

Neural Coding

What is Neural Code - Techniques for recording from brain

- tools for discover how brain rep info
- models that express relatio,
etc

Recording from the Brain

fMRI - Recording someone's brain as they perform task

*Good for finding approx location (avg many neurons)

Faster: EEG, calc. electric field, faster + avg too

Reading Out The Neural Code

- multielectrode array

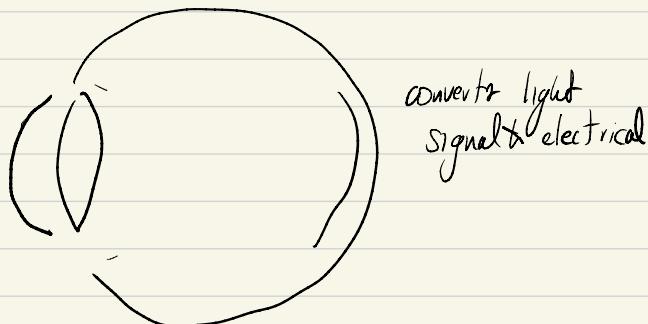
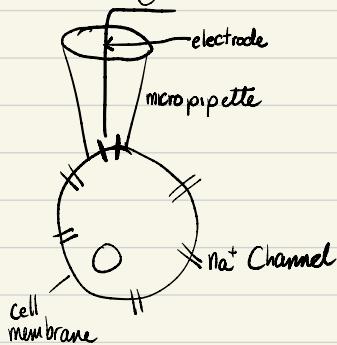
- good for when we can put layers over brain



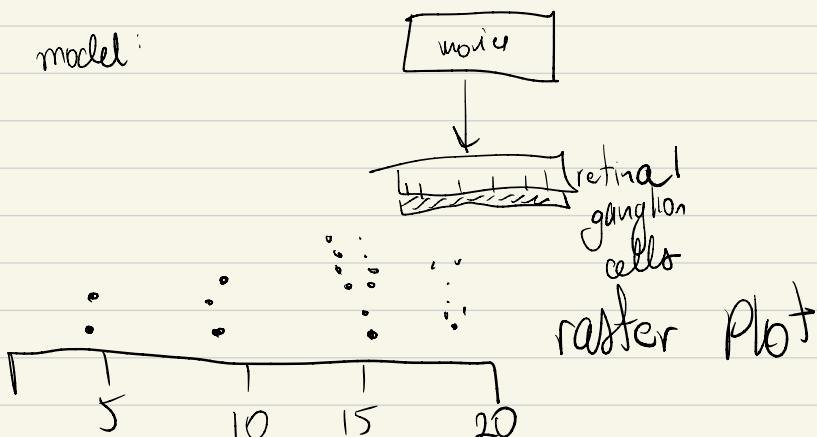
- calcium imaging

↳ Calcium records neural activity since used in procells.

Looking inside single cell



model:



cells fire consistently at app time

suggests that the cell is responding to the particular feature of the stimulus that occurs at this time

Encoding: how does a stimulus cause a pattern of responses
building quasi-mechanistic models

Decoding: what do these responses tell us about the stimulus
• how do we reconstruct what the brain is doing

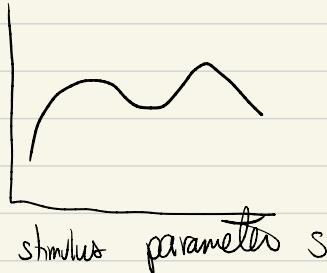
encoding: $P(\text{response} \mid \text{stimulus})$

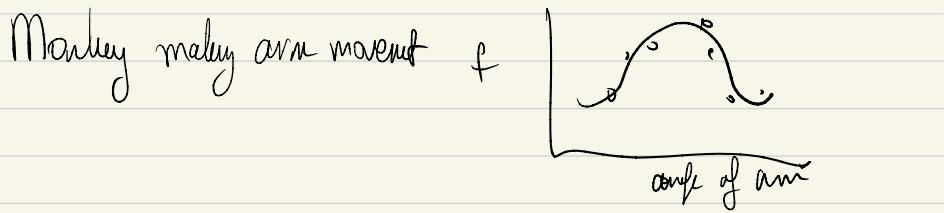
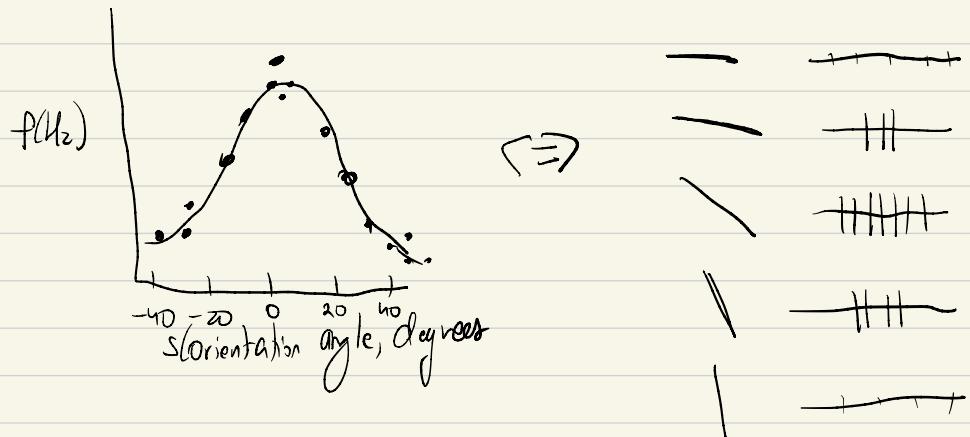
decoding: $P(\text{stimulus} \mid \text{response})$

Q: What is the stimulus? What is the Neural Representation of Information

cavg
prob
rate
spike)

Neural Response





Simple Models

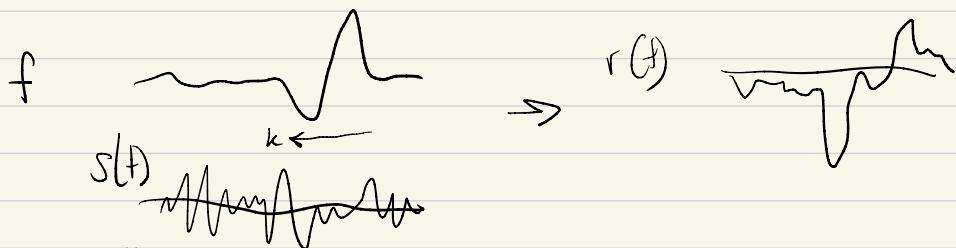
$P(\text{response} \mid \text{stimulus}) \rightarrow r(t)$ given a stimulus

for now, take response as spike in single neuron

$$r(t) = \phi s(t) - \phi s(t-\tau)$$

$$s(t) \rightarrow r(t)$$

Temporal filtering



Linear Filtering:

$$r(t) = \sum_{k=0}^m s_k f_k$$

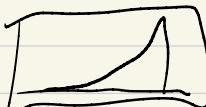
$\overbrace{s_k}$ $\overbrace{f_k}$ weight by factor dep on time

$$r(t) = \int_{-\infty}^t dt' s(t-t') f(t')$$

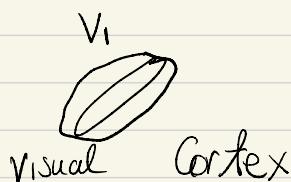
* Convolution !

$$\text{Ex 1: Running Avg: } s(t) \rightarrow \boxed{s(t) - \frac{1}{N} s(t-N)} \Rightarrow r(t)$$

Linear filter: $r(t) = \sum_{k=0}^n s_{t-k} f_k$

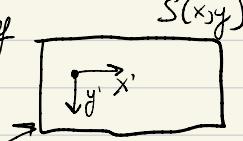
Leaky Avg:  exp. decay into past

Spatial Filtering

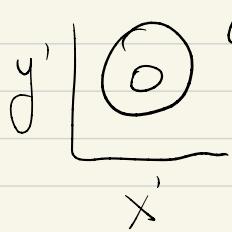


$$r(t) = \sum_{k=0}^n s_{t-k} f_k$$

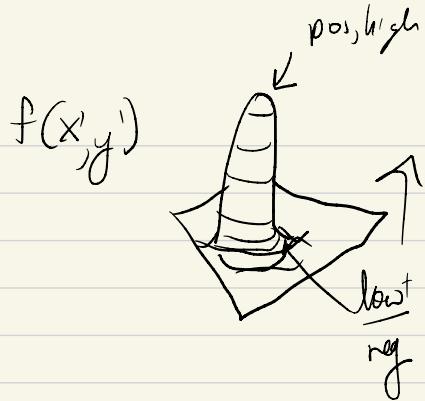
Temporal filtering



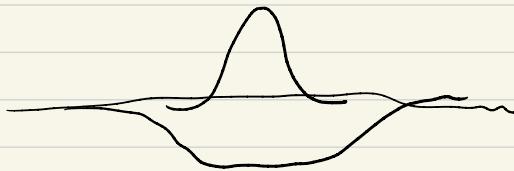
$$r(x, y) = \sum_{x'=-n}^n \sum_{y'=-n}^m s_{x-x', y-y'} f_{x', y'}$$



cartoon $f(x, y')$

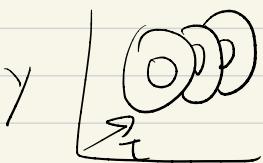


approx as diff by gradients



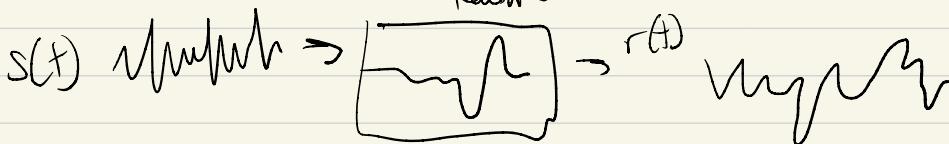
Spatio Temporal filtering

$$r_{xy}(t) = \iiint dx' dy' dz f(x, y, z) s(x-x', y-y', t-z)$$

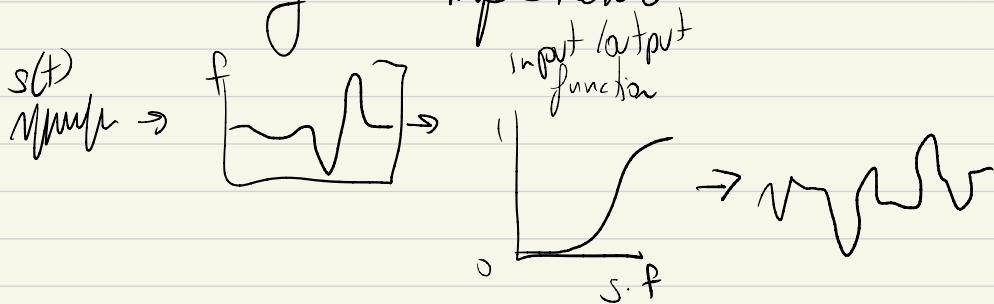


x

y



Finding Components



$P(\text{response} | \text{stimulus})$

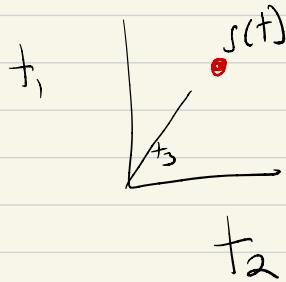
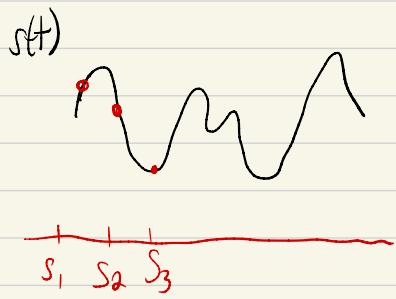
-Our problem is dimensionality!

300,000 per stimulus, comp too heavy.
-data unmanageable.

• We want to sample the responses of the system to many stimuli so that we can characterize what it is about the input that trigger responses

$P(\text{response} | \text{stimulus}) \rightarrow P(\text{response}_i)$

- Start w/ high-dimensional description (image / time-varying waveform) + pick out small set of relevant dims.



dimension becomes
of points in discretization

first axis \rightarrow stimulus @ first point in time,
Second \rightarrow second, ...

$$P(\text{response} | \text{stimuli}) \rightarrow P(\text{response} | s_1, s_2, s_3, \dots, s_m)$$

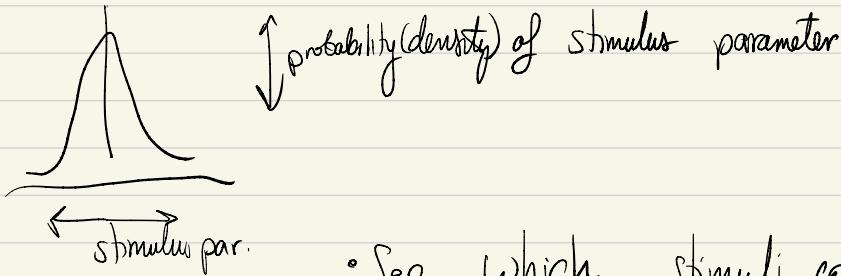
Useful method: Gaussian white noise

as we continue to stimulate w/ white noise,
various

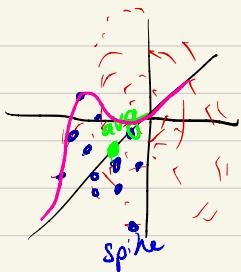
start to fill up distribution.

- Gaussian prior stimulus distribution

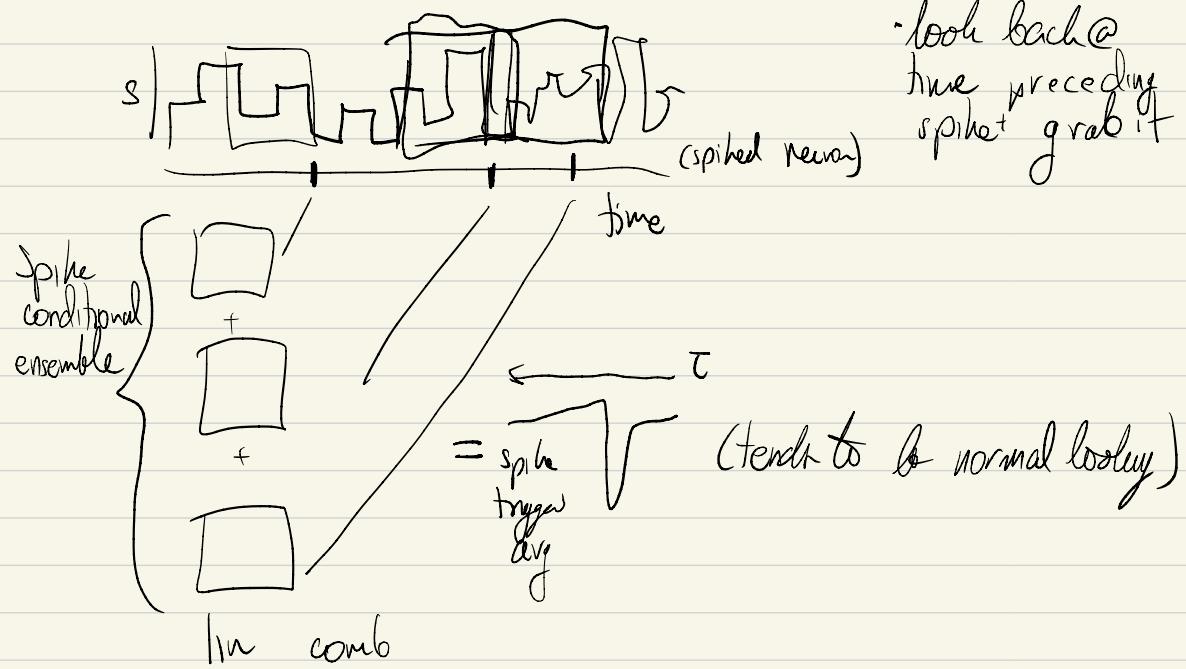
* even along any axis, distribution gaussian
↳ also along lin. comb.



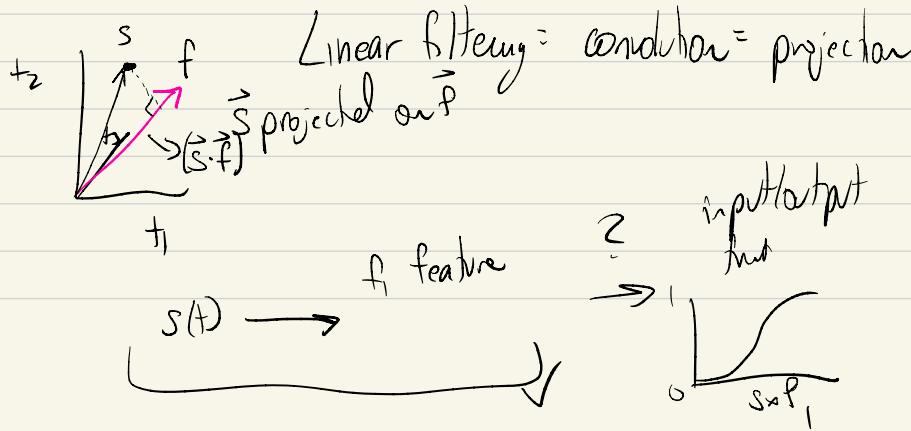
- See which stimuli contributed spikes to occur
- Get spike-conditional distributions
- Calculate spike-triggered avg. and the vector good vector orientation for axis



Reverse correlation: spike triggered avg



* Finding what's common to illicit a spike



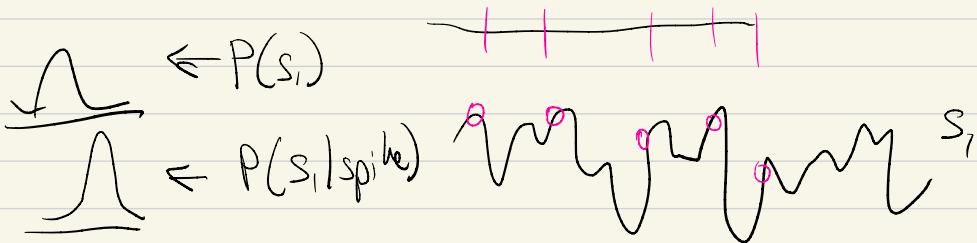
• Determining nonlinear input/output function

Input/output function is:

$$P(\text{spike} \mid \text{stimulus}) \rightarrow P(\text{spike} \mid s_i)$$

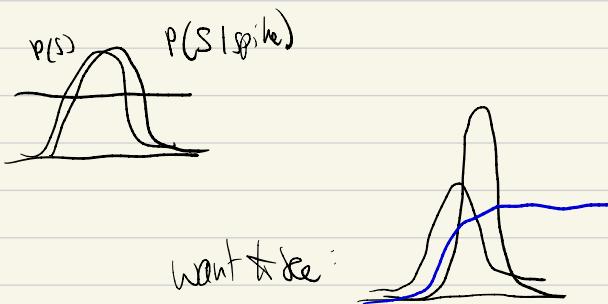
- Found from data using Baye's Rule
spike conditional dist

$$P(\text{spike} \mid s_i) = \frac{P(s_i \mid \text{spike}) \cdot P(\text{spike})}{P(s_i) \leftarrow \text{prior}}$$

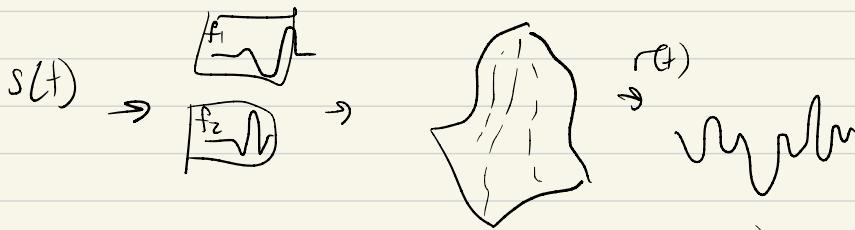


ex : say neuron fires @ random,

$$P(\text{spike} | s_i) = \frac{P(s_i | \text{spike})}{P(s_i)} P(\text{spike})$$

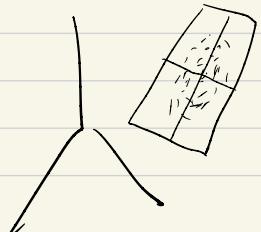


* Consider several filters



$$r(t) = g(f_1 \cdot s, f_2 \cdot s, \dots, f_n \cdot s)$$

Covariance + PCA : discover low-dim structure

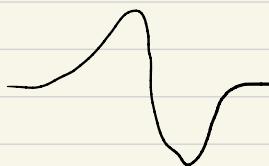
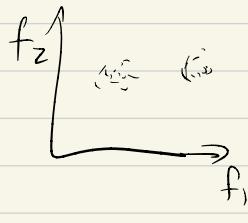


Two principle Components : 2 dimensions perp.

* gives new, sig smaller, basis set

ex: most faces can be reconstructed from like 8 eigenfaces

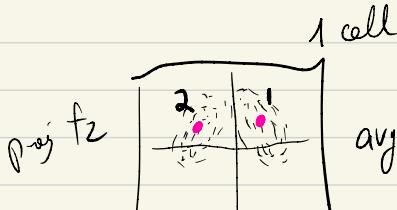
-spike sorting



noise gets put into set
w/ features

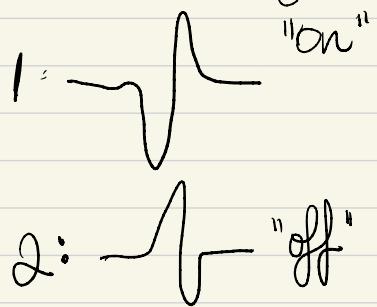
PCA: ① gives representation of data w/ lower dim

② finds vectors w/ maximal variation of data



any 0, but not a story

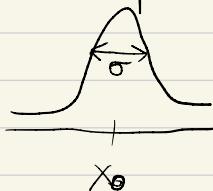
proj. fach 1



neuron likes when light off
& light on.

Variability

Magical Gaussian $P(x) = A \exp[-(x-x_0)^2/2\sigma^2]$

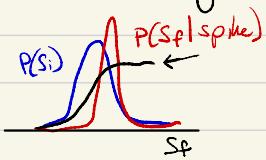


$\langle x \rangle$

$$\bar{x} = \text{mean}(x) = x_0$$

$$\text{var}(x) = \langle (x - \bar{x})^2 \rangle = \sigma^2$$

* You know you found a good feature if input/output curve over your variable looks interesting



* Want two curves as different as possible
 \Rightarrow Kullback-Leibler divergence

$$D_{KL}[P(s), Q(s)] = \int ds P(s) \log_2 P(s)/Q(s)$$

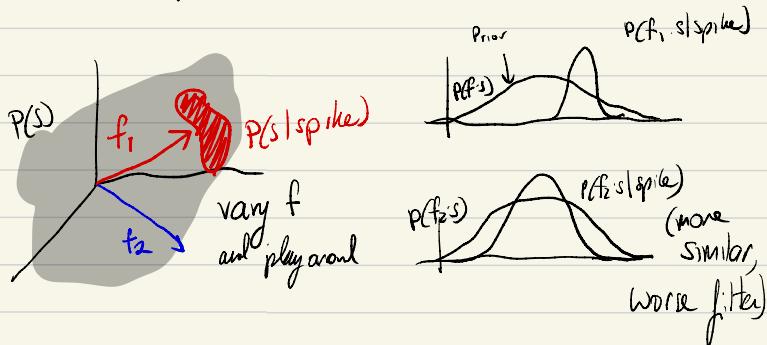
Grochens measure $D_{KL}[P(S_f | \text{spike}), P(S_f)]$ maximal

* Maximally informative dimensions

Max. Inform. Dm cont.

Choose filter in order to maximize DKL b/w

spike conditional & prior distributions



- * Does not depend on what noise
- * Can be used for denoising from natural stimuli

- Advantages:
 - Gives a way of seeking filters that maximize discriminability of spike-conditional dist. & prior
 - does not req. a specific structure for distributions such as Gaussian

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

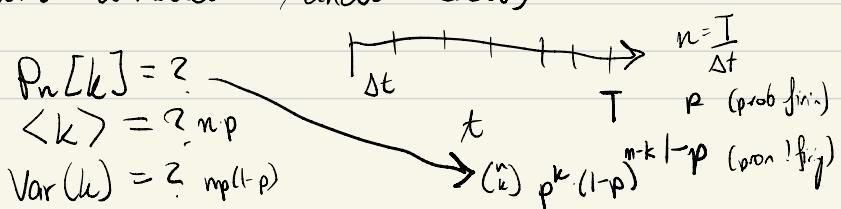
• Bernoulli Trial

- take biased coin w/ small prob. of heads (say a spike occurs whenever lands heads)

Distribution $P_n[k] = ?$

Mean $\langle k \rangle = ?$, $n \cdot p$

Var (k) = ? $n \cdot p \cdot (1-p)$



$$r = \frac{P}{\Delta t}$$

Poisson spiking Dist: $P_T(k) = (rT)^k \exp(-rT)/k!$

Mean: $\langle k \rangle = rT$

Var: rT

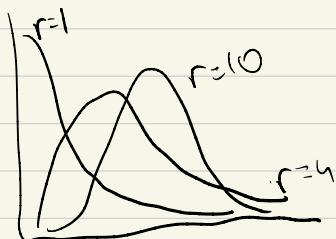
Fano Factor: $F = 1$

Interval dist: $P(T) = r \exp(-rT)$

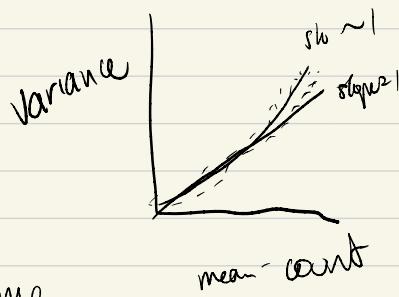
$$P_T(0) = (rT)^0 \exp(-rT) / 0!$$

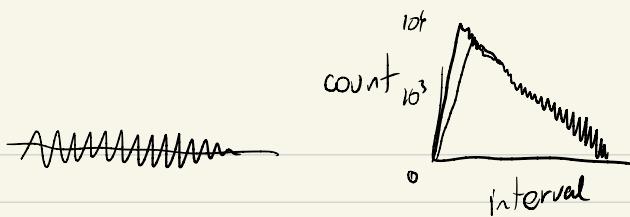
$P_T(k)$ = probability of k spikes in T seconds

$r = \# \text{ spikes} / \text{second}$



*See as r changes w/ Time

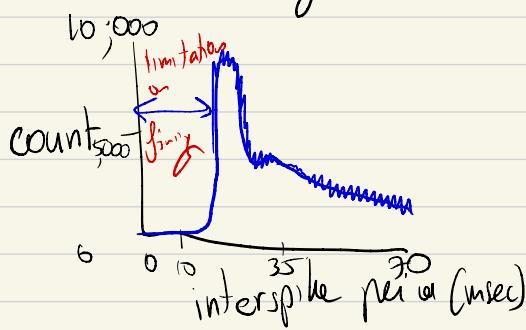




When is a Poisson model poor:

When a neuron fires many hundreds of times per second

- Poisson spiking assumes that each spike time is independent of all others. However, in real neurons, there is a "cooldown" time (refractory period) where a cell cannot fire immediately after first firing ($\sim 1\text{ ms}$), so if a cell is firing hundreds of times, this effect is magnified

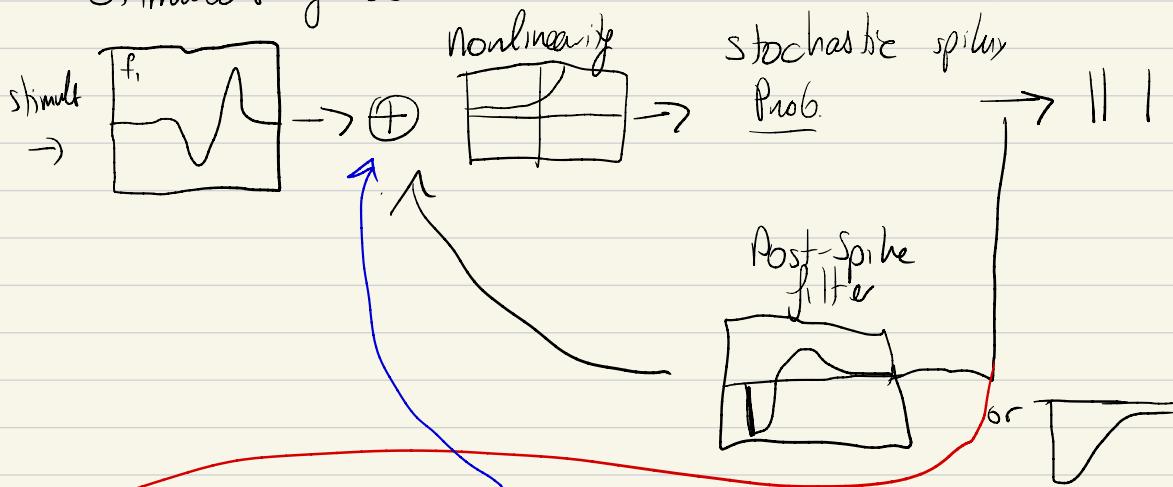


What to add/include into our spike-generation model

- taking into account history of cell's own firing
- taking into account cell's interaction w/ other cells

Generalized linear model

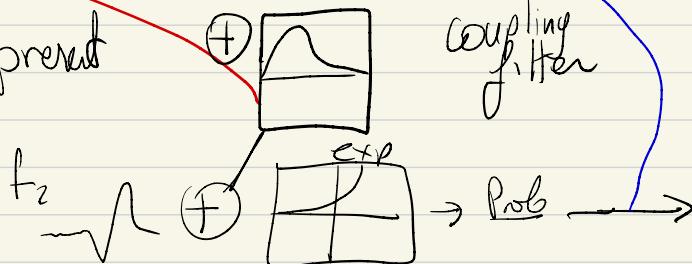
Stimulus filter

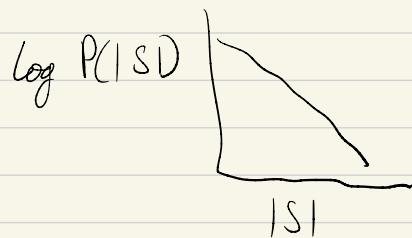
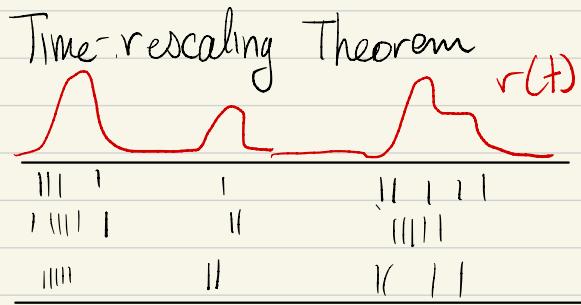


$$GLM : P(\text{spike at } t) \sim \exp(f_i s + h_i r)$$

*Sacrifice some generality for model that is complete, which in optimization is globally convergent.

if Neuron 2 present





exp. dist. of scaled ISI's

$$(t_{i+1} - t_i)r(t)$$