Imperial College London

Signal Propagation through a Lumped Transmission Line

Abstract: In this presentation we will report the findings of an investigation into the properties of a lumped transmission line consisting of forty $330\mu H$ inductors, forty-three $15nF$ capacitors and one 100Ω fixed resistor. The signal used is a sinusoidal alternating current with a peak-to-peak voltage of $4V$ and zero-offset.

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Theory

Wave equation:

$$\frac{\partial V(x,t)}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 (x,t)}{\partial x^2} \qquad (1)$$

Like the EM equation, the constant of proportionality gives us the velocity! In this case the phase velocity!

$$V_{Phase} = \frac{1}{\sqrt{LC}} \qquad (2)$$

Phase velocity: Speed at which the **waves** travel.
Group velocity: Speed at which the **waveforms** travel.

$$Z_0(\omega) = \sqrt{\frac{L}{C}\frac{1}{1-\frac{\omega^2 LC}{4}}} \qquad (3)$$

Cut-off frequency & characteristic impedance:

$$\omega_c = \sqrt{\frac{4}{LC}}, \qquad Z_0 \approx \sqrt{\frac{L}{C}} \qquad (4)$$

Reflection:

$$R = \frac{V_r}{V_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad (5)$$

Summary:

- Higher frequency → Higher impedance
- Higher impedance → Higher attenuation
- Higher attenuation → Waveform dies off faster

We will now investigate how varying frequencies impact the propagation of waves.

Circuit

- The circuit consists of 40 inductors and 43 capacitors arranged in lumps, with each lump consisting of one inductor and one capacitor.
- Each inductor has an inductance of $330\mu H$ and each capacitor has a capacitance of $15nF$.
- A 200Ω fixed resistor is placed in series before the lumped transmission line, and two capacitors are connected in parallel to the lumped transmission line both at the beginning and the end of the line.
- A variable resistor is connected at the end of the circuit, with resistance range of $0 - 5k\Omega$. The circuit could be flipped between the multimeter terminals or the variable resistor by toggling the switch.
- At intervals of 4 lumps, there is a node where we can talke measurements from.

- The circuit + oscilloscope was tested for impedance contribution from other sources before the experiment. These impedance is found to be negligible.
- The source is then set to generate a sine-wave with peak-to-peak voltage of 4V and zero amplitude off-set.
- Probes are connected to the first node and the last, and measurements were taken in channel 1 and channel 2 respectively.
- The frequency of the sine-wave is allowed to be varied.

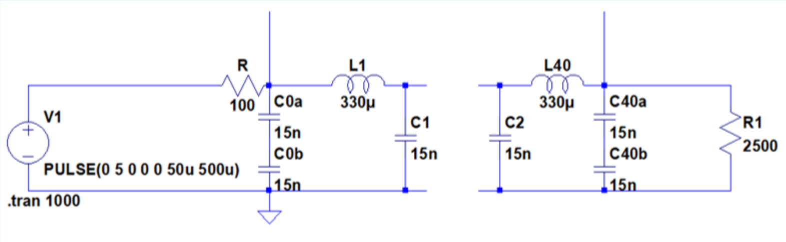

Diagram 1: Circuit diagram of the transmission line.

Methodology

Set Up:
The RTB2004 oscilloscope functions both as a signal generator and a measurement device. The probe from channel 1 is connected to the first node (0 lumps) of the transmission line and the probe from channel 2 is connected to the last node (40 lumps).

Measurement:
To obtain the measurements, we used the "measure" function provided by the oscilloscope. The oscilloscope was set to collect the following four statistics:

1. amplitude of channel 1,
2. amplitude of channel 2,
3. frequency of channel 1, and
4. phase difference of the two waveforms.

When 1000 measurements are taken, we exported the statistics from the oscilloscope as an Excel file.

The Experiment:
Using the "sweep" function in the signal generation, we defined regular 10kHz intervals from 10kHz to 140kHz (near the estimated cut-off frequency) for the oscilloscope to run through. The process was looped until at least 1000 measurement points were taken, where the data is exported. We then set the signal generation to the next 10kHz range, for example from 20kHz to 30kHz, and the process was repeated up to 140kHz. The dataset is then merged into one dataset ranging from 10kHz to 140kHz.

Regular Frequency Points:
To define regular frequency points as the basis for our analysis, we decided to lump each in-phase and out-of-phase datapoints ($\pm 5°$) together and use the mean and standard deviation of each set.

Data & Observations

Figure 1 shows the dataset we obtained at node 0 plotted against frequency. Some main key points from the plot are:

- **The high level of fluctuation:** mainly caused by the reflection from the end of the lumped transmission line.
- **The seemingly discrete vertical distribution:** due to the minimum resolution of RTB2004 [1].
- **Consistency of dataset:** although the waveform fluctuates greatly, the dataset remains in a single line-like form, suggesting high precision of measurement. Note that the dataset consists of 14000 datapoints.

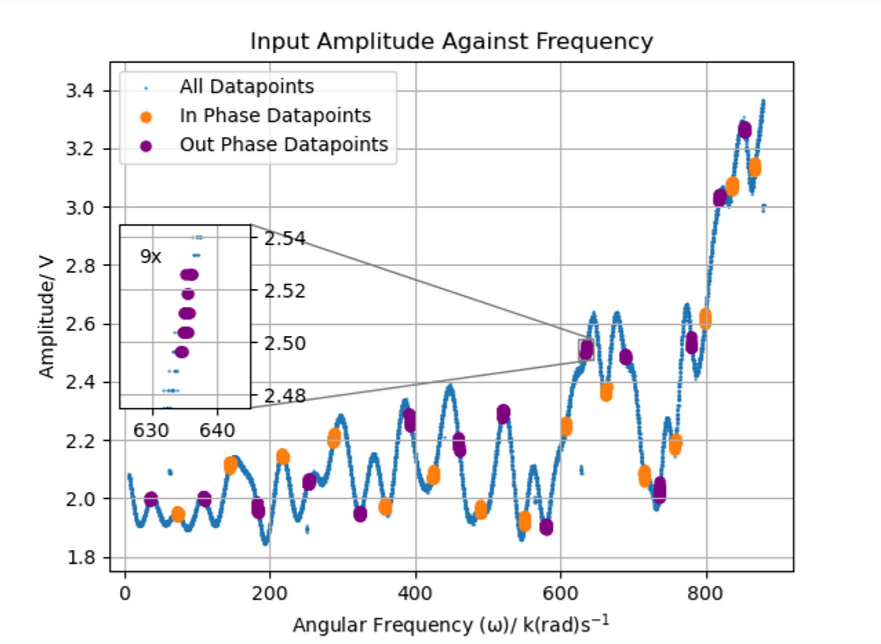

Figure 1: Input amplitude at node 0, measured in channel 1.

[1] "R&S® RTB2000 Oscilloscope Specifications, Data Sheet version 16.00", Rohde & Schwarz.

Data & Observations

Figure 2 shows the dataset obtained at node 40 from channel 2. Key features to note are:

- **Smaller fluctuations:** the data was taken at node 40, which is at the end of the transmission line, thus no reflection present.
- **Unable to obtain data automatically near 0 and cut-off-frequency.**

Next steps:

Our next step after collecting the data would be to use the data for calculations. However, the dataset as it is, is difficult to work with.

Therefore, we lumped each set of in-phase and out-of-phase datapoints together to produce a single mean point with standard deviation. We will work with these datapoint in the later parts of the experiment.

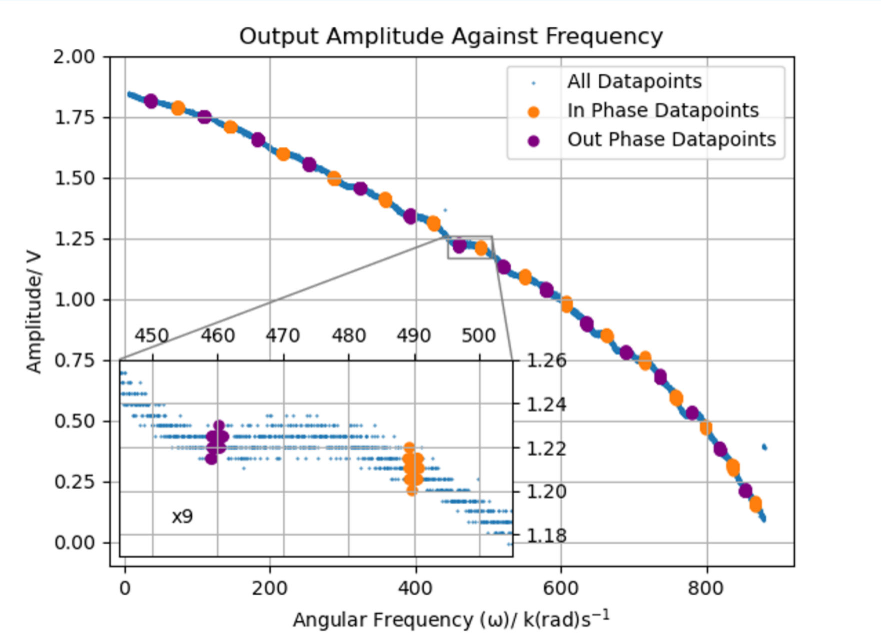

Figure 2: Output amplitude at node 40, measured in channel 2.

Analysis

We merged the previous datasets to produce a plot as shown in Figure 3.

Here, the in-phases and out-of-phase points are merged into a single point with associated error. The original datapoints are kept in the plot for reference but are not used in the calculations.

Using the last six in-phase and out-of-phase datapoints, we extrapolated the curve using a linear assumption.

The cut-off frequency is estimated to be around $873 \pm 10\ krads^{-1}$ or $139 \pm 2\ kHz$.

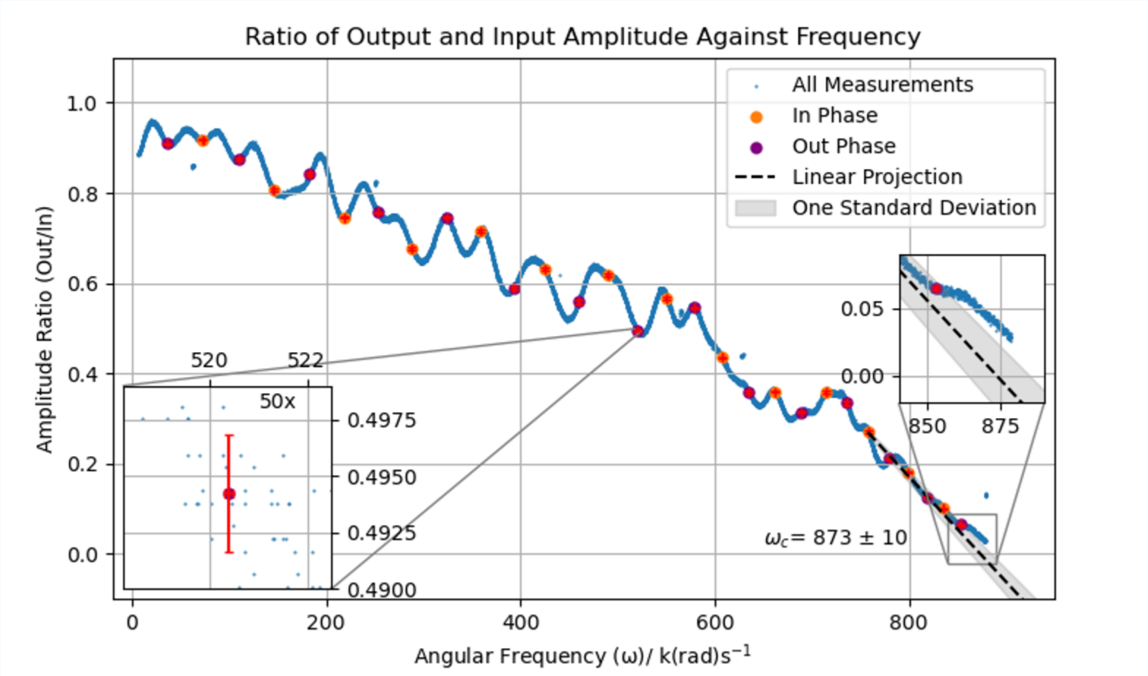

Figure 3: Ratio of output amplitude over input amplitude.

Analysis

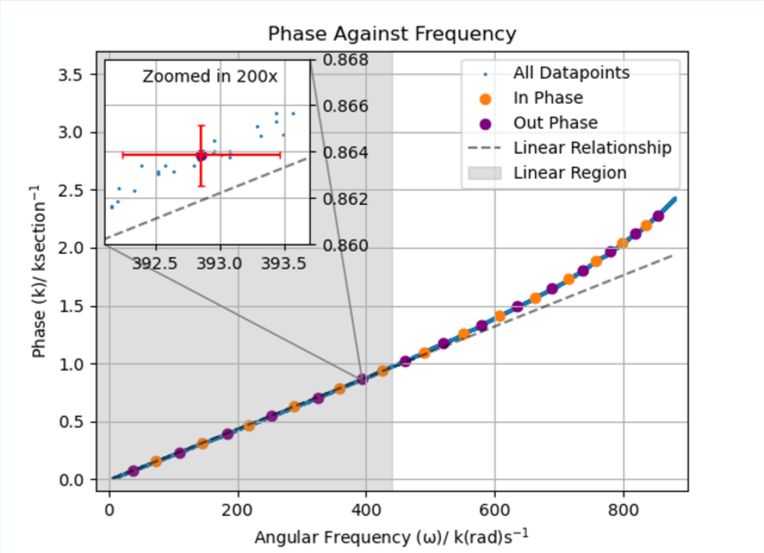

Figure 4: The above plot shows the same datapoints plotted with phase difference against frequency. We can see that for low frequencies, the relationship is approximately linear.

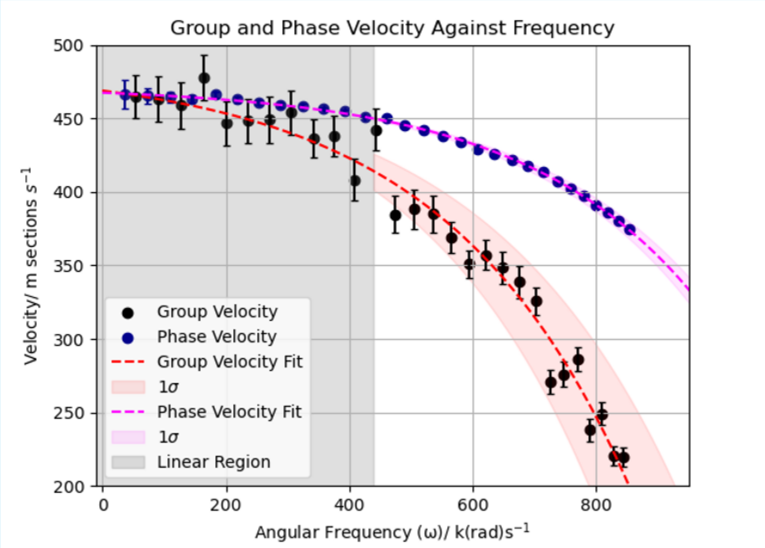

Figure 5: Group velocity and phase velocity plotted against frequency. The cut-off frequency is estimated to be $850 \pm 70\ k(rad)s^{-1}$ or $136 \pm 11kHz$.

Further Investigations

Ways to improve experiment:

1. **Match the impedance for each datapoint:** we could take less number of datapoints, but for each datapoint, match the impedance to remove the reflected wave. **Issue:** This might shift the voltage (requires testing).
2. **Use PyVISA to automate the data collection process:** I have written the code to automate the data collection process so that more precise datapoints could be obtained. It would also greatly save human labour.
3. **Further automate and enable remote use of the oscilloscope:** through the use of a RaspberryPi.
4. **A combination of all the changes outlined above!** PyVISA routine to scan through the range of frequencies slowly, while matching impedance. This would require a lot of time!

Why study transmission lines?

- **The study of waves:** could draw parallel to other forms of wave propagation. Electric waves are quite easily manipulated and variables are easily controlled.
- **Long range communication:** through learning about the properties of transmission lines, we are able to construct high quality long range transmission lines. [2]
- **Recover high quality signals:** By studying the way signals deform and propagate, we can design circuitry or that delivers signals with minimal disruption. [3]

[2] Z.M. Peterson, "*Why Impedance Matching is Important in a Transmission Line*", 28 November 2019, available at: https://www.nwengineeringllc.com/article/.
[3] M. Malinverno, *"What is impedance in audio (and why it is important)"*, 21 January 2022, available at: https://splice.com/blog/.

Summary & Conclusion

- **We first introduced our circuit and methodology:** The circuit consists of 43 $15nF$ capacitors and 40 $330\mu H$ inductors, with nodes spread at regular interval throughout the transmission line. The data is obtained by scanning through the frequencies.
- **Then we displayed some of our results:** We also went through some general observation and explanation on our results.
- **We compared some of our experimental results to the theoretical results:** Using two different methods, we experimentally calculated the cut-off frequency of $873 \pm 10k(rad)s^{-1}$ and $850 \pm 70k(rad)s^{-1}$, compared to the theoretical value of $930 \pm 208k(rad)s^{-1}$. They are found to be in agreement.
- **And finally, we went through some methods of improving the experiment.**

Thank You for Your Attention!

For access to the code and data for the experiment, please scan the QR code below.