COLLABORATIVE SPARSE UNMIXING OF HYPERSPECTRAL DATA USING L2,P NORM

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ABSTRACT

Sparse unmixing is a popular method in remotely sensed hyperspectral imagery interpretation. Recently, the collaborative sparse unmixing model has shown its advantage over the traditional single channel sparse unmixing method since it can utilize the subspace nature of the high-dimensional hyperspectral data to alleviate the difficulty caused by the usually high mutual coherence of the spectral library. However, the existing collaborative sparse unmixing model is constructed in the convex ℓ_1 norm framework while it is known that using the ℓ_p $(0 < p < 1)$ norm can find a sparser solution. In this paper, we propose a collaborative sparse unmixing model and a new algorithm based on the $\ell_{2,p}$ $(0 < p < 1)$ norm. Experimental results on both synthetic and real data demonstrate the effectiveness of the $\ell_{2,p}$ $(0 < p < 1)$ norm collaborative sparse unmixing model.

Index Terms— Hyperspectral unmixing, sparse unmixing, collaborative sparse regression, $\ell_{2,p}$ norm.

1. INTRODUCTION

Due to the limited spatial resolution of the imaging sensors as well as the combination of different materials into one homogeneous mixture, hyperspectral unmixing , which decomposes the mixed pixels into the constituent spectra (endmembers) and their corresponding fractions (abundances), is an important preprocessing step for many hyperspectral imagery applications [1, 2]. For the past a few years, with the linear mixing model, researchers have developed unsupervised and semi-supervised several unmixing approaches [3, 4, 5].

Most recently, by using the subspace nature of the hyperspectral data which means all the pixels in the hyperspectral imagery share the same active set of endmembers included in the spectral library [6, 7, 8], the collaborative sparse unmixing with $\ell_{2,1}$ norm[1] (CLSUnSAL) [9] has received increasing attention due to its superiority over the single channel sparse unmixing algorithms as well as that it is usually combined with the simultaneous greedy algorithms or array processing algorithms to get the state-of-art performance. It is worth mentioning that the CLSUnSAL algorithm is constructed in the convex ℓ_1 norm framework. However, recent studies have shown that using the ℓ_p $(0 < p < 1)$ norm[2] can find sparser solution than using the ℓ_1 norm [10, 11]. The ℓ_p $(0 < p < 1)$ norm is used for sparse unmixing in [12] and shows its advantage in abundance estimation over the ℓ_1 norm algorithm. Thus, it is natural to expect that the $\ell_{2,p}$ $(0 < p < 1)$ norm[3] could be a better alternative of the $\ell_{2,1}$ norm for collaborative sparse unmixing. In this paper, we consider the $\ell_{2,p}$ $(0 < p < 1)$ norm in the collaborative sparse unmixing framework. Then, based on the gradient descent method and the multiplicative update rules, we develop a unified algorithm to solve the new model. Experiments on both synthetic and real data demonstrate that the proposed algorithm outperforms the CLSUnSAL algorithm.

2. A BRIEF INTRODUCTION TO COLLABORATIVE SPARSE UNMIXING MODEL

Suppose that $\mathbf{A} \in R^{L\times m}$ is the given spectral library, where L is the number of spectral bands and m is the number of spectral signatures in the library, and the hyperspectral data set contains K pixels organized in the matrix $\mathbf{Y} \in R^{L\times K}$.

$$\mathbf{Y} = \mathbf{AX} + \mathbf{N} \tag{1}$$

where $\mathbf{X} \in R^{m\times K}$ is the abundance matrix each column of which corresponds with the abundance fractions of a mixed pixel and $\mathbf{N} \in R^{L\times K}$ is the matrix of error term. The collaborative sparse unmixing model assumes that all the pixels in the hyperspectral scene can be constructed by the same active

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[1]The $\ell_{2,1}$ norm of matrix $\mathbf{X} \in R^{m\times K}$ is defined as $\|\mathbf{X}\|_{2,1} = \sum_{k=1}^{m} \|\mathbf{x}^k\|_2$, where $\mathbf{x}^k$ is the k-th row of $\mathbf{X}$.

[2]The ℓ_p $(0 < p < 1)$ norm of a vector $\mathbf{x} \in R^m$ is defined as $\|\mathbf{x}\|_p = (\sum_{i=1}^{m} |x_i|^p)^{\frac{1}{p}}$, where x_i is the i-th element of vector $\mathbf{x}$.

[3]The $\ell_{2,p}$ $(0 < p < 1)$ norm of a matrix $\mathbf{X} \in R^{m\times K}$ is defined as $\|\mathbf{X}\|_{2,p} = (\sum_{k=1}^{m} \|\mathbf{x}^k\|_2^p)^{\frac{1}{p}}$, where $\mathbf{x}^k$ is the k-th row of $\mathbf{X}$.

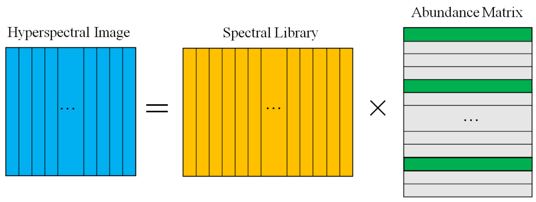

Fig. 1. Illustration of the effect of the $\ell_{2,p}$ $(0 < p < 1)$ norm regularizer. There should be only a few non-zero rows, which are represented in green color, in the abundance matrix. The gray rows in the abundance matrix correspond with the abundances of the non-active members in the spectral library.

subset of the endmembers selected from the spectral library, which means the abundance matrix $\mathbf{X}$ should have only a few nonzero rows. However, the optimization problem of the collaborative sparse unmixing model using the row-ℓ_0 quasi norm is NP-hard. In [9], the row-ℓ_0 quasi norm is relaxed to the $\ell_{2,1}$ mixed norm, i.e. $\sum_{i=1}^{m} \|\mathbf{x}^i\|$. Thus, with an appropriate Lagrange multiplier, the optimization problem becomes:

$$\min_{\mathbf{X}} \frac{1}{2}\|\mathbf{AX} - \mathbf{Y}\|_F^2 + \lambda\|\mathbf{X}\|_{2,1}$$
$$\text{subject to : } \mathbf{X} \geq 0 \tag{2}$$

The CLSUnSAL algorithm [9] has been proposed to solve the problem in Eq. (2).

3. COLLABORATIVE SPARSE UNMIXING USING $\ell_{2,P}$ NORM

As mentioned, the $\ell_{2,p}$ $(0 < p < 1)$ norm could be a better alternative of $\ell_{2,1}$ norm for collaborative sparse unmixing. Thus, the following optimization problem becomes:

$$\min_{\mathbf{X}} g(\mathbf{X}) = \frac{1}{2}\|\mathbf{AX} - \mathbf{Y}\|_F^2 + \lambda\|\mathbf{X}\|_{2,p}^p \tag{3}$$
$$\text{subject to: } \quad \mathbf{X} \geq 0$$

where $\|\mathbf{X}\|_{2,p}^p = \sum_{k=1}^{m} \|\mathbf{x}^k\|_2^p$ and $\lambda > 0$ is the regularization parameter, which controls the row-sparsity of the abundance matrix. Fig. 1 illustrates the effect of the $\ell_{2,p}$ $(0 < p < 1)$ norm regularizer in the proposed model.

Note that the $\ell_{2,p}$ $(0 < p < 1)$ norm problem in Eq. (3) is not convex. Thus, we can only get a local optimum and the many popular convex optimization techniques can not be used. Inspired by the multiplicative update rules, we apply the gradient descent algorithm to solve the $\ell_{2,p}$ $(0 < p < 1)$ norm problem in Eq. (3).

For convenience, assume $\phi(\mathbf{X}) \equiv \lambda\|\mathbf{X}\|_{2,p}^p$. The derivative of $\phi(\mathbf{X})$ with regard to $\mathbf{X}$ can be obtained as:

$$\bigtriangledown\phi(\mathbf{X}) = \frac{\partial\phi(\mathbf{X})}{\partial\mathbf{X}} = \lambda\mathbf{D}\mathbf{X} \tag{4}$$

where

$$\mathbf{D} = \text{diag}\{\frac{p}{\|\mathbf{x}^1\|_2^{2-p}}, \frac{p}{\|\mathbf{x}^2\|_2^{2-p}}, \cdots, \frac{p}{\|\mathbf{x}^m\|_2^{2-p}}\}. \tag{5}$$

Then, the derivative of the objective function $g(\mathbf{X})$ in Eq. (3) can be computed easily:

$$\bigtriangledown g(\mathbf{X}) = \mathbf{A}^T\mathbf{AX} - \mathbf{A}^T\mathbf{Y} + \bigtriangledown\phi(\mathbf{X}) \tag{6}$$

The gradient descent algorithm that minimizes $g(\mathbf{X})$ can be formulated as:

$$\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} - \mathbf{S}^{(k+1)}.*\bigtriangledown g(\mathbf{X}^{(k)}) \tag{7}$$

where $\mathbf{S}^{(k+1)}$ denotes the step size and $.*$ means elementwise multiplication. To guarantee that the nonnegativity constraint in Eq. (3) is satisfied and the objective function $g(\mathbf{X})$ is non-increasing under the update rules in Eq. (7), we define the step size as:

$$\mathbf{S}^{(k+1)} = \mathbf{X}^{(k)}./[\mathbf{A}^T\mathbf{AX}^{(k)} + \bigtriangledown\phi(\mathbf{X}^{(k)})] \tag{8}$$

where $./$ denotes the elementwise division. Finally, putting Eqs. (6) and (8) into Eq. (7), we get:

$$\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)}.*(\mathbf{A}^T\mathbf{Y})./[\mathbf{A}^T\mathbf{AX}^{(k)} + \bigtriangledown\phi(\mathbf{X}^{(k)})] \tag{9}$$

Then, the $\ell_{2,p}$ $(0 < p < 1)$ norm problem in Eq. (3) can be solved.

4. EXPERIMENTS

In this section, we conduct both synthetic and real data experiments comparing with the CLSUnSAL algorithm [9] to see whether the $\ell_{2,p}$ norm could be a better alternative of the $\ell_{2,1}$ norm for collaborative sparse unmixing. Different values of p are considered for the $\ell_{2,p}$ norm: 0.5, 0.2, 0.05.

The root mean square error (RMSE) is used to evaluate the abundance estimations. For the i-th endmember, RMSE is defined as

$$\text{RMSE}_i = \sqrt{\frac{1}{K}\sum_{j=1}^{K}(\mathbf{X}_{ij} - \hat{\mathbf{X}}_{ij})^2} \tag{10}$$

where $\mathbf{X}$ represents the true abundances and $\hat{\mathbf{X}}$ denotes the estimated ones. The mean value of all the endmembers' RMSEs will be calculated. Smaller RMSE means more accurate estimation.

4.1. Synthetic Data Experiment

In our synthetic data experiment, we evaluate the performances of the collaborative sparse unmixing algorithms in situations of different signal-to-noise ratios ($\text{SNR} \equiv 10\log_{10}\frac{\|\mathbf{AX}\|_2^2}{\|\mathbf{n}\|_2^2}$) of noise. Specifically, the synthetic data are corrupted by Gaussian white noise with different levels of SNR: 20, 30 and 40 dB.

The spectral library we use in our synthetic experiment is a subset of the United States Geological Survey (USGS) [13]. Specifically, the spectral library $\mathbf{A} \in R^{224\times 240}$ contains 240 spectral signatures whose reflectance values are measured for 224 spectral bands. Six spectral signatures are chosen from $\mathbf{A}$ to generate the synthetic hyperspectral image: Axinite HS342.3B, Almandine HS114.3B, Acmite NMNH133746, Staurolite HS188.3B, Zoisite HS347.3B, Epidote GDS26.a 75-200um. Finally, following a Dirichlet distribution, the synthetic data sets, each of which contains 900 pixels, are generated using the above signatures (endmember numbers: $k = 6$).

Table 1 shows the RMSEs obtained by different algorithms on the synthetic data corrupted by white noise. The performances of all the algorithms degrade as the noise gets stronger. We can observe that in most cases, the $\ell_{2,p}$ $(0 < p < 1)$ norm collaborative sparse unmixing algorithm outperforms the CLSUnSAL. This phenomenon indicates that the $\ell_{2,p}$ $(0 < p < 1)$ norm could be a better candidate for collaborative sparse unmixing than the $\ell_{2,1}$ norm. Besides, we can also find that as p decreases, the abundances are estimated more accurately.

4.2. Real Data Experiment

The famous AVIRIS Cuprite data set[4] is used to conduct the real data experiment. In our experiment, we use a 204×151 pixels subset with 188 spectral bands (low-SNR bands are removed). The spectral library, which contains 400 spectral signatures, is a subset of the USGS digital spectral library (splib06).

Fig. 2 shows a qualitative comparison among the abundance maps of four highly materials (i.e. Alunite, Buddingtonite, Chalcedony and Montmorillonite) in the considered scene estimated by the considered algorithms. The classification maps of these materials produced by Tricorder software[5] are also displayed. Since the publicly available AVIRIS Cuprite data were collected in 1997 while the Tricorder map was produced in 1995, we only make a qualitative analysis of the performances of different sparse unmixing algorithms by comparing their estimated abundances with the minerals map. It is worth noting that the Tricorder maps are classification maps that consider each pixel in the hyperspectral data pure and classify it as member of a single mineral class while unmixing is a subpixel level classification, which means the abundances for a mixed pixel denote the degree of presence of a mineral in the pixel. Thus, the abundance maps estimated by the sparse unmixing algorithms are somehow different with the Tricorder maps. Nevertheless, we can observe in Fig. 2 that the highest abundances estimated by the sparse unmixing algorithms generally correspond with the pixels belong to the respective class of minerals. Besides, we can also find that the abundances estimated by the $\ell_{2,p}$ $(0 < p < 1)$ norm collaborative sparse unmixing algorithm are generally comparable or higher in the regions classified as respective minerals in comparison to CLSUnSAL. Thus, we can conclude that the $\ell_{2,p}$ $(0 < p < 1)$ norm collaborative sparse unmixing algorithm is an effective tool for sparse unmixing of hyperspectral data.

[4]http://aviris.jpl.nasa.gov/html/aviris.freedata.html
[5]http://speclab.cr.usgs.gov/PAPER/tetracorder

5. CONCLUSION

In this paper, we consider adopting the $\ell_{2,p}$ $(0 < p < 1)$ norm and design a novel algorithm to accomplish the collaborative sparse unmixing task. Experimental results on both synthetic and real data demonstrate that the proposed algorithm is an effective tool for hyperspectral sparse unmixing and that the $\ell_{2,p}$ $(0 < p < 1)$ norm could be a better alternative for collaborative sparse unmixing than the traditional $\ell_{2,1}$ norm.

6. REFERENCES

[1] N. Keshava and J. F. Mustard, "Spectral unmixing," *IEEE Signal Processing Magazine*, vol. 19, no. 1, pp. 44–57, 2002.

[2] Y. Hu, H. Lee, and F. Scarpace, "Optimal linear spectral unmixing," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 37, no. 1, pp. 639–644, 1999.

[3] D. Seung and L. Lee, "Algorithms for non-negative matrix factorization," *Advances in neural information processing systems*, vol. 13, pp. 556–562, 2001.

[4] M.-D. Iordache, J. M. Bioucas-Dias, and A. Plaza, "Sparse unmixing of hyperspectral data," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 49, no. 6, pp. 2014–2039, 2011.

[5] Z. Shi, W. Tang, Z. Duren, and Z. Jiang, "Subspace matching pursuit for sparse unmixing of hyperspectral data," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 52, no. 6, pp. 3256–3274, Jun. 2014.

[6] Y. C. Eldar and H. Rauhut, "Average case analysis of multichannel sparse recovery using convex relaxation," *Information Theory, IEEE Transactions on*, vol. 56, no. 1, pp. 505–519, 2010.

[7] J. Chen and X. Huo, "Theoretical results on sparse representations of multiple-measurement vectors," *IEEE Transactions on Signal Processing*, vol. 54, no. 12, pp. 4634–4643, Dec. 2006.

[8] M.-D. Iordache, J. M. Bioucas-Dias, A. Plaza, and B. Somers, "Music-csr: Hyperspectral unmixing via multiple signal classification and collaborative sparse regression," *IEEE Trans. Geosci. Remote Sens.*, vol. 52, no. 7, pp. 4364–4382, Jul. 2014.

[9] M.-D. Iordache, J. M. B. Dias, and A. Plaza, "Collaborative sparse regression for hyperspectral unmixing," *IEEE Trans. Geosci. Remote Sens.*, vol. 52, no. 1, pp. 341–354, Feb. 2014.

Table 1. RMSEs obtained by different algorithms on the synthetic hyperspectral data corrupted by white noise

	SNR (dB)	CLSUnSAL	$l_{2,p}$ $(p = 0.5)$	$l_{2,p}$ $(p = 0.2)$	$l_{2,p}$ $(p = 0.05)$
	20	0.0540	0.0302	0.0274	0.0257
$k = 6$	30	0.0210	0.0110	0.0104	0.0099
	40	0.0074	0.0042	0.0039	0.0039

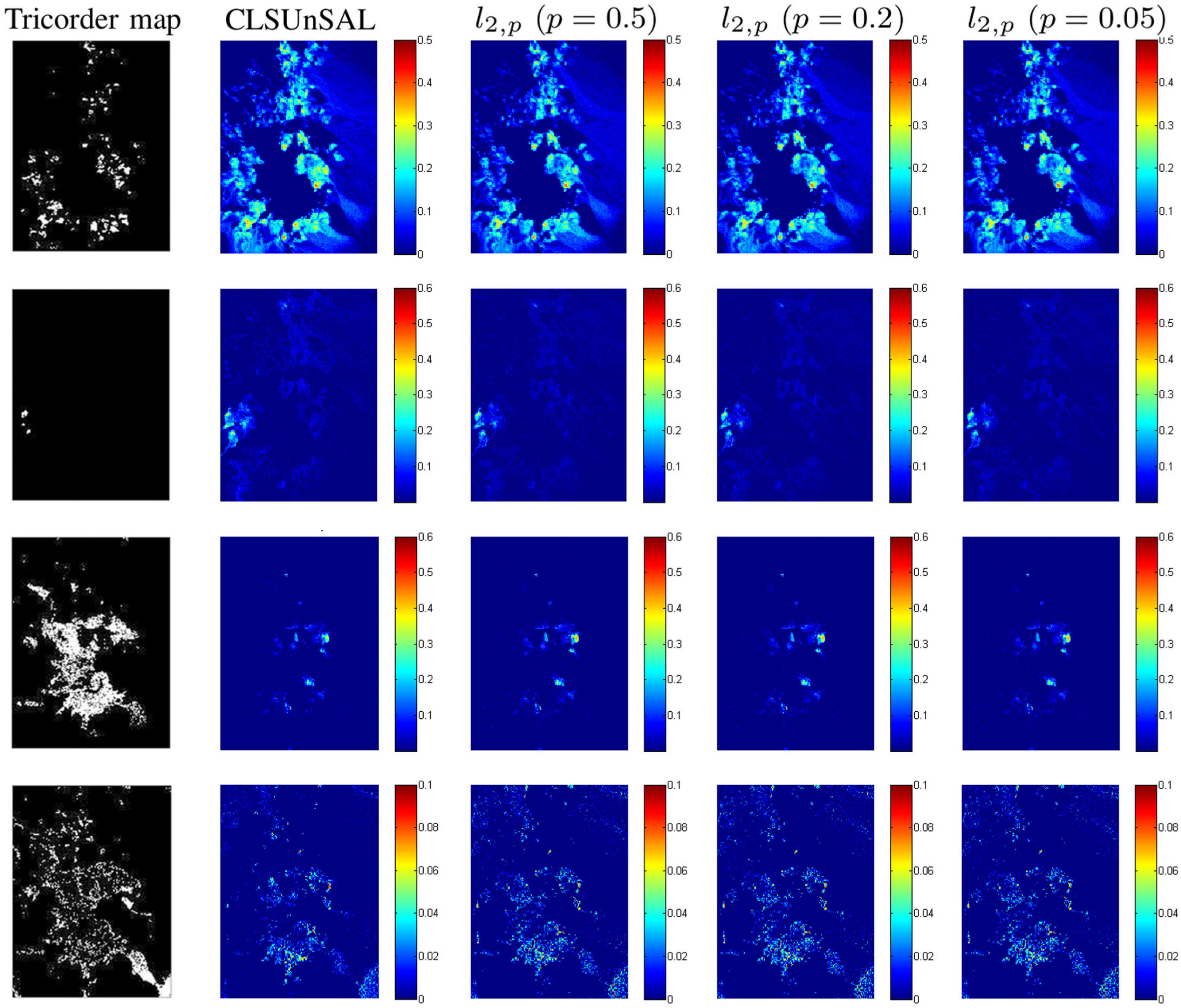

Fig. 2. The fractional abundance maps estimated by CLSUnSAL, the $l_{2,p}$ $(0 < p < 1)$ norm collaborative sparse unmixing algorithm, and the distributed maps produced by Tricorder software for the 204×151 pixels subset of AVIRIS Cuprite scene. From left column to right column are the fractional abundance maps produced or estimated by Tricorder software, CLSUnSAL and our algorithms, respectively. From top row to bottom row are the maps corresponding to Alunite, Buddingtonite, Chalcedony and Montmorillonite, respectively.

[10] R. Chartrand, "Exact reconstruction of sparse signals via nonconvex minimization," *Signal Processing Letters, IEEE*, vol. 14, no. 10, pp. 707–710, 2007.

[11] R. Chartrand and W. Yin, "Iteratively reweighted algorithms for compressive sensing," *in Acoustics, speech and signal processing, 2008. ICASSP 2008. IEEE international conference on. IEEE*, 2008, pp. 3869–3872.

[12] F. Chen and Y. Zhang, "Sparse hyperspectral unmixing based on constrained $l_p - l_2$ optimization," *Geoscience and Remote Sensing Letters, IEEE*, vol. 10, pp. 1142 – 1146, Sept. 2013.

[13] R. N. Clark, G. A. Swayze, R. Wise, E. Livo, T. Hoefen, R. Kokaly, and S. J. Sutley, *USGS digital spectral library splib06a*. US Geological Survey Denver, CO, 2007.