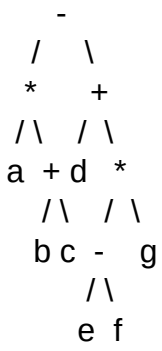
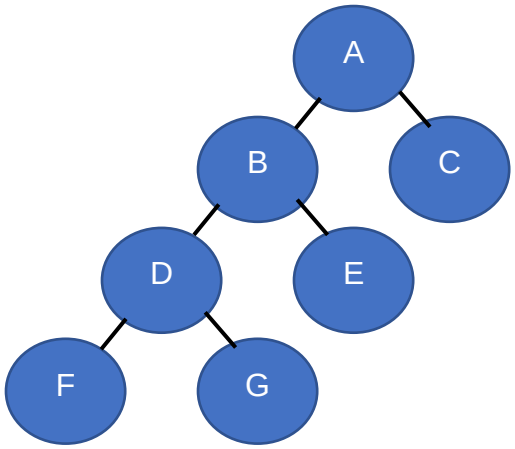
CS32 Final Study Guide Solutions

Trees

2. Draw the expression tree for the expression a*(b+c)-(d+(e-f)*g)



3. For the tree



a. What is the height of node A? **3**
b. What is the height of the tree? **3**
c. What is the depth of node E? **2**
d. What is the parent of node F? **D**
e. Name the descendent(s) of node B. **D, E, F, G**
f. Name the leaves in the tree. **C, E, F, G**

4. Write a C++ function `TreeNode* copy_tree(TreeNode* T)` that returns a copy of binary tree `T`

```
TreeNode* copy_tree(TreeNode *T) {

    if (T == nullptr) {
        return nullptr;
    }
```

```
    return new TreeNode(T->m_data,copy_tree(T->m_left),
copyTree(T->m_right));
}
```

5. Write a C++ function `TreeNode* expand_leaf(TreeNode* node, ItemType x, ItemType y)` that returns a new binary tree that is identical to the binary tree T except that every leaf in `T` now has a left child and a right child whose values are equal to `x` and `y`, respectively. For example, invoking `expand_leaf(T, 9, 12)` on the tree on the left produces the tree on the right.


```
TreeNode* expand_leaf(TreeNode* T, ItemType x, ItemType y)
{
  if (T == nullptr) return T;
  else {
     if (T->m_left == nullptr && T->m_right == nullptr){
        T->m_left = new TreeNode(x);
        T->m_right = new TreeNode(y);
     }
     if (T->m_left != nullptr)
        T->m_left = expand_leaf(T->m_left, x, y);
     if (T->m_right != nullptr)
        T->m_right = expand_leaf(T->m_right, x, y);
    return T; // return the (unchanged) pointer T
  }
}
```

6. Write a C++ function `int height(TreeNode* T)` that returns the height of the binary tree `T`

```
int height(TreeNode* T) {
  if (T == nullptr) return 0;
  else {
    int lHeight = height(T->left);
    int rHeight = height(T->right);
    if (lHeight > rHeight) return lHeight + 1;
    else return rHeight + 1; }
}
```

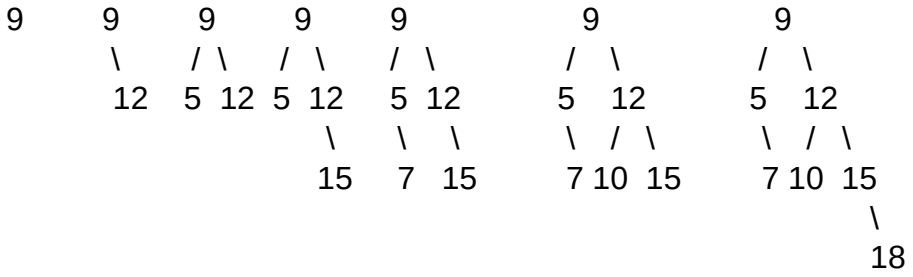
7. Write a C++ function `bool same_tree(TreeNode* T1, TreeNode* T2)` that returns true if binary trees `T1` and `T2` are exactly the same (same values, same structure) and returns false otherwise. You may assume that values of `ItemType` can be compared by means of the == operator.

```
bool sameTree(TreeNode* t1, TreeNode* t2) {
  if (t1 == nullptr && t2 == nullptr) return true;
  else if (t1 != nullptr && t2 != nullptr) {
    return t1->m_data == t2->m_data &&
      sameTree(t1->left, t2->left) &&
      sameTree(t1->right, t2->right);
  }
  else return false;
}
```

8. Write a C++ function `TreeNode* bst_insert(ItemType item, TreeNode* T)` that inserts `item` into the binary search tree `T`. After the insertion, `T` must remain a binary search tree. You may assume that `ItemType` values are comparable using the ==, <, and > C operators.

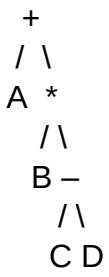
```
TreeNode* bst_insert(ItemType item, TreeNode* T) {
  if (T == nullptr) {
     T = new TreeNode();
     T -> m_data = item;
     T -> m_left = nullptr;
     T -> m_right = nullptr;
  }
  else {
     if (item <= T->m_data) T->m_left = insert(T->m_left,
item);
     else T->m_right = insert(T->m_right, item);
     return T; // return the (unchanged) pointer T
  }
}
```

Draw the binary search tree resulting from sequentially calling `bst_insert` on an initially empty binary tree with the values 9,12,5,15,7,10,18.



Tree Traversal

1. For the tree



   a. Write the node labels in pre-order order. **+ A * B – C D**
   b. Write the node labels in in-order order. **A + B * C – D**
   c. Write the node labels in post-order order. **A B C D – * +**

Searching

1. Given an array containing the sequence `1,5,29,45,67,76,92,104,187,234` (in that order)
   a. State each comparison made in finding the number 234 using linear search. (For example, 234:1 is a comparison of 234 with 1.)

      234:1, 234:5, 234:29, 234:45, 234:67, 234:76, 234:92, 234:104, 234:187, 234:234

   b. State each comparison made in determining that the number 48 is not present using linear search.

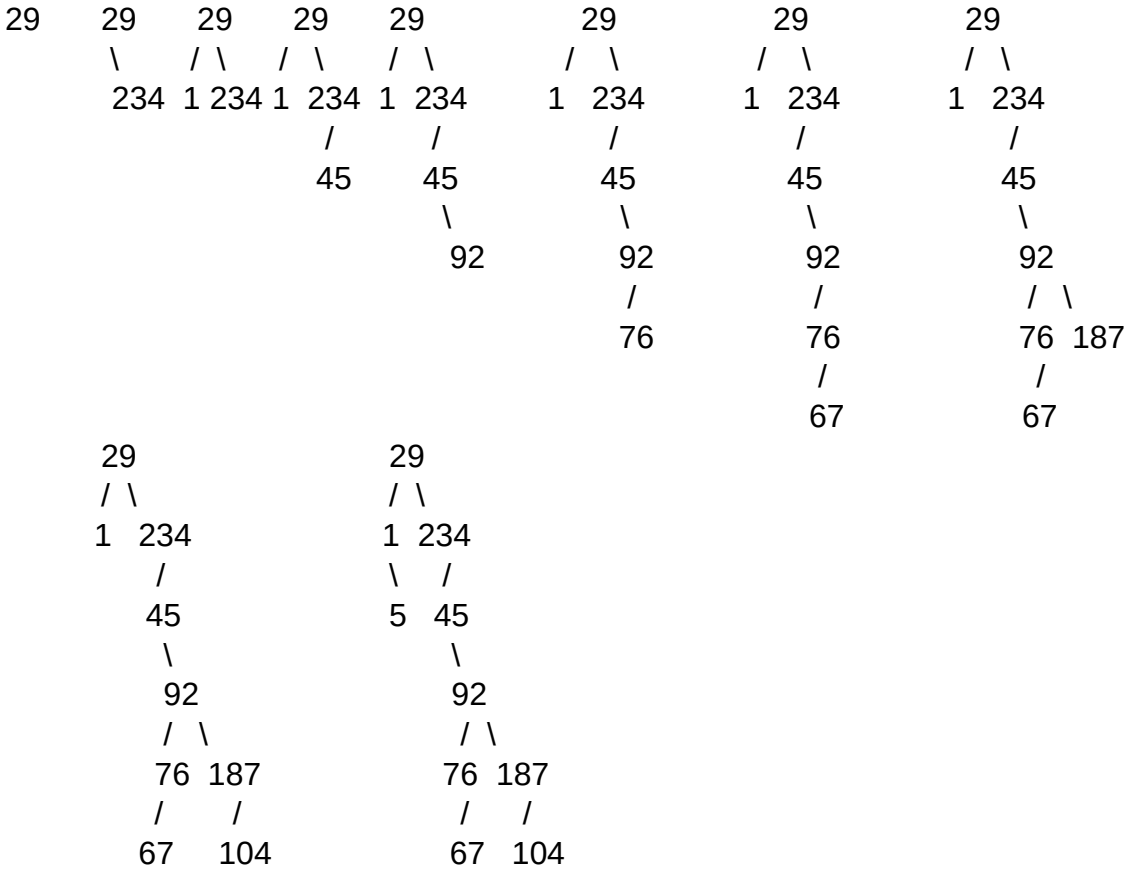
      48:1, 48:5, 48:29, 48:45, 48:67, 48:76, 48:92, 48:104, 48:187, 48:234

2. Given the following sequence of integers (for example, integers received from the keyboard)

   `29,234,1,45,92,76,67,187,104,5`

   (in that order)

   a. Draw the binary search tree that would result from inserting the integers in the order given.



   b. State each comparison made in finding the number 234 using binary search. (For example, 234:1 is a comparison of 234 with 1.)

      234:29, 234:234

   c. State each comparison made in determining that the number 48 is not present using binary search.

      48:29, 48:234, 48:45, 48:92, 48:76, 48:67

d. Write the integers as they would be encountered in an in-order traversal of the tree.

1, 5, 29, 45, 67, 76, 92, 104, 187, 234

3. Linear search can be done on data in an array or in a list. Name an advantage and a disadvantage of the list implementation over the array implementation.

An advantage is that the list implementation does not have a memory restriction, unlike an array implementation. However, a disadvantage of a list implementation is that it is not as simplistic to implement a list as it is an array.

4. Binary search can be done on data in a sorted array or in a binary search tree. Name an advantage and a disadvantage of the array implementation over the BST implementation.

An advantage of an array implementation of binary search is that the code is easier to implement than a BST. It is also a disadvantage in that the array has only a finite number of items you can have, unlike a BST implementation.

Sorting

1. How many permutations of N distinct items are there? **N!**
2. Explain why selection sort is $O(N^2)$ on average. **Selection sort is $O(N^2)$ on average because it takes typically N swap steps with for each of the N items.**
3. Explain why quicksort is O(N log N) on average, but $O(N^2)$ worst case. **Quicksort is typically O(N log N) when the data is randomly distributed. However, when the data is mostly or fully sorted, the worst case is $O(N^2)$.**
4. Given an array containing the integers `12 9 3 6 4 1` (in that order)
   a. Show the array as it is sorted by the selection sort

```
12 9 3 6 4 1
1 9 3 6 4 12
1 3 9 6 4 12
1 3 4 6 9 12
1 3 4 6 9 12
1 3 4 6 9 12
```

   b. Show the array as it is sorted by the mergesort

```
12 9 3 6 4 1
12 9 3    6 4 1
```

```
12   9 3  6    4 1
12   9    3    6    4    1
12   3 9  6    1 4
3 9 12    1 4 6
1 3 4 6 9 12
```

c. Show the array as it is sorted by the insertion sort

```
12 9 3 6 4 1
9 12 3 6 4 1
3 9 12 6 4 1
3 6 9 12 4 1
3 4 6 9 12 1
1 3 4 6 9 12
```

d. Show the array as it is sorted by the bubble sort

```
Iteration 1:
12 9 3 6 4 1
9 12 3 6 4 1 (SWAP)
9 3 12 6 4 1 (SWAP)
9 3 6 12 4 1 (SWAP)
9 3 6 4 12 1 (SWAP)
9 3 6 4 1 12 (SWAP)

Iteration 2:
9 3 6 4 1 12
3 9 6 4 1 12 (SWAP)
3 6 9 4 1 12 (SWAP)
3 6 4 9 1 12 (SWAP)
3 6 4 1 9 12 (SWAP)
3 6 4 1 9 12

Iteration 3:
3 6 4 1 9 12
3 6 4 1 9 12
3 4 6 1 9 12 (SWAP)
3 4 1 6 9 12 (SWAP)
3 4 1 6 9 12
3 4 1 6 9 12

Iteration 4:
3 4 1 6 9 12
3 4 1 6 9 12
3 1 4 6 9 12 (SWAP)
3 1 4 6 9 12
```

```
3 1 4 6 9 12
3 1 4 6 9 12

Iteration 5:
3 1 4 6 9 12
1 3 4 6 9 12 (SWAP)
1 3 4 6 9 12
1 3 4 6 9 12
1 3 4 6 9 12
1 3 4 6 9 12

Iteration 6:
1 3 4 6 9 12
1 3 4 6 9 12
1 3 4 6 9 12
1 3 4 6 9 12
1 3 4 6 9 12
1 3 4 6 9 12
```

5. Write a templated version of both bubble and selection sort.
    a. As the numbers of elements to be sorted increases how do the two sorts compare to one another? **They are both $O(n^2)$**

Asymptotic Analysis

1. Put in increasing order based on the following set: $O(n^3)$, $O(2^n)$, $O(n)$, $O(n \log n)$, $O(\log n)$, $O(1)$, $O(n^2)$ **$O(1)$, $O(\log n)$, $O(n)$, $O(n \log n)$, $O(n^2)$, $O(n^3)$, $O(2^n)$**
2. Linear search can be done on data in an array or in a list. What is the average time complexity for successful search using an array? **O(n)** Using a list? **O(n)**
3. Binary search can be done on data in a sorted array or in a binary search tree. What is the average time complexity for successful search using the array? Using the BST? **O(log n)** What is the worst case time complexity for each? **O(n), which can happen if the data inserted is in fully ascending or descending order**

Hash Tables

1. Identify the differences between open and closed hashing

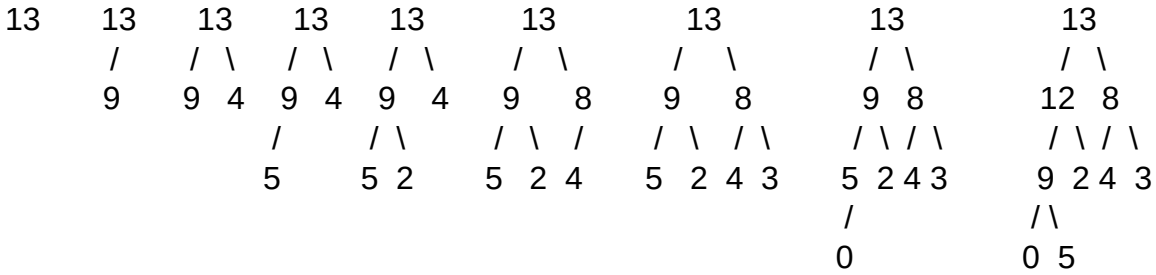
    Closed hashing uses an array, while open hashing uses an array of pointers to linked lists. Closed hashing must be sized appropriately, while open hashing does not have to worry about memory allocation ahead of time.

2. Given the following set `12 9 3 6 4 1` (in that order), in a Closed – Linear Probing hash table, if we apply the hash function h(x) = value % 11, where would 1 be placed? **2**

Array index	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Key value		12	1	3	4		6			9	

Heaps

3. Show a maxheap after inserting each of the following numbers:
`13,9,4,5,2,8,3,0,12`



4. Remove the two biggest numbers from the maxheap, showing the heap structure after each deletion.