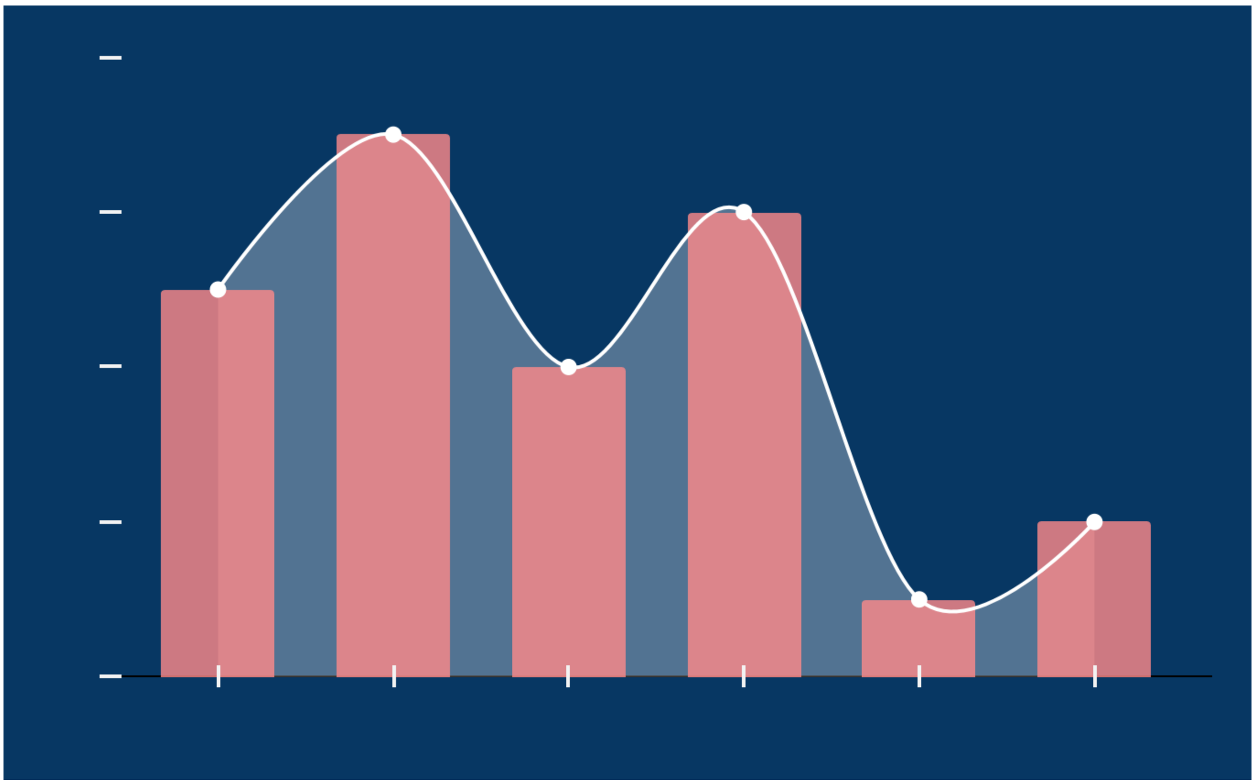
Post Conflict Environmental Assessment: Section IV (HW8)

Combat Zone Lake Nitro-Explosive Compound Content Analysis



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INTRODUCTION

Due to limitations in sensor technology, it's not unusual for certain data points below or above some specified detection limit to be measured with low accuracy, and thus censored for reporting purposes. To exhibit a method for estimation on censored data sets, we've taken samples from 3 lakes close to the border, and we'll be testing for mean concentrations of Nitro-Explosive Compounds (residue compounds from nitrogen based explosives).

Methodology

Because much of the data is censored, we use a Maximum-Likelihood Estimator that assumes the data is Exponentially distributed, then we formulate a Hypothesis using the Fisher Information of the estimator.

Analysis Part I - The Maximum-Likelihood Estimate

The Maximum-Likelihood Estimate is obtained by taking the derivative of the log of the likelihood function, then finding the maxima of that function. Here is the Likelihood function for assumed exponential data with "Left Censored Data", or in this case data below a Lower detection limit.

$$Left\ Censored\ Likelihood\ Function\ =\ [1 - e^{\frac{-c}{\theta}}]\ \cdot \frac{1}{\theta^{n-\pi}} e^{\frac{\sum_{i=\pi+1}^{n} x_i}{\theta}}$$

Here, C = the threshold of the detection limit. And pi is the number of data points that are censored. When data is right-censored, or both left and right censored, you adjust the function such that there is a separate term representing the probability distribution of each interval of the data, and for each the products are taken over their respective number of entries.

For our first sample with Left-Censored data, after taking the derivative of the log of the likelihood function, the equation can be transformed to the following form, from which point we can use an Iterative Method as a root finding technique.

$$11.7 = \theta + \frac{9}{11} \cdot 3\left[\frac{e^{\frac{-3}{\theta}}}{1-e^{\frac{-3}{\theta}}}\right]$$

The iterative method for this entails choosing an estimate of theta < 11.7 , then using it to evaluate the bracketed section, then solving for theta from there to get a new estimate. Then over and over until the estimates converge. This was done with a script in python, and the answer can be seen in the results section.

For Our second dataset, with Right censored data, we are conveniently able to solve for MLE-theta analytically. After taking the derivative of the log-likelihood function and setting it to zero, some algebraic manipulation gives us the following formula. (The answer will be in the results section)

$$\frac{\sum_{i=1}^{13} x_i + 49}{13} = \hat{\theta}_{mle}$$

For our final Dataset with both Left and Right Censored data, we return to the Iterative method, on the following equation.

$$76.689418 = 4\theta + 24\left[\frac{e^{\frac{-3}{\theta}}}{1-e^{\frac{-3}{\theta}}}\right]$$

Unlike with the first data set, the given Iterative Method did not return converging estimates. As a replacement, we used the Wolfram Alpha root finder which uses Newton-Raphson, bisection, and secant methods. Once again the Answer is delivered in the results section.

Analysis II - Mean and Variance

RESULTS

Table1
Maximum-Likelihood Estimates for Theta

Dataset	Iteration(s)	Precision	Estimate
1	25	0.01	7.063
2	N/A	N/A	6.456
3	?	?	7.32

CONCLUSION

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