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Math 3210-001
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Exam 2

All solutions will be my own and I will not consult outside resources. I understand that doing otherwise would be unfair to my classmates and a violation of the U's honor code.

1) Scratch work: $\left|\frac{3n+1}{n-2} - 3\right| \Rightarrow \left|\frac{(3n+1)(1)}{n-2} - \frac{3(n-2)}{n-2}\right|$

$\left|\frac{3n+1-3n+6}{n-2}\right| = \left|\frac{7}{n-2}\right|$ $\quad \frac{7}{n-2} < \varepsilon \quad n-2 > \frac{7}{\varepsilon} \quad n > \frac{7}{\varepsilon}+2$

Proof: Take $N = \frac{7}{\varepsilon}+2$. Then for any $\varepsilon > 0$,

$$\left|\frac{3n+1}{n-2} - 3\right| < \varepsilon \quad \Rightarrow \left|\frac{\frac{21}{\varepsilon}+7}{\frac{7}{\varepsilon}}\right| < \varepsilon$$

$$\Rightarrow \left|\frac{21+7\varepsilon}{7}\right| < \varepsilon$$

This holds $\forall n > N$, thus by definition of convergence, $\lim \frac{3n+1}{n-2} = 3$ ∎

2) Suppose FSOC that $\{a_n + b_n\} \to c \neq a+b$

$\{a_n + b_n\} = \{a+b : a \in a_n, b \in b_n\}$. By the properties of limits we can split this up to get

$\lim\{a_n + b_n\} = \lim a_n + \lim b_n$. $\quad \lim a_n = a$ & $\lim b_n = b$.

Thus $\lim\{a_n + b_n\} = a+b \neq c$. This is a contradiction & $\lim\{a_n+b_n\} = a+b$. ∎

3) If a_n is a Cauchy sequence, then by definition, for any $\varepsilon > 0$ $\exists N$ such that

$$|a_n - a_m| < \varepsilon \quad \text{for all } n,m > N.$$

Then for a_n^2 we have

$$|a_n^2 - a_m^2| = |a_n^2| + |-a_m^2| = |a_n^2| + |a_m^2| =$$
$$= a_n^2 + a_m^2.$$

Take $\varepsilon = a_n - a_m$ then because a_n is Cauchy, we have:

$$|a_n^2 - a_m^2| < \varepsilon.$$

This holds $\forall n, m > a_n$, thus by definition of a Cauchy sequence, a_n^2 is Cauchy if a_n is Cauchy ∎

4) True. By thm, since a_n is convergent, it is bounded. By Balzano-Weirstrass Thm, a_n must have a convergent subsequence since it's bounded.

Then we can take $\{a_n\} = \cos(2\pi n)$. It's easy to see that this sequence converges to 1.

We can take $\{b_n\} = \cos\left(\frac{2\pi n}{4}\right) = \cos\left(\frac{\pi n}{2}\right)$.
$\{b_n\} \to 0$.

Thus by construction, $\exists \{a_n\}$ with the desired properties. ∎