

# Universally Composable Anonymous Broadcast Protocols

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## Abstract

### Anonymous broadcast functionality $\mathcal{F}_R^K$

Initialise:

- (1) a list of pending messages  $L_{pend} \leftarrow []$
- (2)  $status_P \in \{0, 1\} \leftarrow 0$  for party  $P$  indicating whether  $P$  has sent a message in the current round

■ Upon receiving **(sid, WRITE,  $M$ )** from honest party  $P$  or **(sid, WRITE,  $M, P$ )** from  $S$  on behalf of corrupted party  $P$ :

If  $status_P = 0$ , then

- (1) set  $status_P \leftarrow 1$
- (2) append  $M$  to  $L_{pend}$
- (3) if  $|L_{pend}| = K$ , then
  - (a) order the messages lexicographically as  $\langle M_1, \dots, M_K \rangle$
  - (b) set  $L_{pend} \leftarrow []$
  - (c) set  $status_P \leftarrow 0$  for every  $P$
  - (d) send **(sid, BROADCAST,  $\langle M_1, \dots, M_K \rangle$ )** to all parties and **(sid, BROADCAST,  $\langle M_1, \dots, M_K \rangle, P$ )** to  $S$
- (4) else, send **(sid, WRITE,  $|M|, P$ )** to  $S$

### Riposte UC Protocol

Variables:

- $R$  - number of rows in each database table
- $C$  - length of messages
- $e_{l,M}$  -  $R \times C \times 2$  bitstring containing 0 everywhere except in row  $l$  which contains  $(M, M^2) \in \mathbb{F}^k$ , where  $M$  is the message to be sent
- $K$  - message limit in a round

Initialise:

- (1)  $status_P \in \{0, 1\} \leftarrow 0$  for party  $P$  indicating whether  $P$  has sent a message in the current round
- (2)  $count \in \mathbb{N} \leftarrow 0$  indicating the number of valid write requests received this round

■ Upon receiving **(sid, WRITE,  $M$ )** from  $P$

If  $status_P = 0$ , then

- (1) set  $status_P \leftarrow 1$
- (2)  $P$  chooses index  $l \xleftarrow{\$} [0, R)$  and generates bitstring  $e_l$
- (3) generate random  $R \times C \times 2$  bitstring  $r$
- (4) send **(prove,  $P, e_l$ )** to  $\mathcal{F}_{ZK}^{R,R'}$
- (5) send  $r \oplus e_l$  to Server B using  $\mathcal{F}_{\mathcal{AEC}}(\{A, B\})$
- (6) send **(prove,  $P, e_{l,M}$ )** to  $\mathcal{F}_{ZK}^{R,R'}$
- (7)
- (8) send  $r \oplus e_{l,M}$  to Server B using  $\mathcal{F}_{\mathcal{AEC}}(\{A, B\})$
- (9) send  $r$  to Server A using  $\mathcal{F}_{\mathcal{AEC}}(\{A, B\})$
- (10)  $count \leftarrow count + 1$
- (11) if  $count = K$ , then
  - (a) set  $status_P \leftarrow 0$
  - (b) set  $count \leftarrow 0$

■ Upon receiving **(sid, BROADCAST,  $M_A$ )** from Server A and **(sid, BROADCAST,  $M_B$ )** from Server B

- (1) Verify that  $M_A = M_B$
- (2) If  $M_A = M_B$ , forward to  $\mathcal{Z}$

■ Upon receiving **(sid, SEND,  $r \oplus e_l$ )** from  $P$ , if  $P$  has not executed a write request in this phase, then

Server B executes the following:

- (1) Upon receiving **(proof,  $l(y)$ )** from  $\mathcal{F}_{ZK}^{R,R'}$ , if received **(sid, WRITE,  $M$ )** from  $P$ :
- (2) XOR  $r \oplus e_{l,M}$  into its database
- (3) if  $count = K$ , then
  - (a) XOR  $r \oplus e_l$  into its database
  - (b) if  $count = K$ , then
    - (i) combine database with Server A's database
    - (ii) check for collisions
    - (iii) resolve collisions
    - (iv) order messages lexicographically as  $M_B = \langle M_1, \dots, M_K \rangle$
    - (v) broadcast messages to all parties
- (5) Upon receiving **(sid, WRITE,  $M$ )** from  $P$ , if received **(proof,  $l(y)$ )** from  $\mathcal{F}_{ZK}^{R,R'}$ :
  - (a) XOR  $r \oplus e_l$  into its database
  - (b) if  $count = K$ , then
    - (i) combine database with Server A's database
    - (ii) check for collisions
    - (iii) resolve collisions
    - (iv) order messages lexicographically as  $M_B = \langle M_1, \dots, M_K \rangle$
    - (v) broadcast messages to all parties

Figure 1: Anonymous broadcast ideal functionality.

Figure 2: Anonymous broadcast protocol.

AE channel functionality  $\mathcal{F}_{AEC}(\{A, B\})$

Initialise a list  $PendingMsg \leftarrow \emptyset$ .

■ Upon receiving  $(sid, SEND, M)$  from  $P$ , if  $P$  is honest, then:

- (1) If  $\{A, B\} \setminus \{P\}$  is corrupted, then send  $(sid, SEND, M, P)$  to  $S$ .
- (2) If  $\{A, B\} \setminus \{P\}$  is honest, then
  - Choose a random tag  $\overset{\$}{\leftarrow} \{0, 1\}^\lambda$ .
  - Add  $(tag, M, P)$  to  $PendingMsg$
  - Send  $(sid, SEND, tag, |M|, P, \{A, B\} \setminus \{P\})$  to  $S$ .
- (3) Upon receiving  $(sid, ALLOW, tag)$  from  $S$ , if there is a  $(tag, M, P)$  in  $PendingMsg$ , then remove  $(tag, M, P)$  from  $PendingMsg$  and send  $(sid, SEND, M)$  to  $\{A, B\} \setminus \{P\}$

Figure 3: Anonymous broadcast ideal functionality.

Zero-knowledge functionality  $\mathcal{F}_{ZK}^{R, R'}$

- (1) Wait for an input  $(\text{prove}, y, w)$  from  $P$  such that  $(y, w) \in R$  if  $P$  is honest, or  $y, w \in R'$  if  $P$  is corrupt. Send  $(\text{prove}, l(y))$  to  $\mathcal{A}$ . Further, wait for a message **ready** from  $V$ , and send **ready** to  $\mathcal{A}$ .
- (2) Wait for message **lock** from  $\mathcal{A}$ .
- (3) Upon receiving a message **done** from  $\mathcal{A}$ , send **done** to  $P$ . Further, wait for an input **proof** from  $\mathcal{A}$  and send  $(\text{proof}, y)$  to  $V$ .

Corruption rules:

- If  $P$  gets corrupted after sending  $(\text{prove}, y, w)$  and before Step 2,  $\mathcal{A}$  is given  $(y, w)$  and is allowed to change this value to any value  $(y', w') \in R'$  at any time before Step 2.

Figure 4: Zero-knowledge functionality  $\mathcal{F}_{ZK}^{R, R'}$

Broadcast functionality  $\mathcal{F}_{BC}$

- (1) Wait for an input  $(\text{prove}, y, w)$  from  $P$  such that  $(y, w) \in R$  if  $P$  is honest, or  $y, w \in R'$  if  $P$  is corrupt. Send  $(\text{prove}, l(y))$  to  $\mathcal{A}$ . Further, wait for a message **ready** from  $V$ , and send **ready** to  $\mathcal{A}$ .
- (2) Wait for message **lock** from  $\mathcal{A}$ .
- (3) Upon receiving a message **done** from  $\mathcal{A}$ , send **done** to  $P$ . Further, wait for an input **proof** from  $\mathcal{A}$  and send  $(\text{proof}, y)$  to  $V$ .

Corruption rules:

- If  $P$  gets corrupted after sending  $(\text{prove}, y, w)$  and before Step 2,  $\mathcal{A}$  is given  $(y, w)$  and is allowed to change this value to any value  $(y', w') \in R'$  at any time before Step 2.

Figure 5: Zero-knowledge functionality  $\mathcal{F}_{ZK}^{R, R'}$

## 1 Introduction

## 2 Proof

Cases:

- (1) U.r.  $(sid, WRITE, |M|, P)$  from functionality:
  - Simulate a WRITE request on behalf of  $P$  where  $M_{Dummy}$  is all-zeroes
  - Generate  $e_{\ell, M}$
  - $\mathcal{F}_{ZK}$  leaks nothing.  $\mathcal{F}_{AEC}$  leaks the length of the message  $|M|$ , so the simulator sends  $|M|$  to the adversary
- (2) U.r.  $\langle M_1, \dots, M_k \rangle$  from the functionality: If Server A is corrupted, then
  - Simulator sends a dummy message containing all zeroes over  $\mathcal{F}_{AEC}$
  - Extract the subset of honest messages from the set of all messages.
  - Randomly assign honest messages to honest parties
  - Generate  $e_{\ell, M}$  of the corresponding party and send  $r$  to Server A, then  $e_{\ell, M} \oplus r$  is the share of party  $P$  for Server B

If Server B is corrupted, then

  - Simulator equivocates by sending any  $r$  to Server A
  - Extract the subset of honest messages from the set of all messages.
  - Randomly assign honest messages to honest parties
  - Construct a consistent  $e_{\ell, M}$
  - Send  $e_{\ell, M} \oplus r$  of the corresponding party to Server B
- (3) If an adversarial message is received, then
  - Simulator reconstructs  $M$  since it controls  $\mathcal{F}_{AEC}$  and every  $P$  is forced to send messages across the authenticated encrypted channel because all adversaries are passive
  - Tell the functionality to add  $M$  to the list of pending messages

**Case 1:** We first deal with the case where each party  $P_i$  is honest. When  $Z$  provides input to  $P_i$ ,  $S$  begins running  $\pi_R$  on behalf of the honest party  $P_i$ . The protocol execution is easily simulated by  $S$ . In the hybrid world, the authenticated encrypted

channel leaks the length of messages sent through it. In the ideal world,  $S$  emulates this by leaking the length of messages sent through itself when it plays the role of  $\mathcal{F}_{AEC}$ .

**Case 2:** Next we deal with the case where Server A is passively corrupted. Here  $\mathcal{A}$  requests to corrupt Server A in the hybrid world and so  $S$  corrupts Server A in the ideal world. Upon receiving  $(\text{sid}, \text{WRITE}, M_i)$  from honest party  $P_i$ ,  $S$  generates  $e_{L,M_i}$  and random vector  $r$ .  $S$  then sends  $r \oplus e_{L,M_i}$  to Server B and  $r$  to Server A on behalf of  $P_i$ . Upon receiving  $(\text{sid}, \text{BROADCAST}, \langle M_1, \dots, M_k \rangle)$  from  $\mathcal{F}_{BC}$ ,  $S$  extracts the subset of honest messages from  $\langle M_1, \dots, M_k \rangle$  and randomly assigns the honest messages to honest parties.  $S$  orders these messages lexicographically and forwards them to  $Z$ .

**Case 3:** The next case to consider is the case where Server B is passively corrupted. Here  $\mathcal{A}$  requests to corrupt Server B in the hybrid world and so  $S$  corrupts Server A in the ideal world. Upon receiving  $(\text{sid}, \text{WRITE}, M)$  from honest party  $P_i$ ,  $S$  equivocates by sending any random vector  $r$  to Server A.  $S$  then generates  $e_{L,M_i}$  and sends  $r \oplus e_{L,M_i}$  to Server B. When the two servers combine their shares, A can send any share  $g$  instead of  $e_{L,M_i}$  on behalf of Server B. However, when shares are combined,  $r \oplus e_{L,M_i} \oplus r$  and  $g \oplus r$  share the same distribution and are thus indistinguishable. Upon receiving  $(\text{sid}, \text{BROADCAST}, \langle M_1, \dots, M_k \rangle)$  from  $\mathcal{F}_{BC}$ ,  $S$  extracts the subset of honest messages from  $\langle M_1, \dots, M_k \rangle$  and randomly assigns the honest messages to honest parties.  $S$  orders these messages lexicographically and forwards them to  $Z$ .

**Case 4:** Lastly we consider the case where a party  $P^*$  is corrupted.  $\mathcal{A}$  requests to corrupt  $P^*$  in the hybrid world so  $S$  corrupts  $P^*$  in the ideal world.  $\mathcal{A}$  sends  $(\text{sid}, \text{WRITE}, M^*)$  on behalf of  $P^*$ .  $\mathcal{A}$  then continues to execute the protocol, creating shares and sending them to Server A and Server B. Since these shares are sent over  $\mathcal{F}_{AEC}$ ,  $S$  can view these as it emulates the functionality. This allows  $S$  to reconstruct  $M^*$ .  $S$  can then signal its own functionality  $\mathcal{F}_R^K$  to add  $M^*$  to the list of pending messages.

### 3 Background

Perhaps you want to cite the seminal paper of Turing [3], or prior [2] and concurrent [1] work.

### 4 My Amazing System

### 5 Evaluation

#### 5.1 Experimental Setup

#### 5.2 Experimental Analysis

### 6 Conclusions

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### Acknowledgments

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### References

- [1] Alonzo Church. 1936. An Unsolvable Problem of Elementary Number Theory. *American Journal of Mathematics* 58, 2 (1936), 345–363. <http://www.jstor.org/stable/2371045>
- [2] Kurt Gödel. 1931. Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. *Monatshefte für Mathematik und Physik* 38–39, 1 (Dec. 1931), 173–198. doi:10.1007/bf01700692
- [3] Alan M. Turing. 1937. On Computable Numbers, with an Application to the Entscheidungsproblem. *Proceedings of the London Mathematical Society* s2-42, 1 (1937), 230–265. doi:10.1112/plms/s2-42.1.230