COMPUTATION IN SCIENCES-2 · 30 March 2020

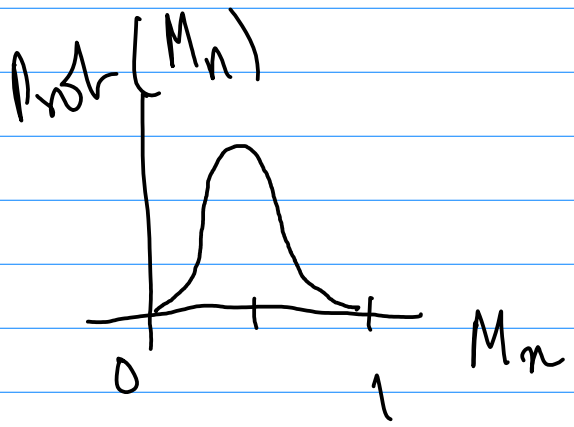
COIN TOSS PROBLEM

$p = p(H)$, $q = p(T)$

1. Fair coin; $p(H) = p(T) = 1/2$.
2. 'N' tosses, independent and identical
3. M_n = fraction of n tosses which are heads.
4. $0 \leq M_n \leq 1$.
5. $\text{Prob}(x \leq M_n < y) = \sum_{r > xn,\ r < yn} P(r|n)$

$P(r|n) = \frac{n!}{r!(n-r)!} p^r q^{n-r}$

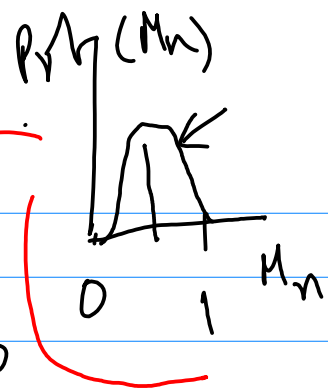
"scipy.misc.comb"

6. 

FAIR COIN $\langle M_n \rangle = \frac{1}{2}$

GAUSSIAN

TASK-1: Compute $P_n(M_n)$ vs M_n ←



$n = 10$
$n = 20$
30
⋮
100

$\sum P_n(M_n) = 1$ for fixed n

$P(M_n)$ is Gaussian

TASK-2: scipy.optimize: curve-fit

$P_n(M_n > 0.9)$ for a range of n.

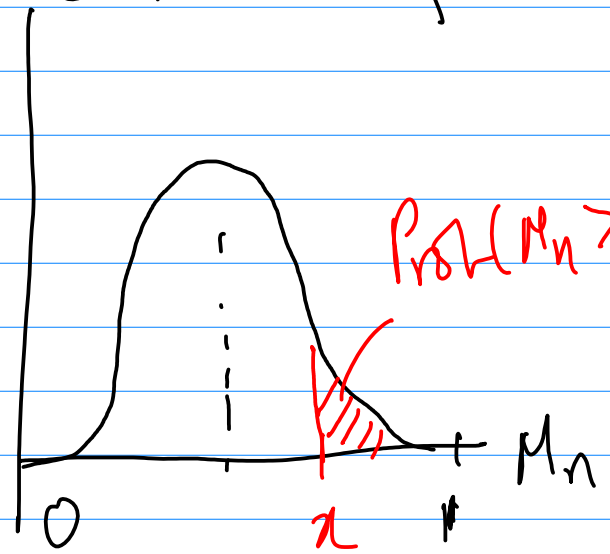
$P_n(M_n > x)$

$x = 0.9$
$x = 0.8$

$y_M(T)$ for RW with trap $x = M$

n

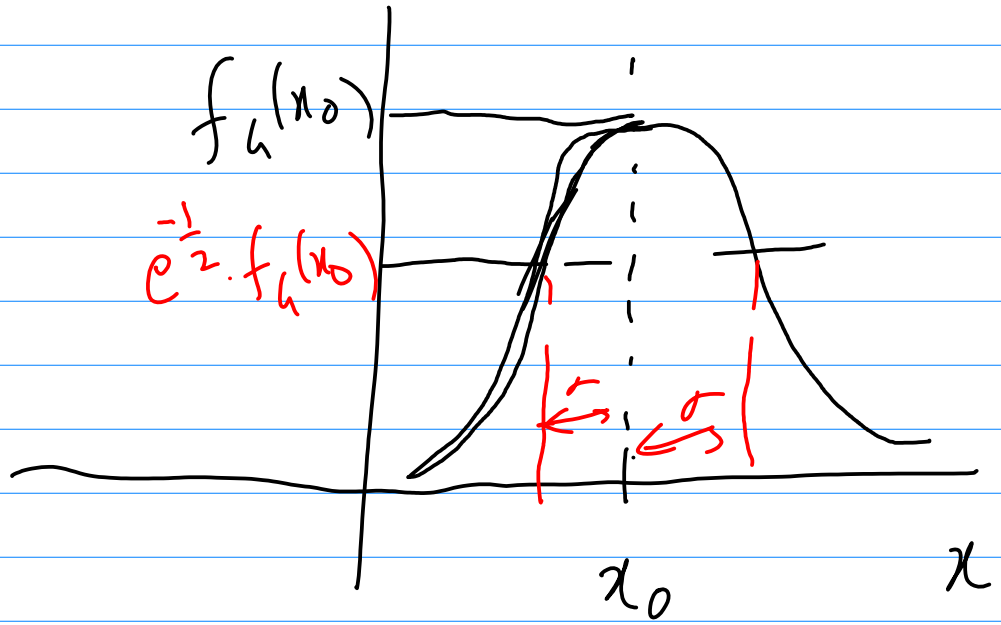
$P_n(M_n)$



Gaussian fn: $f_G(x) = C\, e^{-(x-x_0)^2/2\sigma^2}$

$C(\sigma)$

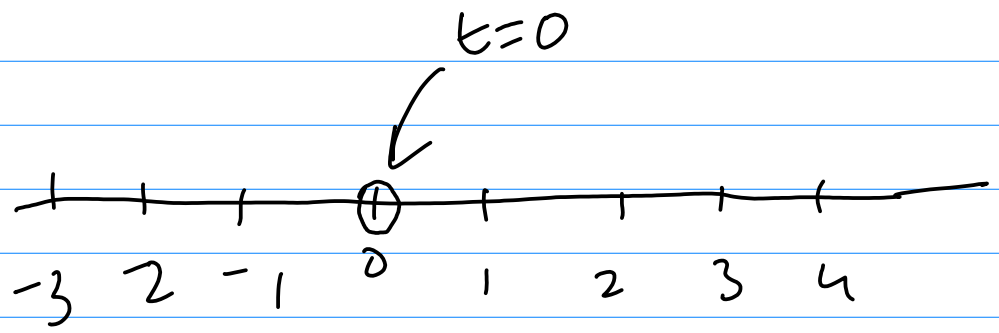
$f_G(x \mid x_0, \sigma)$



$x - x_0 = \pm\sigma$

$f_G(x) = f_G(x_0)$

RANDOM WALK · 1D.



$x(t=0) = 0$

$\rightarrow x(t+1) = x(t) + s(t)$

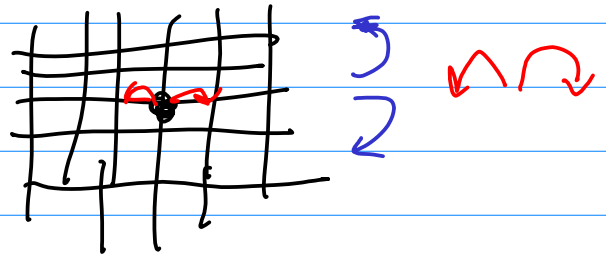
$s(t) = \delta_{c(t),H} - \delta_{c(t),T}$ ← Cointoss

$\delta_{i,j} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$

$\begin{cases} 1 \text{ if } c(t) = \text{Heads} \\ -1 \text{ if } c(t) = \text{Tails} \end{cases}$

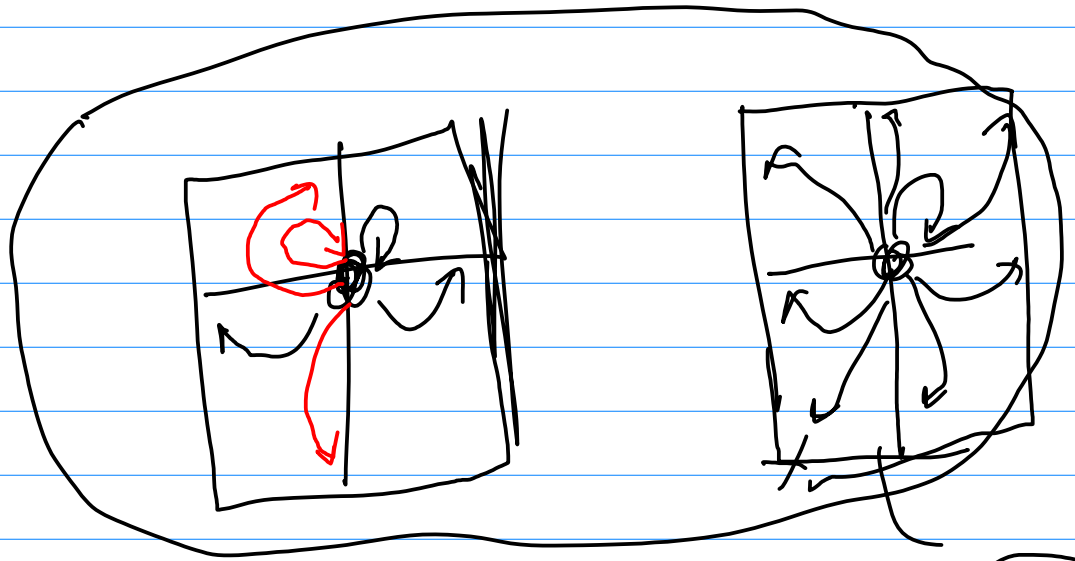
2-D. Red coin ⇔ Left / Right

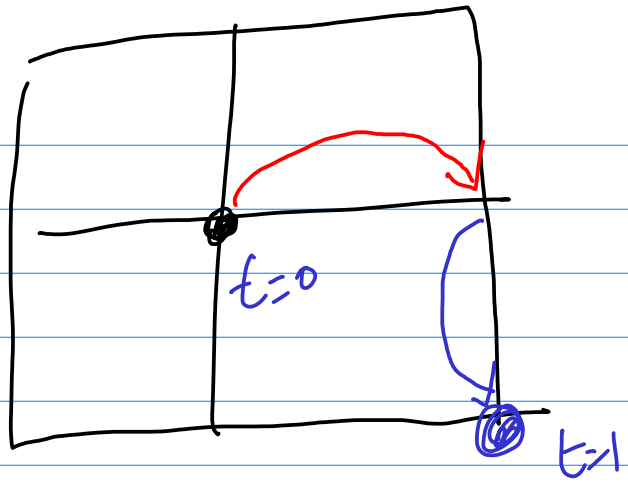
Blue coin ⇔ up / down.

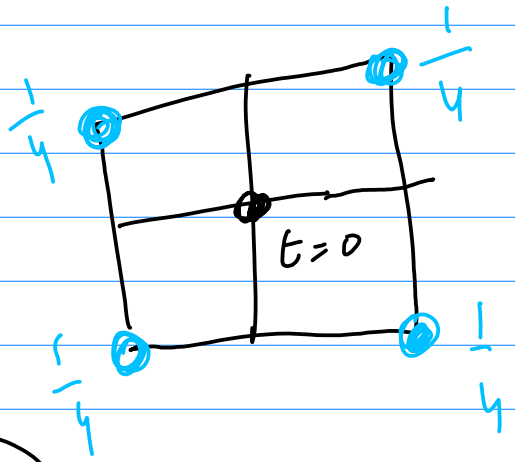


Task-4 : Random Walk

x(t) as fn t





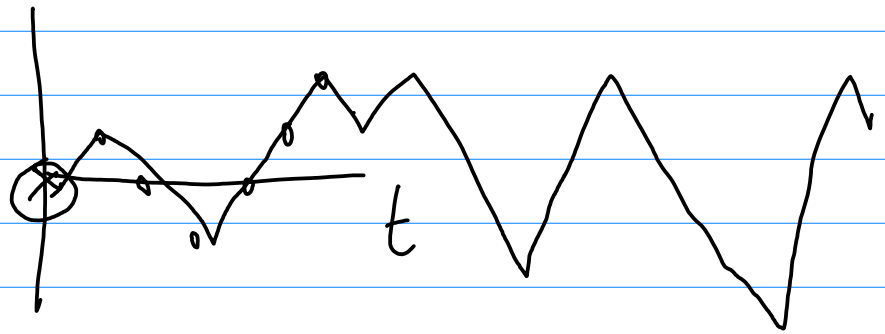


Die - 3 events -1, 0, +1

1 position

1/4

x(t)



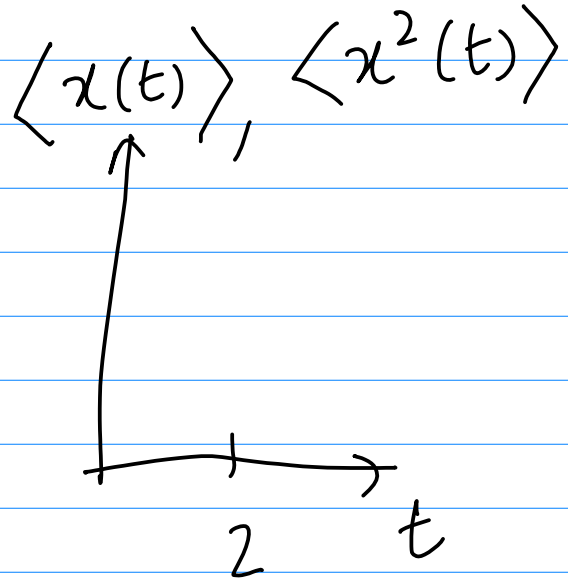
Task 5. Statistics of Random Walk.

$$\begin{cases} x(t+1) = x(t) + \delta_{(t),H} - \delta_{(t),T} \\ x(0) = 0 \end{cases}$$

Traj $x(t)$

(5.1) $\langle x(t) \rangle$ vs t.

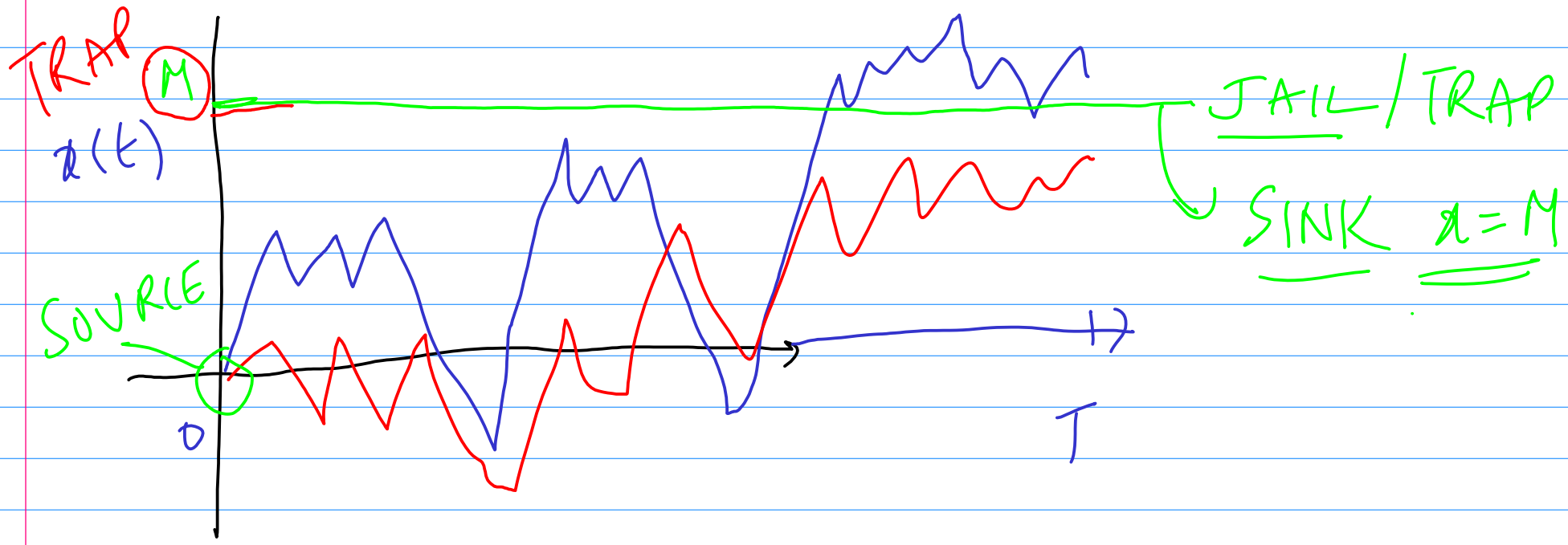
(5.2) $\langle x^2(t) \rangle$ vs t.



"Error in your calculation" $M \to \infty$

(M) trajectories of length t

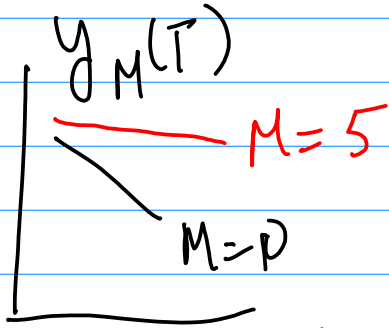
$x(t)$, given that $x(t=0) = 0$, $t \in [0, T]$



Task 6 Random walk with trap at $x = M$

Prob (T, trapped)?
↑
time length

$y_M(T)$



$y_M(T)$ vs T for various M