Experiment Objective

To determine the elementary electric charge, *e*, by measuring the force on an oil drop in an electric field of known strength.

Related Theory

Millikan's Oil Drop Experiment allows the electric charge carried by an oil droplet to be determined by measuring the force experienced by the droplet in an electric field of known strength.

The mass of the drop can be determined from the measured falling velocity of the drop in air, together with the application of Stokes' Law. The observation of the velocity of the drop rising in an electric field then permits a calculation of the force on, and hence, the charge carried by the oil drop.

<u>No field – droplet falling at terminal velocity</u>

Gravitational force: $F_g = mg$

Force due to air resistance: $F_a = kv_f$

where *m* is the mass of the droplet, *g* is acceleration due to gravity, *k* is the coefficient of friction between the air and the droplet, and v_f is the velocity of the falling droplet.

If the droplet has reached terminal velocity, then these two forces are balanced:

$$mg = kv_f \tag{1}$$

<u>Field applied – droplet rising at a constant velocity</u>

Gravitational force: $F_g = mg$

Force due to air resistance: $F_a = kv_r$

Force on charge in an electric field: $F_e = Eq$

where *E* is the magnitude of the electric field and *q* is the charge on the droplet.

Adding these forces vectorially gives:

$$Eq = mg + kv_r \tag{2}$$

Note: in both cases there is a small buoyant force upwards, but this is neglected due to its relative small size in this derivation

An expression for q can then be found by eliminating *k* from (1) and (2), to give:

$$q = mg\frac{(v_r + v_f)}{Ev_f} \tag{3}$$

The mass of the oil droplet can be given as:

$$m = \frac{4}{3}\pi a^3 \rho \tag{4}$$

where a is the radius of the spherical droplet and ρ is the density of the oil.

The radius of a spherical body can be related to its falling velocity in a viscous medium (with the coefficient of viscosity η) through Stokes’ Law, with this radius expressed as:

$$a = \sqrt{\frac{9\eta v_f}{2g\rho}} \tag{5}$$

Viscosity requires a correction factor for v < 0.1 cm/s:

$$\eta_{eff} = \eta\left(\frac{1}{1+\frac{b}{Pa}}\right) \tag{6}$$

where b is a constant, P is atmospheric pressure, and a is the uncorrected radius found using Eq. (5).

$$a = \sqrt{\left(\frac{b}{2P}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \frac{b}{2P} \tag{7}$$

substituting Eq. (6) into Eq. (5) and solving for the radius gives:

$$q = \frac{6\pi}{E}\sqrt{\frac{9\eta^3}{2g\rho\left(1+\frac{b}{Pa}\right)^3}}\left(v_f + v_r\right)\sqrt{v_f} \tag{8}$$

The magnitude of the electric field is:

$$E = \frac{V}{d} \tag{9}$$

where V is the voltage across the plates and d is the plate separation.

Eq.(7) is then substituted into Eq. (8) and rearranged to give:

$$q = \left[\frac{4\pi d}{3}\left(\frac{1}{g\rho}\left[\frac{9\eta}{2}\right]^3\right)^{\frac{1}{2}}\right] \times \left[\left(\frac{1}{1+\frac{b}{Pa}}\right)^{\frac{1}{2}}\right] \times \left[\frac{v_f + v_r\sqrt{v_f}}{V}\right] \tag{10}$$

$$q = \frac{4}{3}\pi\rho g\left[\sqrt{\left(\frac{b}{2P}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \frac{b}{2P}\right]^3 \times \frac{v_f + v_r}{E v_f} \quad (11)$$

Experiment Instructions

Part 1 - Cleaning of the chamber and measurement of plate separation

Disassemble the droplet viewing chamber by removing the housing and then the upper capacitor plate (see Figure 1).
Clean the inside of the chamber thoroughly with paper towel.

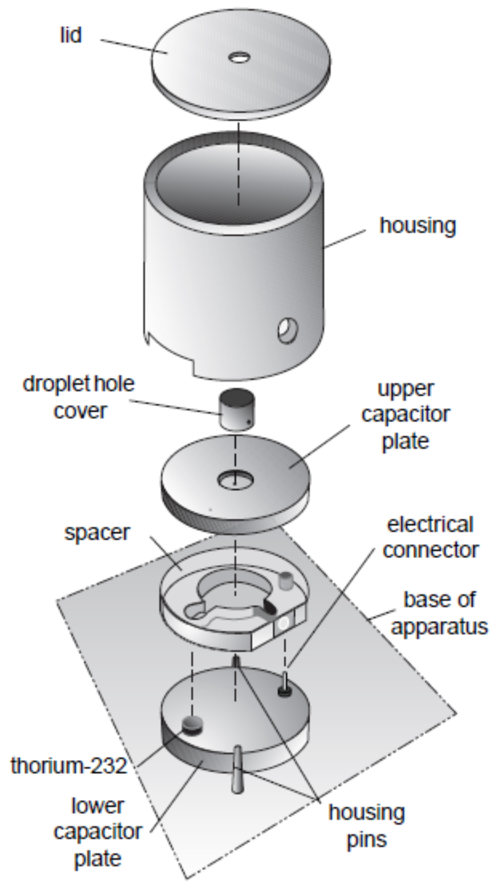

Figure 1 – The droplet viewing chamber

Measure the thickness of the spacer plate, as this represents the plate separation, *d*. Record the data in your lab book. Don't include the raised rim of the spacer plate in your measurement.

Part 2 - Alignment of the optical system

Unscrew the focusing wire (see figure supplied on the lab bench for its location) and carefully insert it into the hole in the centre of the upper capacitor plate. Use the brightness adjustment knob to maximise the contrast.
Focus the reticle by turning the reticle focusing ring and then bring the wire into focus by turning the droplet focusing ring.
Return the focusing wire to its storage location and replace the droplet hole cover and chamber lid.

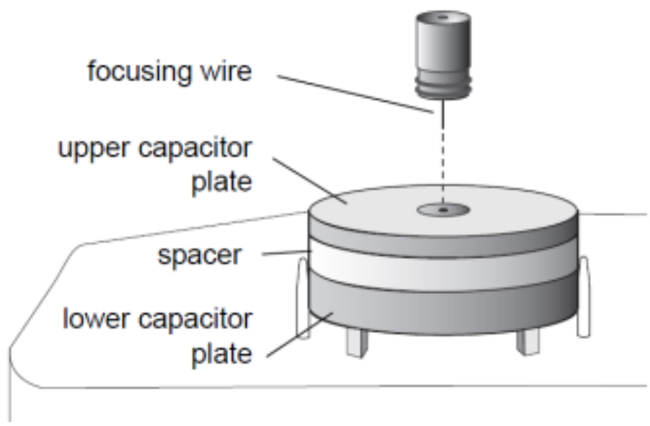

Figure 2 – The focusing wire

Use leads to connect the DC power supply to the plate voltage connectors and, with the aid of a multimeter, adjust the voltage to about 500 VDC. Record this reading.
Measure and record the resistance of the thermistor embedded in the lower capacitor plate by connecting a multimeter to the thermistor connectors (located next to the plate voltage connectors). Use the table located on the platform to determine the viewing chamber temperature. Check the temperature every 15-20 minutes and note of any changes. Use the graph next to the apparatus to determine the air viscosity.
Record the barometric pressure and note any changes during the experiment.
Record the density of the supplied oil and put it into the atomiser.

Part 3 - Measurement of the charge on an oil drop in an electric field

Hold a sheet of paper near the tip of the atomiser and rapidly squeeze the bulb until oil starts spraying. Move the ionisation source lever to the "Spray Droplet Position". This allows air to escape from the chamber while the droplets are being introduced.
Place the tip of the atomiser on hole in the lid of the viewing chamber and squeeze the bulb while looking through the viewing scope. You will need to develop a suitable technique for introducing droplets into the chamber.
Once you see a shower of drops through the viewing scope, move the ionisation source lever to the 'OFF' position.
Select a drop that is falling at 0.02-0.05 mm/s when the plate charging switch is in the "PLATES GROUNDED" position, given that the distance between major reticle lines is 0.5mm. Moving the plate charging switch to "TOP PLATE -" or "TOP PLATE +" should drive the drop up or down.
Once an appropriate drop has been located, adjust the focus of the viewing scope and the brightness until the drop appears as a pinpoint of bright light.

Measure both the rise and fall times (when the plates are charged and grounded, respectively) 10 to 20 times, or until the charge changes spontaneously or it moves out of the field of view.
If the rising velocity of the droplet changes, make again as many measurements of the new rising velocity as you can (10 to 20 measurements). Consider this as a new droplet.
Repeat these measurements as many times as you can. It is desirable to observe as many different oil droplets as possible. 10 oil droplets would be a minimum number.
Take turns in your measurements so everyone in the group have a chance to measure.

Report your velocity data with their errors in an EXCEL table:

DROPLET N	vf (mm/sec)	Δvf (mm/sec)	vr (mm/sec)	Δvr (mm/sec)	d(mm)	T (°C)	p [Pa]	V (Volt)
1								
2								
3								
4								

Include in the table also the distance between plates, the Temperature, pressure and the voltage across the plates. For the errors on d, T p and V you can. Use the convention of the maximum error (+/- the last digit).

Use your scheduled laboratory time effectively. The Millikan oil drop experiment is an excellent introduction to true experimental physics. While the tedium of data collection can seem excessive, the analysis of your data becomes increasingly easier with the amount of data that is collected and the calculated charge of the electron will be much more accurate!

Use the data you have collected and equation 11 below to obtain an experimental value for the charge of an electron.

The charge on a droplet can be expressed as:

$$q = \frac{4}{3}\pi\rho g\left[\sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \frac{b}{2p}\right]^3 \times \frac{(v_f+v_r)}{Ev_f} \qquad (11)$$

Where:
q = charge carried by the droplet [C]
d = separation of the plates in the droplet viewing chamber [m]
ρ = density of oil [kg/m^3]
g = acceleration due to gravity [m/s^2]
η = viscosity of air [N.s/m^2]
b = constant = 8.20×10^{-3} [Pa.m]
p = barometric pressure [Pa]
v_f = velocity of fall [m/s]
v_r = velocity of rise [m/s]
V = potential difference across the plates [V]

Here are three strategies that may help you to analyse the data so as to extract a value of the fundamental charge, e, without biasing your results by prior knowledge of e:

1. Do your values of q fall into groups of integral multiples of a basic charge?
2. If you assume your smallest value of q is a drop with one electron, it should divide integrally into the larger values. If you assume your smallest value of q is a drop with two electrons, then q/2 should divide integrally into the larger values, and so on. Note that this method will not yield a unique solution. Why not?
3. You can also look at the change in ascending sorted values and assume you have drops with n and n + 1 or n + 2 electrons. This is tricky as two similar values might just be drops with the same charge (to within experimental error)

Whatever method you use to determine e, analyse the data **without using the accepted value**. For example, do not divide your q values by the accepted value of e to get a value for n. Once you have determined e for each q value, calculate the mean value of e, along with its standard deviation. Remember to compare your experimental e with the accepted value.

Data that you collect will contribute to a class data set that you will require for your first assessed report. You will be notified when this class data set can be downloaded.

Before leaving the lab, enter the **collected velocity table, the distance between plates, the temperature and the voltage across plates** in the template found on the Blackboard site and upload the file using the following link:

https://www.dropbox.com/request/4CmsqYTWQU5NzvWosAvp

Part 3 – Data analysis for the report

During the data analysis for your report you will consider your own data set and the class data set by using an EXCEL spreadsheet or a computer program created in any programming language.

Calculate *e*

A) Calculate the *q* values for your dataset and for the class dataset. You can sort the *q* values and use a graphic approach to group similar charges.
B) Look at the difference between charges, and try to find the minimum common difference.
C) Your analysis will allow you to determine *e*, as well as the standard deviation in *e*. How does this calculated e compare with the accepted value for *e*?

Calculate the error on *e*

Refer to the lecture slides from Weeks 1 and 2 to conduct this analysis.

A) The error analysis should address first the error in the individual measurements. Use the error propagation formula to calculate the error on each value of *e*.
B) The following points should be considered when evaluating the experimental error:
C) Did you identify systematic errors in the experiment? If so, how did you conclude that the errors were systematic, rather than random? How can you minimise the experimental error?

Discuss the distribution of your data and the statistical error on *e*.

A) Consider the values of *e* from your experiment and from the entire class and the distribution on each dataset.
B) Describe the distribution of the data, what function better describes this distribution? What is the average and the width of the distribution? How do you define the statistical error? What is the error on the mean for your data and for the class data? Comment.