MEC302 – Embedded Computer Systems

2022/23 – Semester 2

Assignment 1 – Modelling continuous, discrete, and hybrid systems

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Remarks:

1. **The assignment is worth 10% of the module mark**.

2. Submission deadline is **April 13, 23:59**.

3. Late submissions will be penalized according to the University policy:

- 5% deduction for each working day after the submission deadline;
- work received more than five working days after the submission deadline will receive a mark of zero.

Instructions:

1. The assignment contains two problems with sets of questions with a total number of 10 questions (six questions for Problem 1 and four questions for Problem 2).

2. Each question is worth 10% of the assignment's total mark.

3. You are expected to provide **exhaustive answers** to all of the questions.

4. The assignment must be completed in handwriting or in a text editor using standard mathematical notations (e.g., using Equation Tools in MS Word).

5. The submitted work needs to be high quality (i.e., readable) scan in **PDF** (for handwritten solutions), or printed in **PDF** format (for solutions written in a text editor).

Good luck!

Problem 1

A DC motor produces a torque that is proportional to the current through the windings of the motor. Neglecting friction, the net torque on the motor, therefore, is this torque minus the torque applied by whatever load is connected to the motor. Newton's second law (the rotational version) gives

$$k_T\, i(t) - x(t) = I \frac{d}{dt}\omega(t), \tag{1}$$

where k_T is the motor torque constant, $i(t)$ is the current at time t, $x(t)$ is the torque applied by the load at time t, I is the rotor moment of inertia, and $\omega(t)$ is the angular velocity of the rotor.

For the setup described above, determine the following:

a) Formulate integral equation of the rotor angular velocity $\omega(t)$ as a function of $i(t)$ and $x(t)$ as variables.

b) Considering that i and x are inputs and ω is an output, draw a block diagram (an actor model) that models the motor. Use only primitive blocks such as integrators (with or without initial value) and basic arithmetic actors (e.g., scale and adder).

c) In practice, the input to a DC motor is not current but rather voltage. Assuming that the inductance of the motor windings is negligible, then the relationship between voltage and current is given by

$$v(t) = R\, i(t) + k_b\, \omega(t), \tag{2}$$

where R is the resistance of the motor windings and k_b is a constant called the motor back EMF (electromagnetic force) constant[1].

Modify your block diagram (actor model) from the previous task so that the inputs are v and x.

d) For the obtained block diagram, formulate the effective integral equation of the rotor angular velocity $\omega(t)$. For $\omega(0) = 0$ and $x(t) = 0$, determine if the system is BIBO stable or not.

Hint: Assume constant input $v(t) = V$.

e) Implement a proportional (P) controller with feedback to maintain the rotor angular velocity ω so that the input is now ψ (angular velocity set point). Draw a block diagram and formulate its effective integral equation. For specific motor parameters in Table I, use your favorite continuous-time modelling software (eg, LabVIEW, Simulink, or Ptolemy II) or any programming software (as was shown on Tutorial 1) to obtain the system response characteristic to a step function $\psi(t) = \begin{cases} 0, t < 0 \\ \Psi, t \geq 0 \end{cases}$. Sketch the response characteristic and explain why the rotor angular velocity ω does not approach setpoint Ψ?

f) Upgrade your P controller to the proportional-integral (PI) controller. Draw a block diagram and sketch the system response characteristic to the step function $\psi(t)$. Does ω approach Ψ now?

Table I – Motor characteristics for numerical modelling

Moment of inertia (I)	0.01 kg m^2
Torque constant (k_T)	0.1 N m/Amp
Resistance (R)	1 Ohm
Back EMF constant (k_b)	0.1 V/rad/sec

[1] Back EMF appears because a rotating motor also functions as an electrical generator, where the voltage generated is proportional to the angular velocity.

Problem 2

An indoor temperature $\tau(t)$ of a household can be represented with the following thermal model:

$$\dot{\tau}(t) = \frac{1}{C_{Th}}\Big(K_{Th}\big(\tau_{Amb} - \tau(t)\big) + P_{Heat}(t)\Big), \tag{3}$$

where C_{Th} is the thermal capacitance of a household, K_{Th} is the effective thermal conductance of walls, windows, and doors, τ_{Amb} is outdoor (ambient) temperature, and $P_{Heat}(t)$ is heating power.

For the setup described above, determine the following:

a) Formulate a discrete difference equation of the indoor temperature $\tau(t)$ as a function of $P_{Heat}(t)$ as a variable. Using your favorite programming software, implement the difference equation in code to model the evolution of the indoor temperature.

Hints: Provide a discrete difference equation and code to model it. The actual modelling (running the code) is not required here.

b) Formulate graphically or mathematically (as a five-tuple) a Finite-State Machine (FSM) for the temperature control system (thermostat) with two states "Heating On" and "Heating Off", where there is one input *temperature* with type $\mathbb{R}$ and two pure outputs *heatOn* and *heatOff*. Consider that FSM changes its state from "Heating Off" to "Heating On" when the *temperature* signal drops below 20 °C and vice versa when the *temperature* exceeds 22 °C.

c) Based on the FSM formulated in task b), develop the thermostat control in code to include in the thermal model developed in task a). Assign $P_{Heat} := \begin{cases} 0, heatOff = present \\ \bar{P}, heatOn = present \end{cases}$. For the specific thermal model parameters provided in Table II, illustrate the performance of the temperature control system (in other words, model the evolution of indoor temperature subject to control for 24 hours).

Hints: 1. *Consider time discrete* $\Delta t = 1/60$ *[hour];*

2. *Make sure there is no chattering.*

d) Considering a single temperature threshold of 20 °C, reformulate your FSM controller to Timed Automata (TA) graphically and in code. To avoid controller chattering, do not allow the state change for at least 15 minutes after the previous one. Similar to the previous task, illustrate the performance of the new temperature control system with TA.

Hint: Consider a clock model $\dot{s}(t) = 1$ *for TA state refinement.*

Table II – Household thermal model characteristics

Thermal conductance (K_{Th})	0.15 kW/°C
Thermal capacitance (C_{Th})	3 kWh/°C
Ambient temperature (τ_{Amb})	5 °C
Heater power capacity ($\bar{P}$)	7 kW