Day 3 of HPC NOTEs

Coalescence in Ecology

Dynamic Equilibrium

- Balance between `immigration` and extinction
- Also, Balance between `speciation` and extinction in
- Species themselves are changing

The Advantages of Coalescence

- Always at equilibrium
- Much faster
- Sampling based

The Disadvantages of Coalescence

- Not ideal for time series
- Complex to program
- Fewer ways in which model can be changed

What's Fractal?

- Two properties:

  1. Self-similar: like the copy of the whole if you look at smaller parts of it.
  2. They have dimension that is not a whole number.

- Example1: **Koch Curve**

Dimension	Width	Size	
1	3	3	= 3^1
2	3	9	= 3^2
x	3	4	= 3^x

4 = 3^x >> log(4) = $x \times log(3)$ >> x = 1.262

What is not fractal?

- A line twice as wide is twice as big --> x^1
- A square twice as wide is four times as big --> x^2
- A cube twice as wide is eight times as big --> x^3

Measuring fractal dimension

The Stick Method

$$c = 2 \times n \times r \times sin(\frac{\pi}{n})$$

$$sin(\theta) \approx \theta$$

$$c \approx 2 \times n \times r \times \frac{\pi}{n} = 2 \times \pi \times r$$

Application of Stick method in Coastlines

- Dimensions = 1 - gradient

$$C(\delta) = K \times \delta^{1-D}$$

$$log(C(\delta)) = log(K) + (1 - D) \times log(\delta)$$

where δ is stick size, K is a constant.

Box Counting Algorithm

- Dimensions = -1 × gradient

$$N(\delta) = K \times \delta^{-D}$$

$$log(N(\delta)) = log(K) + -D \times log(\delta)$$

where $N(\delta)$ is `number of hypercubes needed to cover the object`, K is constant, δ is Hypercube length.

Comparison between two methods

$$C(\delta) = \delta \times N(\delta) = K \times \delta^{1-D}$$

The Mandelbrot set and Chaos

- Mandelbrot set
  - a particular set of complex numbers which has a highly convoluted fractal boundary when plotted.
  - **Two thins may happen for many iterations**
    1. The number becomes infinite and goes outside of the cycle
    2. Or, it stopped in `Limit Cycle`
- Complex Plane
  - a plane on which we can plot the Mandelbrot set

Hausdorff Dimension

- an object has the property that the number of balls of radius r needed to cover the object grows proportionally to r^{-d} as r becomes small

Chaos - the Logistic map

- **Logistic Function**

$$\frac{d}{d_t}P(t) = r \times P(t) \times (1 - P(t))$$

- **Logistic Map**

$$x_{n+1} = r \times x_n \times (1 - x_n)$$

- **Deterministic Chaos**
  - is present in systems where a small change in the initial conditions dramatically changes the outcome