CMP2020M Artificial Intelligence

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CM2020M Artificial Intelligence

- Introduction
- Logic Programming(**1**/3)
- Knowledge Representations
- Games AI & Search(/3)
- Planning I
- Planning II
- Probabilistic AI

My Lectures

1. Propositional Calculus
2. Predicate Calculus
3. Prolog Programming
4. Knowledge Representation

Propositional Calculus

- What is Logic Programming
- The Propositional Calculus
  - Syntax
  - Semantics
  - Proofs
- Next week: Predicate calculus and Unification

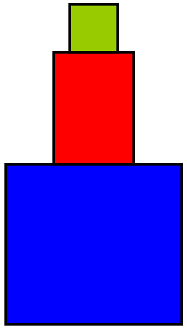
What is Logic Programming?

- In logic programming, you present facts and rules to infer new facts by just ask questions.

- When you asked a question, the run time system searches through the database of facts and rules to determine (by logical deduction) the answer.

Example

- Given:
  - “The red block is above the blue block”
  - “The green block is above the red block”
- Infer?
  - “The green block is above the blue block”
  - “The blocks form a tower”



Logic consists of

- A language
  - tells us how to build up sentences (i.e., *syntax*), and what that sentences mean (i.e., *semantics*)
- An inference procedure
  - which tells us which sentences are valid inference from given sentences

Propositional Logic

- The syntax of propositional logic consists of
  - the propositional symbols:
    - P, Q, R, S, …
  - and connectives:
    - $\wedge$, $\vee$, $\neg$, $\rightarrow$, $\equiv$

- The semantics (interpretation) is assigning a truth value (T or F) to each sentence.

Examples of Propositional Logic sentences

- P means "It is hot."
- Q means "It is humid."
- R means "It is raining."
- $(P \wedge Q) \rightarrow R$

  "If it is hot and humid, then it is raining"
- $Q \rightarrow P$

  "If it is humid, then it is hot"

Propositional calculus semantics

P	$\neg$P
T	F
F	T

Truth table for the operator $\neg$ Negation (not)

Propositional calculus semantics

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth table for the operator $\wedge$ Conjunction (and)

Propositional calculus semantics

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

Truth table for the operator ∨ Disjunction (or)

Propositional calculus semantics

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth table for the operator → Implication

Propositional calculus semantics

P	Q	P≡Q
T	T	T
T	F	F
F	T	F
F	F	T

Truth table for the operator ↔ ⇔ ≡ Equivalence (if and only if)

Truth table is demonstrating the equivalence of $\neg P \lor Q$ and $P \to Q$

P	**Q**	**$\neg P$**	**$\neg P \lor Q$**	**$P \to Q$**	**$(\neg P \lor Q) \equiv (P \to Q)$**

Truth table is demonstrating the equivalence of $\neg P \vee Q$ and $P \rightarrow Q$

P	Q	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q$	$(\neg P \vee Q) \equiv (P \rightarrow Q)$
T	T				
T	F				
F	T				
F	F				

Truth table is demonstrating the equivalence of $\neg P \vee Q$ and $P \rightarrow Q$

P	Q	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q$	$(\neg P \vee Q) \equiv (P \rightarrow Q)$
T	T	F			
T	F	F			
F	T	T			
F	F	T			

Truth table is demonstrating the equivalence of $\neg P \lor Q$ and $P \to Q$

P	Q	$\neg P$	$\neg P \lor Q$	$P \to Q$	$(\neg P \lor Q) \equiv (P \to Q)$
T	**T**	**F**	**T**		
T	**F**	**F**	**F**		
F	**T**	**T**	**T**		
F	**F**	**T**	**T**		

Truth table is demonstrating the equivalence of $\neg P \vee Q$ and $P \rightarrow Q$

P	Q	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q$	$(\neg P \vee Q) \equiv (P \rightarrow Q)$
T	T	F	T	T	
T	F	F	F	F	
F	T	T	T	T	
F	F	T	T	T	

Truth table is demonstrating the equivalence of $\neg P \vee Q$ and $P \rightarrow Q$

P	Q	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q$	$(\neg P \vee Q) \equiv (P \rightarrow Q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Tautology and Contradiction

- A tautology is a formula which is "always true".
  - It is true for every assignment of truth values to its simple components.

- The opposite of a tautology is a contradiction, a formula which is "always false".
  - It is false for every assignment of truth values to its simple components.

Exercise 1

- Use a truth table to demonstrate the equivalence of $\neg(P \vee Q)$ and $(\neg P \wedge \neg Q)$

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
T	T						
T	F						
F	T						
F	F						

Exercise 1 : Answer

- Use a truth table to demonstrate the equivalence of $\neg(P \vee Q)$ and $(\neg P \wedge \neg Q)$

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
T	T	T	F				
T	F	T	F				
F	T	T	F				
F	F	F	T				

Exercise 1 : Answer

- Use a truth table to demonstrate the equivalence of $\neg(P \lor Q)$ and $(\neg P \land \neg Q)$

P	Q	$P \lor Q$	$\neg(P \lor Q)$	$\neg P$	$\neg Q$	$\neg P \land \neg Q$	$\neg(P \lor Q) \equiv (\neg P \land \neg Q)$
T	T	T	F	F	F		
T	F	T	F	F	T		
F	T	T	F	T	F		
F	F	F	T	T	T		

Exercise 1 : Answer

- Use a truth table to demonstrate the equivalence of $\neg(P \vee Q)$ and $(\neg P \wedge \neg Q)$

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
T	T	T	F	F	F	F	
T	F	T	F	F	T	F	
F	T	T	F	T	F	F	
F	F	F	T	T	T	T	

Exercise 1 : Answer

- Use a truth table to demonstrate the equivalence of $\neg(P \vee Q)$ and $(\neg P \wedge \neg Q)$

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

The $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$ is a tautology sentence

Proving things

- A **proof** is a sequence of sentences, where each sentence is either a premise or a sentence derived from earlier sentences in the proof by one of the inference rules.

- The last sentence is the **theorem** (also called goal or query) that we want to **prove**.

Exercise 2

- Symbolise the following propositions. The meaning of symbols chosen for statements also needs to be given:

- Terry likes Science-fiction <u>and</u> going to the Gym.
- Lectures are valuable <u>if and only if</u> they are structured carefully
- Today is Thursday <u>or</u> Yesterday was Wednesday.
- <u>If</u> I do <u>not</u> work hard <u>then</u> I will not pass my AI exam.

Exercise 2: Answer

- Terry likes Science-fiction <u>and</u> going to the Gym.
  - T : Terry likes Science fiction, G : Terry likes going to the gym
  - $T \wedge G$
- Lectures are valuable <u>if and only if</u> they are structured carefully
  - L : Lectures are valuable, S : Lectures are structured carefully
  - $L \equiv S$
- Today is Thursday <u>or</u> Yesterday was Wednesday.
  - T : Today is Thursday, Y : Yesterday was Wednesday
  - $T \vee Y$
- <u>If</u> I do <u>not</u> work hard <u>then</u> I will not pass my AI exam.
  - W : I work hard, A : I will pass my AI exam
  - $\neg W \rightarrow \neg A$

Proofs in Propositional Calculus

- If it is sunny today, then the sun shines on the screen. If the sun shines on the screen, the blinds are brought down. The blinds are not down. (*these are the given premises*)
- Is it sunny today? *(this is the goal*)

- P: It is sunny today.
- Q: The sun shines on the screen.
- R: The blinds are down.
- Given: $P \rightarrow Q$, $Q \rightarrow R$, $\neg R$

Premise

- Question: P

goal

Proof using a truth table

Variables			Given			Trial Conclusions	
P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$\neg R$	P	$\neg P$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

If it is sunny today, then the sun shines on the screen. If the sun shines on the screen, the blinds are brought down. The blinds are not down.

Is it sunny today? *(this is the goal)*

Proof using a truth table

Variables			Given			Trial Conclusions	
P	Q	R	P→Q	Q→R	¬R	P	¬P
T	T	T	T	T	F	T	F
T	T	F	T	F	T	T	F
T	F	T	F	T	F	T	F
T	F	F	F	T	T	T	F
F	T	T	T	T	F	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	F	F	T
F	F	F	T	T	T	F	T

Given:

P→Q

Q→R

¬R

Question: P

Given: $P \rightarrow Q$, $Q \rightarrow R$, $\neg R$
Question: P

Proof using a truth table

Variables			Given			Trial Conclusions	
P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$\neg R$	P	$\neg P$
T	T	T	T	T	F	T	F
T	T	F	T	F	T	T	F
T	F	T	F	T	F	T	F
T	F	F	F	T	T	T	F
F	T	T	T	T	F	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	F	F	T
F	F	F	T	T	T	F	T

Answer: F F F T T T F T

It is not sunny today

Proof procedure

- The problem with proof using truth tables is that the number of rows required grows very quickly as the number of propositional variables increases (2^n).

- A proof procedure is another method of proving statements using inference rules.

Inference rules

- *Modus ponens*

  If P and $P \rightarrow Q$ are true, then infer Q.

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

Inference rules

- *Modus ponens*

  If P and P $\rightarrow$ Q are true, then infer Q.

- *Modus tollens*

  If P $\rightarrow$ Q and $\neg$Q are true, then infer $\neg$P.

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

Inference rules

- *Modus ponens*

  If P and P $\rightarrow$ Q are true, then infer Q.

- *Modus tollens*

  If P $\rightarrow$ Q and $\neg$Q are true, then infer $\neg$P.

- *And elimination*

  If P $\wedge$ Q is true, then infer both P and Q are true

P	Q	P$\wedge$Q
T	T	T
T	F	F
F	T	F
F	F	F

Inference rules

- *Modus ponens*

  If P and P $\rightarrow$ Q are true, then infer Q.
- *Modus tollens*

  If P $\rightarrow$ Q and $\neg$Q are true, then infer $\neg$P.
- *And elimination*

  If P $\wedge$ Q is true, then infer both P and Q are true
- *And introduction*

  If both P and Q are true, then infer P $\wedge$ Q

P	Q	P$\wedge$Q
T	T	T
T	F	F
F	T	F
F	F	F

The Rules of Inference

Rule of inference	Tautology	Name
$p \to q$ $\underline{p \qquad}$ $\therefore q$	$[p \land (p \to q)] \to q$	Modus ponens
$\neg q$ $\underline{p \to q}$ $\therefore \neg p$	$[\neg q \land (p \to q)] \to \neg p$	Modus tollen
$p \to q$ $\underline{q \to r}$ $\therefore p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\underline{\neg p \quad}$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
$\underline{p \qquad}$ $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$\underline{p \land q}$ $\therefore p$	$(p \land q) \to p$	Simplification
p $\underline{q \qquad}$ $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\underline{\neg p \lor r}$ $\therefore q \lor r$	$[(p \lor q) \land (\neg p \lor r)] \to (p \lor r)$	Resolution

Proof using inference rules

- P: It is sunny today.
- Q: The sun shines on the screen.
- R: The blinds are down.
- Given: $P \rightarrow Q$, $Q \rightarrow R$, $\neg R$
- Question: P

Modus ponens: If P and P $\rightarrow$ Q are true, then infer Q.
Modus tollens: If P $\rightarrow$ Q and $\neg$Q are true, then infer $\neg$P.
And elimination: If P $\wedge$ Q is true, then infer both P and Q are true
And introduction: If both P and Q are true, then infer P $\wedge$ Q

Proof using inference rules

- $P \rightarrow Q$, $Q \rightarrow R$, $\neg R$ — Given

$\neg$ Q — Modus Tollens

$\neg$ P — Modus Tollens

So P is false

It is not sunny today

Modus ponens: If P and P $\rightarrow$ Q are true, then infer Q.
Modus tollens: If P $\rightarrow$ Q and $\neg$Q are true, then infer $\neg$P.
And elimination: If P $\wedge$ Q is true, then infer both P and Q are true
And introduction: If both P and Q are true, then infer P $\wedge$ Q

Example 2:

- Consider the following statements for the "weather problem".

1.	Humid	Premise	"It is humid"
2.	Humid$\rightarrow$Hot	Premise	"If it is humid, it is hot"
3.	(Hot$\wedge$Humid)$\rightarrow$Rain	Premise	"If it's hot & humid, it's raining"

Is it raining? Goal

4.	Hot	Modus Ponens (1, 2)	"It is hot"
5.	Hot$\wedge$Humid	And Introduction (1, 4)	"It is hot and humid"
6.	Rain	Modus Ponens (3, 5)	"It is raining"

Exercise 3

- Consider the following statements:
- IF food is older than 7 days THEN it is unsafe to eat. The food is 8 days old.
- Is it save to eat the food? why?

- What inference rule has been used?

Exercise 3: Answer

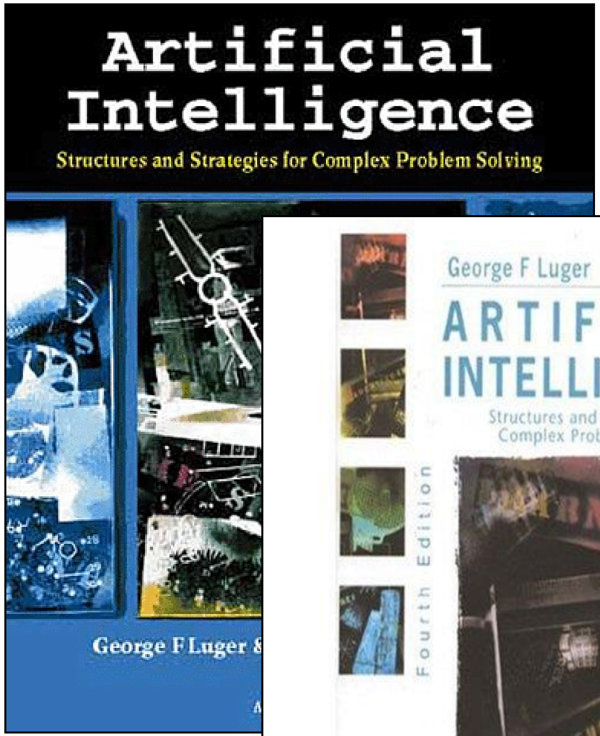
- Consider the following statements:
- IF food is older than 7 days THEN it is unsafe to eat. The food is 8 days old.
- Is it save to eat the food? why?
  - F: food is older than 7 days.
  - E: food is safe to eat
  - given:
    - $F \rightarrow \neg E$
    - F
  - Infer
    - $\neg E$
    - The food is unsafe to eat
- What inference rule has been used?
  - Modus Ponens : If P and P $\rightarrow$ Q are true, then infer Q.

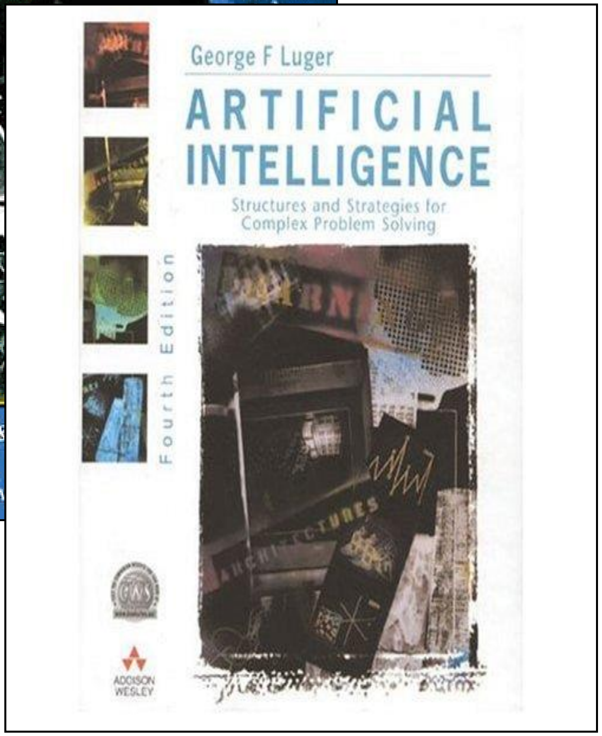
Next lecture: Predicate Calculus

- Predicate Calculus
- Unification
- Substitution

A little reading

- Luger, G.F. and Stubblefield, W.A., **Artificial Intelligence – Structures and Strategies for Complex Problem Solving**, (Addison-Wesley, 1998).
- Chapter 2 (3rd edition) 'The Predicate Calculus'
- take care with alternative editions – chapter numbers differ, and later editions are by Luger only





Thank you for listening!