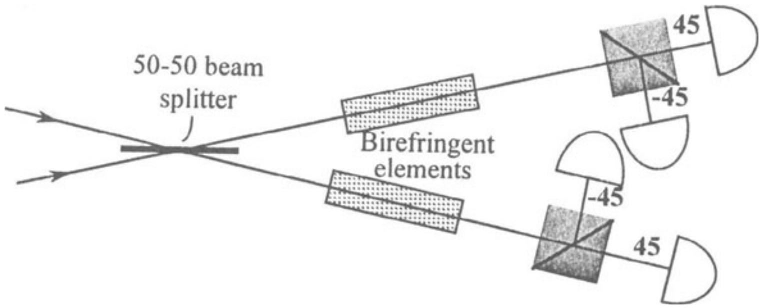

Fig. 6. Improved Bell-state analysis using doubly-entangled photons.

through very birefringent elements, so that the horizontal components are slowed relative to the vertical. In this case, the state ψ^+ (for which the photons exit with a relative delay) becomes distinguishable by timing from the states $\phi^{\pm}$ (for which the photons exit with no relative delay). Finally, by analyzing the pair with a polarizing beam splitter at 45°, we can distinguish the final two: for ϕ^+ both photons will go to the same detector, while for ϕ^- each photon will go to a different detector [19]. Thus, by taking advantage of the extra time-energy correlations of the down-conversion photons, one is able to solve an otherwise difficult problem.

4.3. Hyper-Entangled States

Finally, we consider the quantum state associated with photons emitted along the directions 4-4′ − 5-5′. They are apparently in a momentum-entangled state of the sort described above. They are also polarization-entangled, because each photon belongs to both an extraordinary cone and an ordinary cone. And of course they are automatically energy-entangled due to the down-conversion process. Therefore, the quantum state of these pair of photons is triply- or "hyper-entangled":

$$|\Psi\rangle = \left(\int_0^{E_p} dEA(E)|E\rangle_s|E_p - E\rangle_i\right) \otimes \frac{(|1_s, 1_i'\rangle + e^{i\phi}|2_s, 2_i'\rangle)}{\sqrt{2}} \otimes \frac{(|H_s, V_i\rangle + |V_s, H_i\rangle)}{\sqrt{2}} \tag{6}$$

All the advantages of this sort of state, and the benefits it engenders are not yet known to us, but it is our hope that, just as the Franson experiment and quantum dense-coding are improvable using doubly-entangled states, other experiments in quantum information may be simplified or enabled using such hyper-entanglements. In particular, there may be a way to allow purely optical quantum gates, requiring only linear optical elements (aside from the down-conversion crystal itself).

5. NON-MAXIMALLY ENTANGLED STATES

Until now we have concerned ourselves implicitly with entangled states in which the magnitudes of the contributing terms were equal. There is, however, a more general state in which this is not the case; such states are known as "non-maximally" or "partially"entangled, and have the form: $|\psi\rangle \propto |H_1, V_2\rangle + \epsilon|V_1, V_2\rangle$. In the limit of $\epsilon \to 1$, we recover a standard entangled state, while $\epsilon = 0$ gives a product state.

The desired state should be obtainable from type-II phase-matching by selecting the points 6 and 6′, as shown in Fig. 2. First, notice that the photons again each belong to both an extraordinary cone and an ordinary one, as in the previously discussed situation for points 3 and 3′. However, because the size of the cones is very different, so too should be the relative contributions of them. In particular, if a photon is detected along direction 6, it is more likely to have come from the *smaller* cone – the extraordinary-polarized one – because the angular density of emitted photons should be higher than for the larger ordinary- polarization cone. If the density scales simply as the reciprocal of the cone circumference, then we would obtain a value for ϵ of about 0.5.[4]

Such partially-entangled states have been shown to be useful in *gedanken* experiments demonstrating the nonlocality of quantum mechanics without inequalities [20]. More importantly for experiments, Eberhard has shown that by using a non-maximally entangled state, one can reduce the required detector efficiency for a loophole-free test of Bell's inequalities from 83% to 67% (in the limit of no background) [11]. This, in turn, is relevant because we now have single photon detectors with efficiencies of 75-80% [21], so there is hope of completing a true test of quantum nonlocality.

6. CONCLUSION

While we do not claim to understand all the possible ramifications or benefits of multiply-entangled states, it is clear that they enable a wealth of new phenomena. For example, using the accompanying energy-entanglement, it is possible to distinguish all four of the polarization-Bell states – otherwise, only three may be distinguished [17]. Such states may also simplify experimental implementations of quantum teleportation, and true loophole-free tests of quantum nonlocality.

Finally, there exists an even more general class of entangled states, even for a single degree of freedom, in which the amplitudes of the contributing terms have different magnitudes. Such states have been called "non-maximally entangled", and have been shown at least to reduce the experimental constraints in tests of Bell's inequalities. Using novel phase-matching possibilities it should be possible to produce such states, and even to produce and study non-maximal hyper-entangled states, the most general possible quantum state of two particles. Some of the above results have already been observed experimentally [6,17,18], while results for the other schemes are expected in the next months.

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NOTES

1. There has been some work looking at two particles with a mixed entanglement, e.g., where the polarization of one particle is entangled with the direction of the other [9]. Also, one group has examined the case where one variable is truly entangled, and another *emulates* an entanglement by means of a post-selection [10].
2. We are considering here only the case of a cw (continuous wave) pump, therefore assumed to be nearly monochromatic. If, on the other hand, one were to employ a very short pulsed pump, then the time correlations of the down-conversion photons would remain, but the energy correlations would be smeared out by the inherent spectral width of the pump.
3. It might seem that we *are* actually performing a post-selection on our data by using a polarizer at 45°; indeed, this only transmits half of the light. However, just as the transmitted photons display perfect 100%-visibility fringes (as one of the phases is changed), so too would the absorbed photons. If we were to use instead a polarizing beam splitter, oriented in the 45°/-45° basis, then out one port we would obtain fringes while out the other we would obtain anti-fringes.
4. It does not matter that we are looking at a non-degenerate solution — the color of the light in no way yields information about whether the photon along direction 6 (or 6′) came from the ordinary or the extraordinary cone.

Ode to Entangled States

Photons twins, at birth separated
And yet they remain so well correlated
Their colors, directions and spins synchopated
No wonder these states are so celebrated

If that one goes this way, this one goes that
If this one comes early, that one comes late
Like two random roulette wheels, yet somehow both "fixed"
To hit the same number though they're never mixed

They drove EPR to say "It's incomplete"
They've got the Bell inequalities beat
When factoring primes they allow you to cheat
Who knows what new marvel is next at our feet

Just out of reach were problems that dangled
Current attempts to solve them seemed wangled
Perhaps what's required is something new-fangled
Enter the states called *hyper-entangled*