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Using revenue management to improve pricing and capacity management in programme management

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This paper presents revenue management models for pricing, capacity planning, and capacity reallocation and demonstrates their applicability for programme (project) management. In programme management, the allocation of capacity (resource time) to schedule activities requires the resolution of time *versus* revenue trade-offs. Thus, capacity planning and scheduling present a hierarchical problem for programme managers. Furthermore, current programme management methods do not consider the issue of price sensitivity exhibited in many programme management situations. Because of this omission, critical linkages between capacity management and scheduling of activities among programmes have not been addressed. Specifically, the issue of the reservation of capacity specifically for higher revenue generating activities has been omitted from programme management research. This paper asserts that, through capacity planning and scheduling, specific capacity should be reserved for customers willing to pay higher prices to have critical activities, for example, change orders, expedited. This capacity has scheduling effects that impact the programme NPV. This paper proposes potential solutions to capacity and programme scheduling problems using revenue management techniques.

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Introduction

Programme (project) managers frequently confront the inter-related problems of capacity planning and scheduling resource-constrained project activities. The allocation of capacity (resource time) to schedule activities requires the resolution of time *versus* revenue trade-offs. One approach to this problem has been to schedule resource-constrained projects to maximize net present value (NPV). See Abbasi and Arabiat[1] and Herroelen *et al*[2] for an overview of current research on that problem. Furthermore, Vanhoucke *et al*[3] consider the effects of the timing of project progress payments on maximizing project NPV. Subsequently, Herroelen *et al*[4] extend that modelling as linkage between critical chain scheduling/buffer management. This paper utilizes changes in project NPV resulting from scheduling changes as a means of implementing revenue management techniques for project capacity planning and scheduling.

While appropriate for their specific project settings, such programme management methods do not consider the issue of price sensitivity exhibited in many programme management situations. Because of this omission, critical linkages between capacity management and scheduling of activities among projects have not been addressed. Specifically, the issue of the reservation of capacity specifically for higher revenue generating activities has been omitted from programme management research. This paper asserts that, through capacity planning and scheduling, specific capacity should be reserved for customers willing to pay higher prices to have critical activities, for example, change orders, expedited. This capacity has scheduling effects that impact the project NPV. This paper proposes potential solutions to capacity and project scheduling problems using revenue management techniques.

Consider the hierarchical problem faced by a grading contractor that performs earthwork on a variety of construction projects. In the aggregate, capacity for an assortment of earthmovers must be determined first. This capacity, as well as the pricing of this capacity, is determined periodically as changes in the economy and demand dictate. Subsequently, the contractor must allocate capacity in order to schedule new and on-going activities. In addition to construction, this problem is encountered in a wide variety of project settings such as developing new products or services, managing marketing campaigns, managing corporate mergers and acquisitions, software design, and healthcare management.

Airlines regularly make pricing and capacity allocation decisions by deciding whether to accept discount seat

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booking requests, or to refuse these requests in hope of later, higher fare, bookings. This pricing and capacity allocation is accomplished through *revenue management* (also known as *yield management*). Revenue management (RM) is an order acceptance and refusal process that integrates the marketing, financial, and operations functions to maximize revenue from capacity (see Cross[5]). It combines a differential pricing strategy with proactive capacity planning and an order booking process to manage demand, delivery reliability, and realize additional revenue from change orders.

While the use of revenue management techniques are commonly associated with airlines,[6–8] they have also been successfully applied to the railway, cruise ship,[6] restaurant, hotel, and resort,[9–11] health care, electric utility, car rental advertising, entertainment, printing, publishing, and telecommunications industries.[15] Harris and Pinder[16] demonstrate the use of revenue management for assemble-to-order (ATO) manufacturing; a setting with operations characteristics similar to those found in project settings. See Weatherford and Bodily,[17] Petruzzi and Dada,[18] and McGill and Van Ryzin[19] for comprehensive overviews and surveys of revenue management.

A major characteristic of revenue management is the time-dependent value of the capacity; for example, an airline seat has no value once the flight has taken off; this characteristic is frequently referred to as *asset perishability*.[17] Revenue management works well when short-run capacity is fixed and orders are time-sensitive. This time-sensitivity typically arises when a segment of customers have price (low cost) as an order winner and can place orders early; meanwhile, other customers have requirements that prevent them from placing orders within specific production time fences. In such settings, delivery speed, delivery reliability, and change order responsiveness become those segment's order winners. Thus, the grading contractor's customers can be segmented according to price sensitivity based upon those delivery preferences; preferences that have scheduling consequences.

Just as the unused airline seat has no value once the flight has departed, unused capacity for the grading contractor's equipment represents lost revenue opportunity. Because programme management involves the allocation of time-dependent, perishable, capacity (resources) by scheduling activities to resource conflicts, revenue management techniques are well-suited to many programme management situations. Scheduling activities is essentially an order acceptance/refusal process. As such, revenue management models for pricing, capacity planning, and capacity reallocation in programme management settings are developed and demonstrated in this paper.

The next section characterizes business environments in which revenue management has been successful and makes comparisons to programme management settings. The third section presents optimal pricing, capacity, and capacity reallocation (scheduling) models. A numerical example and describes managerial implications of adopting revenue management for programme management is presented in the fourth section. A summary and conclusions are provided in the final section.

Revenue management concepts

Kimes[10] and Weatherford and Bodily[17] have identified several common characteristics of service operations environments that successfully use revenue management: perishability, fixed capacity, high capacity change costs, stochastic demand, demand/market segmentation, historical sales data with forecasting capabilities, and advance sales/bookings. These characteristics are also found in programme management environments. Owing to the stochastic nature of activity durations as well as stochastic demand for resources in programme management, there will nearly always be forecasting errors; hence the need to balance the cost of excess capacity *versus* the cost of providing a specified level of service.

(1) *Perishability.* All services are perishable; due to the time effects of event scheduling, capacity utilization in programme management is also perishable. Just as revenue from an empty airline seat is lost at departure, unused capacity is lost in programme management environments.

(2) *Fixed capacity.* Revenue management practitioners agree that RM is most applicable to environments with short-run fixed capacity.[8,20] While overtime and subcontracting are sometimes available, there are limits to these temporary capacity extensions. Because of the significant costs of capital equipment expansion, firms in programme management environments typically operate under conditions of short-run fixed capacity.

(3) *High capacity change costs.* The advantage of a revenue management system emerges when capacity change costs are high relative to incremental production and marketing costs. The nature of programme management requires fine-tuning the scheduling and marketing of under-utilized or over-utilized processing time and is relatively inexpensive compared to the cost of acquiring and maintaining additional capacity or liquidating excess capacity. As an example, in the construction industry the cost of heavy construction equipment (eg cranes and earth moving equipment) is extremely high and thus such capacity is not easily reduced.

(4) *Stochastic demand.* While fluctuations in demand create problems for efficient capacity management, these same fluctuations create revenue opportunities, that is, the price premium for expediting orders. Thus, demand fluctuations enhance the value of change order responsiveness and due date reliability for programme management. In general, optimal margins increase as the variability of demand increases. This is because revenue management is designed to capture the willingness of customers to pay for expedited orders, especially in tightly scheduled plans.

(5) *Segmentable demand.* Revenue management is most effective when demand can be segmented and price sensitivity varies across market segments. Segmenting project/activity demand can be difficult; if restrictions appear unjustified and arbitrary, then customers may reject price differentials. The airline industry relies primarily on time-sensitive (eg, business traveler change orders) and destination-sensitive (eg, vacation cities) restrictions to segment demand. In principle, programme management settings can segment in the same manner. Customers needing their projects (or change orders) expedited are typically price-insensitive customers, while customers able to book their projects in advance are typically price-sensitive (cost minimizing) customers. Demand segmentation has largely been ignored by programme management and is critical to revenue management.

(6) *Historical sales data with forecasting capability.* Customer data, preferably transaction-specific records, are required to realize the full potential of revenue management. This information is used for demand forecasting, estimation of segment-level order delay sensitivity, and price sensitivity. Analysis of booking patterns at various price points, subject to various fencing restrictions is performed on customer segments at a range of time periods. Capturing this data and making it accessible in a management information system is crucial to the implementation of RM. Project managers wishing to implement revenue management need to develop and maintain market information for estimating the effects of price and delivery changes on demand for different market segments.

(7) *Advance sales/bookings.* Another key element of programme management is the order acceptance and refusal process for both advance sales and change order requests. Advance sales assure some capacity utilization and allow updating of long- and medium-term forecasts. In a revenue management system, booking policies determine the amount of capacity to reserve for later-arriving high-margin demand by stopping advance sales in the discount segments at predetermined optimal levels. The trade-off between booking firm orders at a discount and reserving capacity for later arriving customers willing to pay a premium for change orders is a central concept in the creation of revenue management policies.

As an illustration of a programme management situation that is suitable for the application of RM, consider the grading contractor mentioned in the introduction. The contractor is frequently engaged in several projects simultaneously. Because the earthwork tasks are typically early in the projects and are predecessors to many other tasks, they are often critical to the customers' projects. The cost of any single piece of earthmoving equipment (eg, scrapers, loaders, bulldozers, excavators, and dump trucks) can range a quarter-million dollars to over a half-million dollars, thus making idle equipment an expensive opportunity cost. In this situation, because short-term capacity is fixed and revenues from unused capacity are lost and because some customers have flexibility in their scheduling requirements while other customers are willing to pay a premium for either specific due-date reliability or for having their projects expedited, revenue management could be successfully employed.

In many operations settings (eg, airlines, hotels, car rental agencies) the management and segmentation of demand are viewed as marketing functions. Thus programme management research and practice has not focused on this aspect of linking these marketing functions with the operations functions of capacity management and scheduling. Revenue management relies not only on these linkages, but the information technology linkages afforded by the current volume of demand information available. Thus, in order for revenue management to be implemented in a programme management setting, the segmentation mechanisms previously described must be developed. Subsequently, forecasting and booking procedures can then be established and implemented.

Theoretical framework

Revenue management is predicated upon market segmentation creating customer classes. In the programme management setting, these classes can be based on delivery speed (earliest possible completion of project or activity), due-date reliability, or change order responsiveness. As an example, consider two classes: the first class paying a premium for specific project activities expedited, while the second class pays the standard rate and accepts the schedule specified in the contract. Thus, a wide variety of customer segmentations are possible. Given such classes, demand curves for each product/market segment can be estimated and subsequently used to determine the optimum pricing and capacity. The revenue management model presented in this paper has three components: a simultaneous pricing and capacity model, a differential pricing policy, and a multi-class capacity allocation/reallocation process.

Optimal pricing and capacity

Operationalizing a revenue management system for programme management first requires class-specific demand specifications. Given classes specified according to segment attributes (eg, on-time activity completion), consider price dependent, decreasing, demand functions with a random component that is independent of price. As such, stochastic demand for class *i* may be specified as a function of the class price. In this manner, stochastic demand can be modelled in either an additive[21] or multiplicative manner.[18] An additive model is used in this paper. Given this setting, the following terms are defined:

p_i rate for class *i*; that is, unit-capacity price paid by customers in class *i*. $p_1 > p_2$

q_i units of class i capacity

s_i units of capacity in excess of expected demand for class i (eg a safety stock)

$Q_i(p_i)$ expected demand for class i given price p_i; a decreasing function capturing the dependency between demand and price. Note that $q_i = Q_i(p_i) + s_i$

ε_i random component of demand defined over the range (a, b)

$f_i(\varepsilon_i)$ probability density function of ε_i; let μ_i and σ_i represent the mean and standard deviation of ε_i, respectively

$F_i(\varepsilon_i)$ cumulative distribution function of ε_i

$D_i(p_i, \varepsilon_i)$ stochastic demand for class i: $D_i(p_i, \varepsilon_i) = Q_i(p_i) + \varepsilon_i$

vc variable cost of the resource, this can be a rental rate

g_i per-unit penalty cost of shortage

h per-unit cost of excess units of capacity (ie, holding or disposal cost)

Given these terms, p_i and q_i are decision variables with the artificial decision variable, s_i, being used to facilitate calculation of q_i. The cost variables, vc, g_i, and h, are used to define the expected profit function. Note that vc is a per time period unit cost for the use of a resource and is recognized as a rental rate.

The linkage between revenue management's pricing and capacity decisions and programme management's scheduling decisions is the per-unit penalty cost of shortage, g_i. This term is the marginal cost of shortage incurred under a resource constrained schedule. To use a linear programming analogy, g_i is like a shadow price for the resource based upon being able to reschedule the project with 1 additional resource unit. One method of determining g_i, is to set g_i equal to $NPV(q_i) - NPV(q_i - 1)$; where $NPV(q_i)$ and $NPV(q_i - 1)$ are the net present values (NPVs) obtained from solving the resource-constrained NPV scheduling problem given q_i, and $q_i - 1$, units of capacity respectively. See Pinder and Marucheck[22] for a formulation of this problem. Note that q_i is initialized for calculating g_i. After the optimum p_i and q_i have been determined, g_i is recalculated and iteration is used until satisfactory convergence is reached. Because g_i is a function of the schedule, it provides the direct means by which project scheduling is determined from, and linked to, the aggregate capacity and pricing decisions. See Pinder[23] for further examples of how g_i may be determined.

In a simplified example, consider the grading contractor undertaking a project using 29 days of earthmoving capacity with a resultant project NPV of \$1,200,000. Suppose the project was rescheduled using 28 days of earthmoving capacity and, due to changes in the timing of payments, the project NPV became \$1,199,000. Then, the penalty of a unit shortage, g_i, would equal \$1,000.

In practice, extra units of capacity can incur costs (storage, insurance, taxes, etc.) or provide revenue (be rented, leased, or sold to other firms). As such, the range of h is $(-\infty, \infty)$ and a negative value of h reflects a salvage value.

Profit, $\Pi(p_i, q_i)$, is the difference between sales revenue and the sum of the costs and can be expressed as:

$$\Pi(p_i, q_i) = \begin{cases} p_i D_i(p_i, \varepsilon_i) - vc\, q_i - h(q_i - D_i(p_i, \varepsilon_i)), \\ \quad D_i(p_i, \varepsilon_i) \leqslant q_i \\ p_i q_i - vc\, q_i - q_i - g_i(D_i(p_i, \varepsilon_i) - q_i), \\ \quad D_i(p_i, \varepsilon_i) > q_i \end{cases} \tag{1}$$

Substituting $D_i(p_i, \varepsilon_i) = Q_i(p_i) + \varepsilon_i$ and $q_i = Q_i(p_i) + s_i$ in Equation (1) yields:

$$\Pi(p_i, s_i) = \begin{cases} p_i(Q_i(p_i) + \varepsilon_i) - vc(Q_i(p_i) + s_i) - h(s_i - \varepsilon_i), \\ \quad \varepsilon_i \leqslant s_i \\ p_i(Q_i(p_i) + s_i) - vc(Q_i(p_i) + s_i) - g_i(\varepsilon_i - s_i), \\ \quad \varepsilon_i \leqslant s_i \end{cases} \tag{2}$$

This substitution provides an alternative interpretation of the capacity decision variable. When the choice of s_i is larger than ε_i, then idle capacity occurs; when s_i is smaller than ε_i, then shortages occur. Thus, the optimal safety stock (s_i^*) is used to determine the optimal capacity (q_i^*). Rearranging the terms in Equation (2) to express profit in marginal terms yields:

$$\Pi(p_i, s_i) = \begin{cases} (p_i - vc)(Q_i(p_i) + \varepsilon_i) - (vc + h)(s_i - \varepsilon_i), \\ \quad \varepsilon_i \leqslant s_i \\ (p_i - vc)(Q_i(p_i) + \varepsilon_i) - (p_i - vc + g_i)(\varepsilon_i - s_i), \\ \quad \varepsilon_i > s_i \end{cases} \tag{3}$$

Let

$$I_i(s_i) = \int_a^{s_i} (s_i - \varepsilon_i) f_i(\varepsilon_i) \mathrm{d}\varepsilon_i \text{ and}$$

$$S_i(s_i) = \int_{s_i}^{b} (\varepsilon_i - s_i) f_i(\varepsilon_i) \mathrm{d}\varepsilon_i$$

Thus, expected profit is:

$$E[\Pi(p_i, s_i)] = (p_i - vc)(Q_i(p_i) + \mu_i) - (vc + h) I_i(s_i) - (p_i - vc + g_i) S_i(s_i) \tag{4}$$

Note that $I(s_i)$ is the expected number of excess (idle) units of capacity if s_i is chosen too high. Similarly, $S(s_i)$ is the expected shortage if s_i is chosen to low. Note that $(p_i - vc)(Q_i(p_i) + \mu_i)$ is the profit thatwould occur in the absence of uncertainty; that is, the riskless profit.[21] Thus, Equation (4) is the riskless profit less the expected costs of excess capacity and shortage.

To maximize $E[\Pi(p_i, s_i)]$ over p_i and s_i, consider the first and second partial derivatives of $E[\Pi(p_i, s_i)]$ taken with respect to p_i and s_i:

$$\frac{\partial E[\Pi(p_i, s_i)]}{\partial p_i} = (p_i - vc)\frac{dQ_i(p_i)}{dp_i} + (Q_i(p_i) + \mu_i) - S(s_i) \tag{5}$$

$$\frac{\partial^2 E[\Pi(p_i, s_i)]}{\partial^2 p_i} = (p_i - vc) \frac{d^2 Q_i(p_i)}{d^2 p_i} + 2\frac{dQ_i(p_i)}{dp_i} \tag{6}$$

$$\frac{\partial E[\Pi(p_i, s_i)]}{\partial s_i} = -(vc + h)F_i(s_i) + (p_i - vc + g_i)(1 - F_i(s_i)) \tag{7}$$

$$\frac{\partial^2 E[\Pi(p_i, s_i)]}{\partial^2 s_i} = -(p_i + g_i + h)f_i(s_i) \tag{8}$$

When $Q_i(p_i)$ is either linear downward sloping or exponential then, from Equation (6), then $E[\Pi(p_i, s_i)]$, Equation (4), has a maximum with respect to p_i (a full mathematical appendix is available from the author). Similarly, Equation (8) shows that $E[\Pi(p_i, s_i)]$ is concave down for s_i for a given p_i. Thus, the simultaneous solution of Equations (5) and (7) determine price and capacity values that maximize expected profit.

Furthermore, Equation (7) can be arranged to provide s_i^*:

$$F(s_i^*) = (p_i^* - vc + g_i)/(p_i^* + g_i + h) \tag{9}$$

where the value of the cumulative distribution, $F(s_i^*)$, is the optimum service level resulting from the optimum s_i^* units of capacity above (or below) the expected demand ($\hat{Q}_i(p_i^*)$). The service level is the ratio of the marginal opportunity cost resulting from a shortage of one unit to the total of the marginal costs of excess capacity and shortage. Thus, service level increases as potential contribution margin (p_i-vc) increases, or as the penalty cost of shortage (g_i) increases. In other words, capacity is increased to avoid lost profit opportunities (to capture higher contribution margins) and to avoid incurring liquidated damages or lost time value of money due to project and activity delays. In this manner, programme management scheduling concerns are linked to the revenue management decisions of pricing and capacity. Operationally, Equations (5) and (9) can be used simultaneously to set price and capacity.

To further illustrate the role of the demand curve in the programme management setting, let $Q_i(p_i)$ be estimated over a linear range by: $\hat{Q}_i(p_i) = \beta_{0i} + \beta_{1i}p_i$. Operationally, in estimating $D_i(p_i, \varepsilon_i) = \hat{Q}_i(p_i) + \varepsilon_i$, the uncertainty term ($\varepsilon_i$) would be the distribution of the error term and ε_i should be unbiased with $\mu_i = 0$. To begin, consider the riskless price, p_i^0, derived from the riskless profit portion of Equation (4):

$$p_i^0 = \frac{\beta_{1i}vc - \beta_{1i} - \mu_i}{2\beta_{1i}} \tag{10}$$

Using this result and substituting the linear estimation, $\hat{Q}_i(p_i) = \beta_{0i} + \beta_{1i}p_i$, into Equation (5) provides

$$\frac{\partial E[(\Pi(p_i, s_i)]}{\partial p_i} = 2\beta_{1i}(p_i - p_i^0) - S(s_i) \tag{11}$$

Checking for the second-order optimality condition:

$$\frac{\partial^2 E[\Pi(p_i, s_i)]}{\partial^2 p_i} = 2\beta_{1i} \tag{12}$$

Since $Q_i(p_i)$ is a downward sloping function, then $\beta_1 < 0$, thus Equation (12) shows that for a piecewise linear demand function that the expected profit Equation (4) is concave down with a maximum at p_i^*. Setting Equation (11) $\equiv 0$ and solving for p_i yields

$$p_i^* = p_i^0 + S(s_i^*)/2\beta_{1i} \tag{13}$$

In this manner, Equations (9) and (13) can be used to simultaneously determine the optimum price (p_i^*) and capacity ($q_i^* = Q_i(p_i^*) + s_i^*$) for piecewise linear demand functions. Methods of solution include grid search, successive approximation (iteration) and simulation. These methods are demonstrated in a numerical example in the next section.

Differential pricing

After determining the optimum price and capacity, a differential pricing policy can be established. Differential pricing is the mechanism by which different rates are applied to the segmented classes to reflect different levels of service, delivery date certainty, or change order responsiveness needed by the different classes. These pricing policies maintain a constant differential rate while providing short-term pricing adjustments due to short-term fixed capacity.

Differential pricing allows programme management to charge the appropriate price for the resources required to complete the activities/project according to clients' demand. In this manner, higher revenues are obtained from clients willing to pay a premium for different levels of service, delivery date certainty, or change order responsiveness.

To determine the pricing for each class, set Equation (7) $\equiv 0$ and rearrange, yielding

$$(vc + h) = (p_i + g_i + h)(1 - F_i(s_i)) \tag{14}$$

Note that Equation (14) is true for is true for each class, thus

$$\begin{aligned}(vc + h) &= (p_1 + g_1 + h)(1 - F_1(s_1)) \\ &= (p_2 + g_2 + h)(1 - F_2(s_2))\end{aligned} \qquad (15)$$

Consequently, pricing for each class must maintain the following proportionality:

$$\frac{p_1 + g_1 + h}{p_2 + g_2 + h} = \frac{1 - F_2(s_2)}{1 - F_1(s_1)}. \qquad (16)$$

Optimal capacity reallocation

Keeping in mind the hierarchical nature of the capacity process, capacity is fixed in the short-run and must be adjusted, or reallocated, to provide for moderate fluctuations in demand. In revenue management systems, this reallocation process is managed according to an optimal order acceptance and refusal process. This process is typically facilitated through the use of booking limits and threshold curves; see Cross[20] for an example.

Allocating and reallocating optimal capacities depends upon determining the optimal protection level, Γ_1^*, for the higher margin class under perishability and stochastic demand. Note that $\Gamma_1^* = Q_1(p_1) + s_1^*$. Reserving too much capacity for class 1 expedited activities (projects) results in lost sales to class 2 advance contract orders. Similarly, failing to reserve enough capacity for the expedited class results in the displacement of higher contribution expedited orders; that is, *high yield spill.*

The expected opportunity cost is expressed as

$$\begin{aligned}E[L(s_1)] = &\int_a^{s_1} (p_2 - vc + g_2)(s_1 - \varepsilon_1) f_1(\varepsilon_1)\mathrm{d}\varepsilon_1 \\ &+ \int_{s_1}^{b} ((p_1 + g_1) - (p_2 + g_2))(\varepsilon_1 - s_1) f_1(\varepsilon_1)\mathrm{d}\varepsilon_1\end{aligned} \qquad (17)$$

The first integrand in Equation (17) is the expected lost class 2 contribution margin resulting from reallocating (reserving) too much capacity for class 1; that is, lost sales to class 2 customers. In contrast, the second integrand is the expected lost class 1 premium resulting from high yield spill; that is, lost sales to class 1 customers (a failure to reallocate enough class 1 capacity).

Differentiating (17) with respect to s_1 yields

$$\begin{aligned}\frac{\partial E[L(s_1)]}{\partial s_1} &= (p_2 - vc + g_2)F(s_1^*) \\ &- ((p_1 = g_1) - (p_2 + g_2))(1 - F(s_1^*))\end{aligned} \qquad (18)$$

with a second derivative of

$$\begin{aligned}\frac{\partial^2 E[L(s_1)]}{\partial^2 s_1} &= ((p_2 - vc + g_2) \\ &+ ((p_1 + g_1) - (p_2 + g_2)))f_1(s_1)\end{aligned} \qquad (19)$$

where $F(s_1^*)$ is the cumulative distribution of demand at the optimal protection level, s_1^*. Note that s_1^* depends on the previously determined initial capacity and rates. Since the two multiplicands of (19) are $\geqslant 0$, then (18) can be set$\equiv 0$ to minimize (17). Setting (18)$\equiv 0$ and rearranging yields:

$$\begin{aligned}(p_2 - vc + g_2)F(s_1^*) &= ((p_1 + g_1) \\ &- (p_2 + g_2))(1 - F(s_1^*))\end{aligned} \qquad (20)$$

Another way of interpreting (20) is that the l.h.s. is the expected marginal opportunity cost of excess capacity reserved for class 1 (which could have been sold to class 2 customers), while the r.h.s. is the expected marginal opportunity cost of the shortage of capacity for class 1 (ie, the displacement of capacity sold to class 2, which should have been reserved and sold to class 1).

To further interpret the scheduling and capacity reallocation implications of this model, Equation (20) can be rearranged as

$$(p_1 - vc + g_1)(1 - F(s_1^*)) = (p_2 - vc + g_2) \qquad (21)$$

Since $F(s_1^*)$ is the service level for expedited activities, $(1 - F(s_1^*))$ is the probability of shortage in the high yield segment. Equation (21) shows that the optimal protection level for the expedited class, derived from s_1^*, reserves capacity such that the expected marginal contribution foregone by denying service to even one class 1 order should be no greater than the marginal contribution generated by the last class 2 order. That is, beginning from certainty of shortage in the lowest rate class and proceeding to protect the next higher rate class leads to equilibrium for the expected marginal contributions across all customer classes. This condition from (21) is consistent with maintaining equilibrium in (7) and (16). In this manner, a project manager can employ knowledge of contribution margins associated with competing activities to prioritize activity schedules. This also provides a heuristic similar to the solution results of turnpike problems.

Solution methodologies and numerical example

The following example illustrates the use of the models from the previous section. Consider a grading contractor with several projects under way. One item of equipment used for many activities is a small front-end loader commonly referred to as a Bobcat. The programme manager must periodically (in practice, this period can be monthly) determine the price and capacity of resources using an aggregate model. Furthermore, the manager faces decisions regarding the acceptance of expedited orders (jobs) that affect scheduling.

Consider two classes of demand, the first class being customers willing to pay a premium to have their activities expedited, the second class paying the standard rate.

For this example the rental rate (vc) is \$200/day and the storage cost (h) is \$20/day. Empirical data from the contractor was used to determine the demand curves. The demand curves (in units/day) for the two classes are illustrated in Figure 1 and are estimated to be

$$Q_1(p_1) = 100 - 0.1p_1 \tag{22}$$

$$Q_2(p_2) = 320 - 0.5p_2 \tag{23}$$

with the estimation errors for each class being normally distributed with an average of 0 and standard deviations of 20 and 15 days, respectively. Thus, $f_1(\varepsilon_1)$ is normal (0,20) and $f_2(\varepsilon_2)$ is normal (0,15). Operationally, the uncertainty term (ε_i) results from the estimation error of the demand curves. Furthermore, assume that a unit shortage causes an average delay of 3 weeks on receiving \$40,000 from the class 1 customer projects and an average delay of 1 week on receiving \$20,000 from the class 2 customer projects. The per-unit costs of shortage of each class (g_1 and g_2) were then calculated using a rate of return of 15%; thus, g_1 equals \$340/day and g_2 equals \$80/day (see Pinder and Marucheck[22] for scheduling problem formulation and solution methodology).

Three solution methodologies were used to solve the example problem: a grid search; a successive approximation method, beginning with the risk free price; and a simulation using stochastic demand. Solutions for these methods were obtained using a standard spreadsheet program with nearly instantaneous recalculation times.

The first method of solution was a grid search over successively smaller response surface of the expected profits computed using pairs of price and capacity. Equation (4),

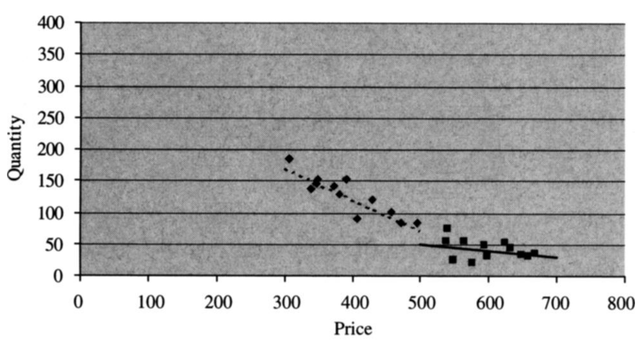

Figure 1 Demand curves for front-end loader.

along with Simpson's rule to estimate $I(s_i)$ and $S(s_i)$, was used to calculate expected profit for combinations of price and capacity (p_i and s_i). The scale of the grid was successively refined to achieve a satisfactory level of precision. A benefit of this method is that the method is not restricted to linear demand functions.

The second method was a successive approximation method. This method made use of the piecewise linearity of the demand curves. The first step of this method was to use Equation (10) to calculate the risk-free price. Next, the service level was determined via Equation (9) and the corresponding capacity, s_i, determined from the service level. The successive estimate of price was computed by using Equation (13) along with Simpson's rule to approximate $S_i(s_i)$. This updated price was then used to re-estimate the service level (Equation (9)) and the corresponding capacity, q_i, and safety stock, s_i. These steps were repeated until satisfactory convergence for both price and safety stock (p_i and s_i) were achieved.

Table 1 shows the solution provided by both methods. Thus, to operationalize the policy, the contractor would have a total capacity of 171 loader-days available, with 115 loader-days available for early booking (scheduling) at a rate of \$415.32 per loader-day. The remaining 56 of loader-days would be reserved for expedited orders that would be charged \$586.45.

The response surface, $E[\Pi(p_i, s_i)]$, was plotted so that the sensitivity of the expected profit to the decision variables could be examined. The response surface is show in percentage terms relative to the optimum expected profit; that is, $E[\Pi(p_i, s_i)]/E[\Pi(p_i^*, s_i^*)]$. Figure 2 shows the deviations from the optimum expected value for class 1 as a function of p_i and s_i, while Figure 3 illustrates the same information for class 2. The price dimension of the two graphs demonstrates the price sensitivity of class 2 relative to class 1. Thus indicating the class 2 solution is more sensitive to small changes in price due to their price sensitivity. This is to be anticipated given that class 2 is expected to book early in order to obtain the lower price. The safety stock dimension of the two graphs exemplifies the relative sensitivity of class 1 to availability of capacity. Thus, expected profits are sensitive to the reservation of capacity for class 1. This is also expected given class 1's penchant to pay a premium for use of capacity that could be otherwise

Table 1 Solution to numerical example

	Grid search and successive approximation		
	Class 1	*Class 2*	*Total*
p_i^*	\$586.45	\$415.32	
s_i^*	14.66	2.76	
q_i^*	56.01	115.11	171.12
$E[\Pi(p_i^*, s_i^*)]$	\$10,200.12	\$21,157.19	\$31,357.31

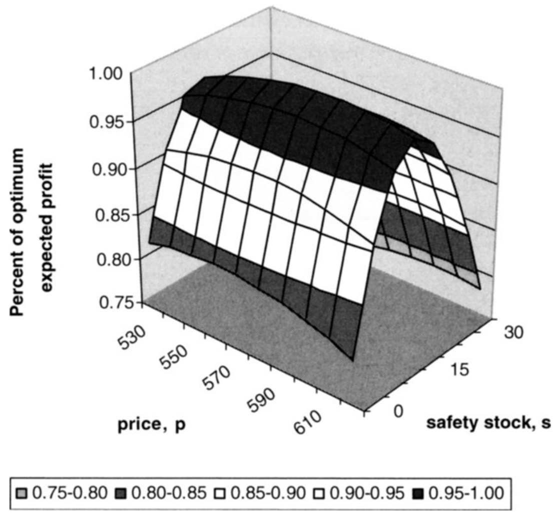

Figure 2 Deviations from optimum expected profit for class 1.

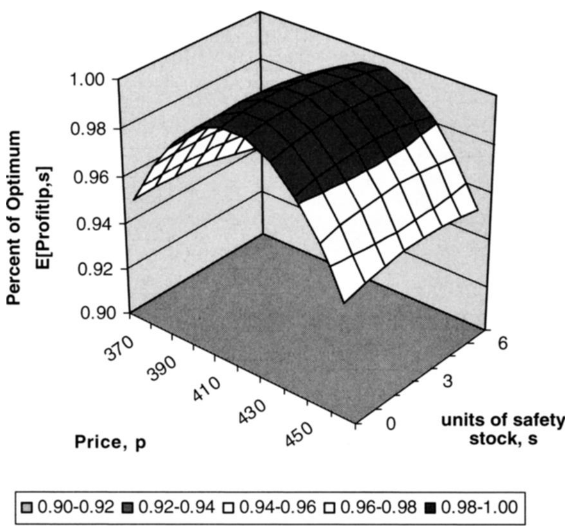

Figure 3 Deviations from optimum expected profit for class 2.

utilized by class2. Further demonstrating that class 1 must bear the cost of idle capacity in order to have capacity available for expediting projects.

In the third method, price and capacity were determined through a Monte Carlo simulation. In this simulation, a single trial consisted of simulated demand for each class given price and capacity combinations. For each trial, the excess, or shortage, and resulting profit were calculated. A grid search of expected profits, in a manner similar to the grid search described above, provided the optimum price and capacity. This method yielded the same solution as given in Table 1. For optimum price and quantity, the expected overage and shortage ($I(s_i)$ and $S(s_i)$) were also calculated over the 100 runs of 5000 trials and used to cross check the Simpson's rule approximations for the first two methods.

A second simulation was constructed to estimate the value of using these RM models in this programme management setting. This simulation was constructed to study the effect of not using the proposed models. For this simulation it was assumed that the decision maker was ignorant of the existence of class1. In such a setting, no class 1 capacity is reserved but class 1 customers can purchase any capacity remaining after class 2 demand has been met. Thus, the simulation uses p_2^*, s_2^*, and q_2^*, and attempts to satisfy expedited orders, that is, the late arriving class 1 customers, with capacity that is left over after class 2 demand has occurred. The expected profit estimated from this simulation was again expressed as a percentage of the optimum expected profit, $E[\Pi(p_i^*, s_i^*)]$. The percentage of the optimum expected profit was 78.8% with a margin of error of 0.1% (based again upon 100 runs of 5000 trials). Thus, the use of this model improved expected profits in this setting on the order of 25% (approximately equal to (1−0.788)/0.788).

Managerial implications and conclusions

The most significant benefit for programme management firms using revenue management is increased revenue. This revenue is directly attributable to pricing based on the value perceived by customers.

Fixed short-run capacity to meet stochastic demand causes expected capacity shortage in lower rate classes; this, in turn, creates the potential for price premiums for due-date reliability, activity speed (expedited activities), and change order responsiveness in higher rate classes. By effecting increased revenues from these bases of competition, revenue management provides several benefits for firms using revenue management and their customers.

Improved capacity planning is another important benefit derived by firms employing revenue management. From a revenue management perspective, capacity reserved for expedited orders is idle rather than excess. Such reserved capacity is required to demonstrate scheduling flexibility in order to attract potential higher price customers with last-minute change orders. Availability of this reserved capacity also creates enhanced delivery speed and reliability for booked orders and provides a means of production smoothing. Finally, optimal price premiums in a revenue management system identify the appropriate rate at which to recover fully allocated costs. This view of capital cost recovery provides more accurate net cash flow and pro forma operating statements as inputs for the capacity planning process. Thus, revenue management can serve as a significant component, linking demand management and capacity planning, in the planning and control system for a project-oriented operation.

The most apparent costs associated with revenue management are those associated with collecting and maintaining the information used in the revenue management decision models. A fallacious cost associated with installing a revenue management system, is that of idle capacity reserved for expedited orders. While revenue management pricing recovers fully allocated costs, management often perceives the opportunity cost of the reserved capacity as an out-of-pocket cost. It should not do so; the reserved capacity is required to attract higher-price customers with last-minute orders or change orders.

There are several useful insights for the implementation and management of a revenue management system. First, price differentials between orders booked in advance and expedited orders should be significant enough to ensure that customers will accept delivery date promises and not cancel orders or refuse delivery. It is important to recognize that booked orders are not realized as revenue until receipts appear and substantial deposits may be required. In operations with significant cancellations, substantial overbooking can be appropriate.[17]

Second, without suitable protection levels (stop sales mechanisms), accepting profitable occasional low-margin accounts may irreparably damage the premium pricing of other profitable repeat purchase accounts. Repeat purchase loyalty in the high-margin expedited class with high delivery reliability and change order responsiveness is the key to achieving sustainable price premiums.

Thus, revenue management provides several benefits for programme management. First, increased revenue and profits are realized from appropriate pricing mechanisms, an area often neglected by programme management. Second, capacity planning is improved by reserving capacity for a segment of higher paying customers. Finally, capacity reallocation provides an improved capacity/scheduling linkage. Thus, the revenue management based models for pricing, capacity planning, and capacity reallocation in programme management settings developed and demonstrated in this paper can provide significant benefits for programme management.

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Appendix A

The bold numbers in parenthesis, for example, **(1)**, correspond to the equation numbers in the text.

Optimal pricing and capacity

Profit, $\Pi(p_i, q_i)$, is the difference between sales revenue and the sum of the costs:

$$\Pi(p_i, q_i) = \begin{cases} p_i D_i(p_i, \varepsilon_i) - vcq_i - h(q_i - D_i(p_i, \varepsilon_i)), \\ \quad D_i(p_i, \varepsilon_i) \leqslant q_i & \mathbf{(1)} \\ p_i q_i - vcq_i - g_i(D_i(p_i, \varepsilon_i) - q_i), \\ \quad D_i(p_i, \varepsilon_i) > q_i \end{cases} \tag{A.1}$$

Substituting $D_i(p_i,\varepsilon_i)=Q_i(p_i)+\varepsilon_i$ and $q_i=Q_i(p_i)+s_i$ in Equation (1) yields

$$\Pi(p_i,s_i)=\begin{cases} p_i(Q_i(p_i)+\varepsilon_i)-vc(Q_i(p_i)+s_i)-h(s_i-\varepsilon_i), \\ \quad \varepsilon_i\leqslant s_i \qquad (\mathbf{2}) \\ p_i(Q_i(p_i)+s_i)-vc(Q_i(p_i)+s_i)-g_i(\varepsilon_i-s_i), \\ \quad \varepsilon_i>s_i \end{cases} \tag{A.2}$$

Rearranging the terms in Equation (2) to express profit in marginal terms yields:

The steps for rearranging the two components of Equation (A.2) into the comparable component of Equation (A.14) are as follow:

$$p_i(Q_i(p_i)+\varepsilon_i-vc(Q_i(p_i)+s_i)-h(s_i-\varepsilon_i) \tag{A.3}$$

$$p_i(Q_i(p_i)+\varepsilon_i)-vcQ_i(p_i)-vcs_i+vc\varepsilon_i-vc\varepsilon_i \\ -h(s_i-\varepsilon_i) \tag{A.4}$$

$$p_i(Q_i(p_i)+\varepsilon_i)-vcQ_i(p_i)-vc\varepsilon_i-(vc+h)(s_i-\varepsilon_i) \tag{A.5}$$

$$p_i(Q_i(p_i)+\varepsilon_i)-vc(Q_i(p_i)+\varepsilon_i)-(vc+h)(s_i-\varepsilon_i) \tag{A.6}$$

$$(p_i-vc)(Q_i(p_i+\varepsilon_i)-(vc+h)(s_i-\varepsilon_i) \tag{A.7}$$

$$p_i(Q_i(p_i)+s_i)-vc(Q_i(p_i)+s_i)-g_i(\varepsilon_i-s_i) \tag{A.8}$$

$$p_i(Q_i(p_i)+p_is_i-vcQ_i(p_i)-vcs_i-g_i(\varepsilon_i-s_i) \tag{A.9}$$

$$p_iQ_i(p_i)+p_is_i-vcQ_i(p_i)-vcs_i+p_i\varepsilon_i \\ -p_i\varepsilon_i+vc\varepsilon_i-vc\varepsilon_i-g_i(\varepsilon_i-s_i) \tag{A.10}$$

$$p_iQ_i(p_i)+p_i\varepsilon_i-vcQ_i(p_i)-vc\varepsilon_i \\ -p_i\varepsilon_i+p_is_i+vc\varepsilon_i-vcs_i-g_i(\varepsilon_i-s_i) \tag{A.11}$$

$$(p_i-vc)(Q_i(p_i)+\varepsilon_i)-p_i(\varepsilon_i-s_i) \\ +vc(\varepsilon_i-s_i)-g_i(\varepsilon_i-s_i) \tag{A.12}$$

$$(p_i-vc)(Q_i(p_i)+\varepsilon_i)-(p_i-vc+g_i)(\varepsilon_i-s_i) \tag{A.13}$$

Rearranging the terms in Equation (2) to express profit in marginal terms yields

$$\Pi(p_i,s_i)=\begin{cases} (p_i-vc)(Q_i(p_i)+\varepsilon_i)-(vc+h)(s_i-\varepsilon_i), \\ \quad \varepsilon_i\leqslant s_i \qquad (\mathbf{3}) \\ (p_i-vc)(Q_i(p_i)+\varepsilon_i)-(p_i-vc+g_i)(\varepsilon_i-s_i), \\ \quad \varepsilon_i>s_i \end{cases} \tag{A.14}$$

Thus, expected profit is

$$E[\Pi(p_i,s_i)]=(p_i-vc)(Q_i(p_i)+\mu_i) \\ -(vc+h)\int_a^{s_i}(s_i-\varepsilon_i)f_i(\varepsilon_i) \\ \mathrm{d}\varepsilon_i-(p_i-vc+g_i) \\ \int_{s_i}^{b}(\varepsilon_i-s_i)f_i(\varepsilon_i)\mathrm{d}\varepsilon_i \tag{A.15}$$

Letting $I_i(s_i)=\int_a^{s_i}(s_i-\varepsilon_i)f_i(\varepsilon_i)\mathrm{d}\varepsilon_i$ and $S_i(s_i)=\int_{s_i}^{b}(\varepsilon_i-s_i)f_i(\varepsilon_i)\mathrm{d}\varepsilon_i$ yields

$$E[\Pi(p_i,s_i)]=(p_i-vc)(Q_i(p_i)+\mu_i)-(vc+h)I_i(s_i) \\ -(p_i-vc+g_i)S_i(s_i) \quad (\mathbf{4}) \tag{A.16}$$

To determine price, p_i, consider the first and second partial derivatives of $E[\Pi(s_i,p_i)]$ (Equation (A.16)) taken with respect to p_i:

$$\frac{\partial E[\Pi(p_i,s_i)]}{\partial p_i}=(p_i-vc)\frac{\mathrm{d}Q_i(p_i)}{\mathrm{d}p_i} \\ +(Q_i(p_i)+\mu_i)-S_i(s_i) \quad (\mathbf{5}) \tag{A.17}$$

$$\frac{\partial^2 E[\Pi(p_i,s_i)]}{\partial^2 p_i}=(p_i-vc)\frac{\mathrm{d}^2Q_i(p_i)}{\mathrm{d}^2p_i}+\frac{\mathrm{d}Q_i(p_i)}{\mathrm{d}p_i} \quad (\mathbf{6}) \tag{A.18}$$

Since $Q_i(p_i)$ is a continuously decreasing, function (4) is maximized.

To determine capacity adjustment, s_i, consider the first and second partial derivatives of $E[\Pi(s_i,p_i)]$ (Equation (A.16)) taken with respect to s_i:

$$\frac{\partial E[\Pi(p_i,s_i)]}{\partial s_i}=-(vc+h)\frac{\mathrm{d}I_i(s_i)}{\mathrm{d}s_i}-(p_i-vc+g_i)\frac{\mathrm{d}S_i(s_i)}{\mathrm{d}s_i} \tag{A.19}$$

$$\frac{dI_i(s_i)}{ds_i} = \int_a^{s_i} \frac{\partial(s_i - \varepsilon_i)f_i(\varepsilon_i)}{\partial s_i} d\varepsilon_i + (s_i - s_i)f_i(s_i)(1) - (s_i - a)f_i(a)(0) \quad (A.20)$$

$$= \int_a^{s_i} ((1)f_i(\varepsilon_i) + (s_i - \varepsilon_i)(0))d\varepsilon_i \quad (A.21)$$

$$= \int_a^{s_i} f_i(\varepsilon_i)d\varepsilon_i = F_i(s_i) \quad (A.22)$$

$$\frac{dS_i(s_i)}{ds_i} = \int_{s_i}^{b} \frac{\partial(\varepsilon_i - s_i)f_i(\varepsilon_i)}{\partial s_i} d\varepsilon_i + (s_i - b)f_i(b)(0) - (s_i - s_i)f_i(s_i)(1) \quad (A.23)$$

$$= \int_{s_i}^{b} ((-1)f_i(\varepsilon_i) + (\varepsilon_i - s_i)(0))d\varepsilon_i \quad (A.24)$$

$$= -\int_{s_i}^{b} f_i(\varepsilon_i)d\varepsilon_i = -(1 - F_i(s_i)) \quad (A.25)$$

Substituting Equations (A.22) and (A.25) into Equation (A.19) yield

$$\frac{\partial E[\Pi(p_i, s_i)]}{\partial s_i} = -(vc + h)F_i(s_i) + (p_i - vc + g_i)(1 - F_i(s_i)) \ (\mathbf{7}) \quad (A.26)$$

with a second derivative of

$$\frac{\partial^2 E[\Pi(p_i, s_i)]}{\partial^2 s_i} = -(p_i + g_i + h)f(s_i) \ (\mathbf{8}) \quad (A.27)$$

<0 thus, Equation (4) is maximized

Thus, setting (A.26)$\equiv$0 yields

$$0 \equiv -(vc + h)F_i(s_i) + (p_i - vc + g_i)(1 - F_i(s_i)) \quad (A.28)$$

$$(vc + h)F_i(s_i^*) = (p_i - vc + g_i)(1 - F_i(s_i^*)) \quad (A.29)$$

$$F_i(s_i^*) = (p - vc + g_i)/(p_i - vc + g_i + vc + h) \quad (A.30)$$

$$F_i(s_i^*) = (p - vc + g_i)/(p_i + g_i + h) \ (\mathbf{9}) \quad (A.31)$$

Other operationally useful forms include:

$$(1 - F_i(s_i^*)) = (vc + h)/(p_i + g_i + h) \quad (A.32)$$

$$(vc + h) = (p_i + g_i + h)(1 - F_i(s_i)) \quad (A.33)$$

As an example, let $Q_i(p_i)$ be estimated over a linear range by $Q_i(p_i) = \beta_0 + \beta_1 p_i$. From Equation (A.16), the special case of the riskless (no excess or shortage of capacity) version of expected profit is:

$$E[\Pi(p_i)] = (p_i - vc)(Q_i(p_i) + \mu_i) \quad (A.34)$$

Substituting the linear estimation for $Q_i(p_i)$ in (A.34) and differentiating yields:

$$\frac{\partial E[\Pi(p_i)]}{\partial p_i} = (p_i - vc)\beta_1 + (\beta_0 + \beta_1 p_i + \mu_i) \quad (A.35)$$

and a second derivative of

$$\frac{\partial^2 E[\Pi(p_i, s_i)]}{\partial^2 p_i} = 2\beta_1 \quad (A.36)$$

Since $Q_i(p_i)$ is a continuously decreasing function, then $\beta_1 < 0$ and (A.35) is maximized.

Setting Equation (A.35)$\equiv$0

$$0 = \beta_1 p_i - \beta_1 vc + \beta_0 + \beta_1 p_i + \mu_i \quad (A.37)$$

and solving for p_i results in

$$p_i^0 = \frac{\beta_1 vc - \beta_0 - \mu_i}{2\beta_i} \ (\mathbf{10}) \quad (A.38)$$

The riskless price, p_i^0, is used to solve for the price under risk. Substituting the linear estimation for $Q_i(p_i)$ in Equation (A.17) and taking the first derivative yields

$$\frac{\partial E[\Pi(p_i, s_i)]}{\partial p_i} = (p_i - vc)\beta_1 + (\beta_0 + \beta_1 p_i + \mu_i) - S_i(s_i) \quad (A.34')$$

$$= \beta_1 p_i - \beta_1 vc + \beta_0 + \beta_1 p_i + \mu_i - S_i(s_i) \quad (A.35')$$

$$= 2\beta_1 p_i - (\beta_1 vc - \beta_0 - \mu_i) - S_i(s_i) \quad (A.36')$$

$$= 2\beta_1 p_i - 2\beta_1 \frac{\beta_1 vc - \beta_1 - \mu_i}{2\beta_1} - S_i(s_i) \quad (A.37')$$

$$= 2\beta_1 p_i - 2\beta_1 p_i^0 - S_i(s_i) \quad (A.38')$$

$$= 2\beta_1 (p_i - p_i^0) - S_i(s_i) \ (\mathbf{11}) \quad (A.39)$$

$$0 \equiv 2\beta_1 (p_i - p_i^0) - S_i(s_i) \quad (A.40)$$

To check the second-order optimality:

$$\frac{\partial^2 E[\Pi(p_i, s_i)]}{\partial^2 p_i} = 2\beta_{1i} \quad \mathbf{(12)} \tag{A.41}$$

Since β_{1i} is <0, the special case of the linear form of (A.16) is maximized by

$$p_i^* = p_i^0 + S(s_i)/2\beta_1 \quad \mathbf{(13)} \tag{A.42}$$

Differential pricing policy

Starting with Equation (A.26):

$$\frac{\partial E[\Pi(p_i, s_i)]}{\partial s_i}$$

$$= -(vc + h)F_i(s_i) + (p_i - vc + g_i)(1 - F_i(s_i)) \tag{A.2'}$$

$$\begin{aligned} = &- (vc + h)F_i(s_i) + (p_i - vc + g_i)(1 - F_i(s_i)) \\ &+ (vc + h)(1 - F_i(s_i)) - (vc + h)(1 - F_i(s_i)) \end{aligned} \tag{A.43}$$

$$\begin{aligned} = &- (vc + h)F_i(s_i) - (vc + h)(1 - F_i(s_i)) \\ &+ (p_i - vc + g_i)(1 - F_i(s_i)) + (vc + h)(1 - F_i(s_i)) \end{aligned} \tag{A.44}$$

$$\begin{aligned} = &- (vc + h)(F_i(s_i) + (1 - F_i(s_i)) \\ &+ ((p_i - vc + g_i) + (vc + h))(1 - F_i(s_i)) \end{aligned} \tag{A.45}$$

$$= -(vc + h) + (p_i + g_i + h)(1 - F_i(s_i)) \tag{A.46}$$

Setting Equation (A.52)≡0:

$$0 = -(vc + h) + (p_i + g_i + h)(1 - F_i(s_i)) \tag{A.47}$$

$$\begin{aligned} (vc + h) &= (p_i + g_i + h)(1 - F_i(s_i)) \\ &\text{for each class } i \quad \mathbf{(14)} \end{aligned} \tag{A.48}$$

$$\begin{aligned} (vc + h) &= (p_1 + g_1 + h)(1 - F_1(s_1)) \\ &= (p_2 + g_2 + h)(1 - F_2(s_2)) \quad \mathbf{(15)} \end{aligned} \tag{A.49}$$

$$\frac{p_1 + g_1 + h}{p_2 + g_2 + h} = \frac{1 - F_2(s_2)}{1 - F_1(s_1)} \quad \mathbf{(16)} \tag{A.50}$$

Optimal capacity reallocation

$$\begin{aligned} E[L(s_1] = &\int_a^{s_1} (p_2 - vc + g_2(s_1 - \varepsilon_1)f_1(\varepsilon_1)\mathrm{d}\varepsilon_1 \\ &+ \int_{s_2}^{b} ((p_1 + g_1)(p_2 + g_2))(\varepsilon_1 - s_1)f_1(\varepsilon_1)\mathrm{d}\varepsilon_1 \quad \mathbf{(17)} \end{aligned} \tag{A.51}$$

$$\begin{aligned} E[L(s_1)] = &(p_2 - vc + g_2)I_1(s_1) \\ &+ ((p_1 + g_1) - (p_2 + g_2))S_1(s_1) \end{aligned} \tag{A.52}$$

See Equations (A.15) and (A.16) for the definitions of $I(s_1)$ and $S(s_1)$ and Equations (A.22) and (A.25) for their derivatives.

$$\begin{aligned} \frac{\partial E[L(s_1)]}{\partial s_1} = &(p_2 - vc + g_2)\frac{\mathrm{d}I_1(s_1)}{\mathrm{d}s_1} \\ &+ ((p_1 + g_1) - (p_2 + g_2))\frac{\mathrm{d}S_1(s_1)}{\mathrm{d}s_1} \end{aligned} \tag{A.53}$$

$$\begin{aligned} = &(p_2 - vc + g_2)F_1(s_1) - (p_1 + g_1) \\ &- (p_2 + g_2))(1 - F_1(s_1)) \quad \mathbf{(18)} \end{aligned} \tag{A.54}$$

$$\begin{aligned} \frac{\partial^2 E[L(s_1)]}{\partial^2 s_1} = &((p_2 - vc + g_2) + ((p_1 + g_1) \\ &- (p_2 + g_2)))f_1(s_1) \geqslant 0 \quad \mathbf{(19)} \end{aligned} \tag{A.55}$$

Thus, Equation (A.52) is minimized. Setting Equation (A.54)≡0 and rearranging terms yields

$$\begin{aligned} &(p_2 - vc + g_2)F_1(s_1^*) \\ &= ((p_1 + g_1) - (p_2 + g_2))(1 - F_1(s_1^*)) \quad \mathbf{(20)} \end{aligned} \tag{A.56}$$

$$\begin{aligned} &(p_2 - vc + g_2)F_1(s_1^*) + (p_2 - vc + g_2)(1 - F_1(s_1^*)) \\ &= ((p_1 + g_1) - (p_2 + g_2))(1 - F_1(s_1^*)) \\ &\quad + (p_2 - vc + g_2)(1 - F_1(s_1^*)) \end{aligned} \tag{A.57}$$

$$\begin{aligned} &(p_2 - vc + g_2)(F_1(s_1^*) + 1 - F_1(s_1^*)) \\ &= ((p_1 + g_1) - (p_2 + g_2) + (p_2 - vc + g_2))(1 - F_1(s_1^*)) \end{aligned} \tag{A.58}$$

$$(p_2 - vc + g_2)(1) = (p_1 + g_1 - p_2 - g_2 + p_2 - vc + g_2)(1 - F_1(s_1^*)) \quad (A.59)$$

$$(p_2 - vc + g_2) = (p_1 - vc + g_1)(1 - F_1(s_1^*)) \quad (A.60)$$

$$(p_1 - vc + g_1)(1 - F_1(s_1^*)) = (p_2 - vc + g_2) \quad (\mathbf{21}) \quad (A.61)$$

Solving for $F_1(s_1^*)$ yields the service level version of (A.61):

$$F_1(s_1^*) = ((p_1 + g_1) - (p_2 + g_2))/(p_1 - vc + g_1) \quad (A.62)$$

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