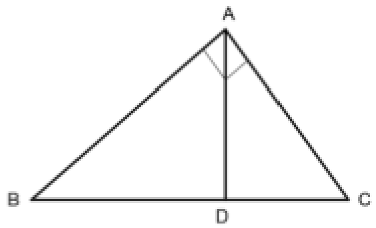
- **State and Prove that Pythagoras Theorem:**

OR

- **State and Prove that Baudhayan Theorem:**

It States that

"In right angle triangle, Square of length of the hypotenuse is equal to the sum of square of length of other two sides.



Given Data: $\angle A$ is a right angle in ΔABC.

To Prove: $BC^2 = AB^2 + AC^2$

Proof: Let $\overline{AD} \perp \overline{BC}$, D $\in \overline{BC}$

$\angle B$ and $\angle C$ are acute angle in ΔABC.

$\therefore B - D - C$ $\qquad$ ($\because$ ***Co-linear***)

$\therefore BC = BD + DC$ $\qquad$ **(1)**

Now, Using Corollary

$\therefore AB^2 = BD \times BC$ and $AC^2 = DC \times BC$

$\therefore AB^2 + AC^2 = BD \times BC + DC \times BC$

$= BC(BD + DC)$ $\qquad$ [***From 1***]

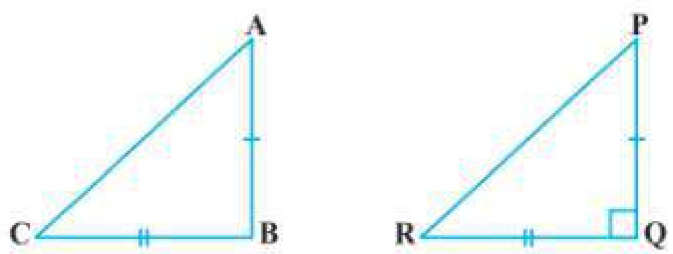
$= BC^2$

Hence Proved

- **Converse of Pythagoras Theorem:**

It States that

"In a triangle, if square of one side is equal to the sum of squares of the other two sides, then the angle opposite to the first side is a right angle.



s

Given Data: $AC^2 = AB^2 + BC^2$

To Prove: $\angle B$ is a right angle in ΔABC.

Proof: We can construct a ΔPQR right angled at Q such that $PQ = AB$ and $QR = BC$.

Now, from ΔPQR, we have:

$PR^2 = PQ^2 + QR^2$ (Pythagoras Theorem, as $\angle Q = 90°$)

Or,

$PR^2 = AB^2 + BC^2$ $(By\ Construction)$ (1)

But $AC^2 = AB^2 + BC^2$ $(Given)$ (2)

So, AC = PR $[From\ (1) \&\ (2)]$ (3)

Now, in ΔABC and ΔPQR,

AB = PQ $(By\ Construction)$

BC = QR $(By\ Construction)$

AC = PR $[Proved\ in\ (3) above]$

So, $\Delta ABC \cong \Delta PQR$ $(SSS\ Congruence)$

Therefore, $\angle B = \angle Q$ $(CPCT)$

But $\angle Q = 90°$ $(By\ Construction)$

So, $\angle B = 90°$

Hence Proved

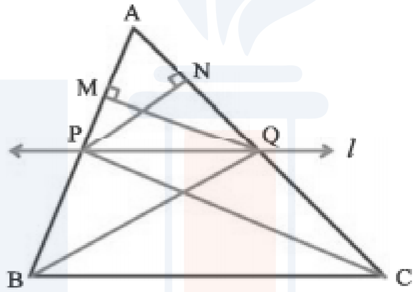
- **Basic Proportionality Theorem OR Thales Theorem**

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: In the plane of ΔABC, a line $l || \overline{BC}$ and l intersects $\overline{AB}$ and $\overline{AC}$ at points P and Q respectively.

To Prove: $\frac{AP}{PB} = \frac{AQ}{QC}$

Proof: Let $\overline{QM} \perp \overline{AB}$, and $\overline{PN} \perp \overline{AC}$. Construct $\overline{BQ}\ and\ \overline{CP}$.



Area of triangle = $\frac{1}{2} \times base \times altitude$

$\therefore Area\ of\ \Delta APQ = \frac{1}{2} AP \times QM$

$Area\ of\ \Delta PBQ = \frac{1}{2} PB \times QM$

$\therefore \frac{Area\ of\ \Delta APQ}{Area\ of\ \Delta PBQ} = \frac{\frac{1}{2} AP \times QM}{\frac{1}{2} PB \times QM} = \frac{AP}{PB}$ **(i)**

Also Area of $\Delta APQ = \frac{1}{2} AQ \times PN$

Area of $\Delta CPQ = \frac{1}{2} QC \times PN$

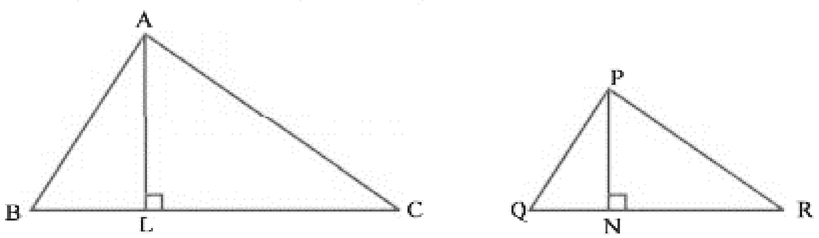
$\therefore \frac{Area\ of\ \Delta APQ}{Area\ of\ \Delta PCQ} = \frac{\frac{1}{2} AQ \times PN}{\frac{1}{2} QC \times PN} = \frac{AQ}{QC}$ **(ii)**

$\Delta PBQ\ and\ \Delta PCQ$ are having common base $\overline{PQ}$ and they are lying between two parallel lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{BC}$

Area of $\Delta PBQ = Area\ of\ \Delta PCQ$ **(iii)**

From $(i), (ii)\ and\ (iii) \frac{AP}{PB} = \frac{AQ}{QC}$.

- **Areas of two similar triangles are proportional to squares of corresponding sides.**



Given: Correspondence ABC $\leftrightarrow PQR\ of\ \Delta ABC\ and\ \Delta PQR\ is\ a\ similarity.$

To prove: $\frac{ABC}{PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

Proof: Draw altitudes $\overline{AL}\ and\ \overline{PN}$.

The correspondence ABC $\leftrightarrow PQR$ is a similarity. **(AA)**

$\therefore \angle B \cong \angle Q$

And $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ gives $\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ **(i)**

In ΔABL and ΔPQN,

$\angle B \cong \angle Q$

$\angle ALB \cong \angle PNQ$ **(Right Angle)**

$\therefore$ The correspondence $ABL \leftrightarrow PQN$ is a similarity.

$\therefore \frac{AB}{PQ} = \frac{AL}{PN}$

$\therefore \frac{AL}{PN} = \frac{AB}{PQ} = \frac{BC}{QR}$ **(ii)**

Now, area of triangle $= \frac{1}{2} base \times altitude$

$$\frac{ABC}{PQR} = \frac{\frac{1}{2} BC \cdot AL}{\frac{1}{2} QR \cdot PN}$$

$$= \frac{BC}{QR} \times \frac{AL}{PN} \quad = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2}$$ **[Using (ii)]**

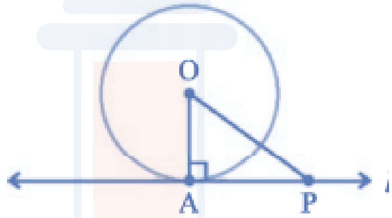
$$\therefore \frac{ABC}{PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

- **A tangent to a circle is perpendicular to the radius drawn from the point of contact.**

Given:- Line *l* is tangent to the circle with centre O radius *r* at point A.

To Prove: $\overline{OA} \perp l$

Proof: Let $P \varepsilon l, P \neq A$.



If P is in the exterior of circle with centre O radius *r*, then the line *l* will be a secant of the circle and not a tangent. But *l* is a tangent of the circle, so P is not in the interior of the circle. Also $P \neq A$.

$\therefore$ P is the point in the exterior of the circle.

$\therefore$ OP > OA. ($\overline{OA}$ is the radius of the circle)

Therefore each point $P \varepsilon l$ except A satisfies the inequality OP > OA.

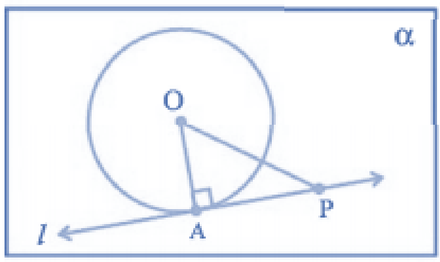
Therefore OA is the shortest distance of line *l* from O.

$\overline{OA} \perp l$

Hence Proved

- **If a line is in the plane of a circle such that it is perpendicular to the radius of the circle at its end point on the circle, then the line is a tangent to the circle.**

In the figure line *l* and circle with centre O and radius *r* in plane α and the line *l* is perpendicular to radius $\overline{OA}$ at the end point A which is on the circle.



If P is any point on *l*, then

OA < OP because $\overline{OA} \perp l$

$\therefore$ OP > OA. Therefore OP > *r*

Therefore all point like P on *l* are in the exterior of circle with centre O radius *r*.

$\therefore$ Line *l* intersect the circle with centre O radius *r*. Hence *l* is a tangent to the circle at O.

Hence Proved