Discussion: 20.10.2025 until 24.10.2025

1. Conversions between units

Calculate the following expressions explicitly in the SI system:

(a) $\hbar c$ (also in units of MeV fm).
Use Heisenberg's uncertainty principle or the relation for the de Broglie wavelength to establish a relationship between energies in MeV and lengths in fm.

(b) the de Broglie wavelength $\lambda = h/p$ of an electron, accelerated from rest by a potential difference of 2 MV.

(c) Assume a proton in the LHC collider – accelerated to an energy of 6.5 TeV – competes in a race against a beam of light. By what speed difference (in km/h) are the LHC protons slower than the photons of the light beam?
Hint: You can use a Taylor expansion for an occurring square root to reduce the numerical calculation error.

2. Kinematics

At the HERA storage ring, electrons with an energy of 27.5 GeV collided head-on with protons circling in the opposite direction with an energy of 920 GeV.

(a) Calculate the total energy in the centre-of-mass system of the electron and proton.

(b) What energy do electrons that collide with a proton at rest need in order to achieve the same centre-of-mass energy as in part (a)?

3. Cross section and luminosity

Consider the production of $\mu^+\mu^-$-pairs using the example of the electron–positron collider LEP (operation: 1989–2000, maximum beam energy: 109.5 GeV).

The differential cross section for the process $e^+e^- \to \mu^+\mu^-$ mediated by a virtual photon is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cdot \hbar^2}{4 \cdot s \cdot c^2}(1 + \cos^2\theta),$$

with the approximate assumption $m_e = 0$. The detection of $\mu^+\mu^-$-pairs takes place on a sphere of radius $R = 1\,\text{m}$ (assuming the detection efficiency for μ^+ and μ^- is 100 % on the entire sphere).

(a) Assuming a beam energy of 40 GeV, how large does the storage-ring luminosity need to be in order to register at least one muon or anti-muon per cm^2 per hour on average on the entire detector surface?
Hint: For simplicity just use the minimum of the differential cross section.

(b) How high are the electric currents of the particle beams at the typical LEP luminosity of $\mathcal{L} = 20 \cdot 10^{30}\,\text{cm}^{-2}\text{s}^{-1}$ (Assumptions: $I_{e^+} = I_{e^-}$, two particle bunches each, beam dimensions: $\sigma_x = 13\,\mu\text{m}$, $\sigma_y = 430\,\mu\text{m}$, ring circumference: 26.6 km)? How many electrons (positrons) are in each particle bunch?

4. Rutherford (Geiger-Marsden) Scattering

At the institute of Ernest Rutherford in Manchester back in 1909, Hans Geiger and Ernest Marsden investigated the scattering of α particles on thin foils of gold ($^{197}_{79}$Au, density: $\rho_{\mathrm{Au}} = 19.29\,\mathrm{kg/dm^3}$). The α particles created in the decay of Radium-C (^{214}Bi) had a kinetic energy of about $E_{\mathrm{kin}} = 5\,\mathrm{MeV}$. These measurements were described by Rutherford's atomic model in 1911.

(a) At which scattering angles do the nuclei reach the smallest possible distance? How large is the kinetic energy of the α particles at the turning point?

(b) Calculate the smallest possible distance d_{min} of nuclei centres in the scattering!

(c) Geiger and Marsden observed that when scattering off gold foils with a thickness of $d = 0.4\,\mu\mathrm{m}$ roughly one in 20000 α particles were scattered off by an angle of more than 90°. Calculate the exact rate using the Rutherford scattering formula. Use the integrated cross section for scattering angles larger than ϑ:

$$\sigma = \pi b^2 \text{ with } b = \frac{1}{4\pi\epsilon_0}\frac{zZe^2}{E_{\mathrm{kin}}}\frac{1}{2\tan\vartheta/2}$$