MTH 3105: Discrete Mathematics

Take Home Assignment I

1.

(a) Define the sequence $c_0, c_1, \ldots$ by the equations $c_0 = 0$ and $c_n = c_{\lfloor n/2 \rfloor} + 3$ for all $n > 0$. Prove that $c_n \leq 2n$ for all $n \geq 3$. (5 Marks)

(b) Prove that any positive integer N is divisible by 11 if and only if the difference between the sum of odd digits and the sum of even digits is divisible by 11. (5 Marks)

2.

(a) Assume that $\forall x \exists y P(x, y)$ is false and the domain of them is nonempty. Which of the following must be false? (2 Marks)

(i) $\forall x \forall y P(x, y)$

(ii) $\exists x \forall y P(x, y)$

(iii) $\exists x \exists y P(x, y)$

(b) Let $P(x)$ denote the statement "x is an accountant" and let $Q(x)$ denote the statement "x owns a Porsche". Write each statement below in first order logic.

(i) All accountants own Porsches. (2 Marks)

(ii) Some accountant owns a Porsche. (2 Marks)

(iii) All owners of Porsches are accountants. (2 Marks)

(iv) Someone who owns a Porsche is an accountant. (2 Marks)

3.

(a) Prove that $\sqrt{6}$ is irrational. (4 Marks)

(b) A detective has interviewed four witnesses to a crime. From their stories, the detective has concluded that:

(i) If the butler is telling the truth, then so is the cook.

(ii) The cook and the gardener cannot both be telling the truth.

(iii) The gardener and the handyman are not both lying.

(iv) If the handyman is telling the truth then the cook is lying.

Who must be lying? There may be more than one liar. Show your steps. (6 Marks)

4.

(a) Simplify the following statement: (2 Marks)

$\neg (\neg q \wedge \neg (\neg q \vee s)) \vee (q \wedge (r \rightarrow r))$

(b) Determine whether or not the following arguments are valid:

(i) $\dfrac{\neg p \rightarrow q, \neg q}{p}$ (4 Marks)

(ii) $\dfrac{\neg p \rightarrow \neg q}{p \rightarrow q}$ (4 Marks)

This assignment is due 1 week from the 17th of October, 2017.