Homework One

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Question 1

Problem

$(a) Pr(Y \neq 1)$
$(b) Pr(X = 1 \cap Y = 0)$
$(c) Pr(X = 1|Y = 0)$
(d) Are X and Y independent? Explain why.

Solution

(a) $P(Y \neq 1) = P(X = 0 \cap Y = 0) + P(X = 1 \cap Y = 0) = 0.4 + 0.1 = \boxed{0.5}$

(b) From the table, $P(X = 1 \cap Y = 0) = \boxed{0.1}$

(c) From the table, $P(X = 1|Y = 0) = \boxed{0.1}$

(d) By definition, two random variables are independent iff $P(A|B) = P(A)$. From the previous answers, X and Y because $P(X|Y) = P(X)$.

Question 2

Problem

(a) What is $E(C_i)$ in the early season where $i \in \{1, 2, 3, 4, 5, 6, 7, 8\}$?
(b) Using (a), find the expected number of field goals Natasha makes in total from playing in games 1 through 8.
(c) What is $E(C_i)$ at the end of the season, where $i \in \{9, 10, 11, 12\}$?
(d) Using (c), find the expected number of field goals Natasha makes in total from playing in games 9 through 12.
(e) What is the expected number of field goals made by Natasha this season?
(f) In football, a field goal is worth 3 points. What is the expected number of points Natasha scores this season?

Solution

(a) From the problem, for $i = 1, ..., 8$ and $j = 1, ..., A_i$, M_{ji} is 1 with probability 0.7. Therefore the expected number of field goals Natasha makes in a game i where $i = 1, ..., 8$ is 0.7. I.e $E(C_i) = \boxed{0.7}$

(b) By definition, $E[X] = \sum\limits_{\omega \in \Omega} (\omega * P[X = \omega])$. From part $(a), E(C_i) = 0.7$. Now add these up for values $i = 1, ..., 8$ to get $0.7 * 8 = \boxed{5.6}$

(c) Using the same logic as part (a) with $i = 9, ..., 12$ and $j = 1, ..., A_i$, M_{ji} is 1 with probability 0.6 (from the problem). Therefore the expected number off field goals Natasha makes in a game i where $i = 9, ..., 12$ is 0.6. I.e $E(C_i) = \boxed{0.6}$

(d) By definition, $E[X] = \sum_{\omega \in \Omega} (\omega * P[X = \omega])$. From part $(c), E(C_i) = 0.6$. Now add these up for values $i = 9, ..., 12$ to get $0.6 * 4 = \boxed{2.4}$

(e) The expected number of field goals for the season is the sum of the expected number of goals for games $1, ..., 8$ and $9, ..., 12$, which is $5.6 + 2.4 = \boxed{8}$

(f) The expected number of points is the expected number of field goals $*$ the number of points per field goal, which is $8 * 3 = \boxed{24}$

Question 3

Problem

(a) What is a reasonable prior distribution Isidor can use to model his current cluelessness about p? [Hint: Keep it simple and uninformative.]
(b) Isidor now conducts 10 procedurally identical flips of the coin. On each flip, the coin has probability p of landing heads up. And when it does, Isidor records a 1. Otherwise he records a 0. The result is $\{1, 0, 1, 1, 1, 1, 1, 0, 1, 1\}$. Let the ith flip be denoted X_i. Given this setup, a reasonable likelihood model for the flips is: $X_1, \ldots, X_{10}$ are independent (given p) and identically distributed under a *Bernoulli distribution with parameter* p, specifically where $Pr(X_i|p) = p$ and $Pr(X_i = 0|p) = 1 - p$. Using this likelihood and the prior in (a), what value of p gives the most likely model?

Solution

(a) A reasonable prior distribution Isidor can use to model his current cluelessness about p is a uniform distribution with value 0.5. This assumes that he was given a fair coin with probability of heads $H = 0.5$ and probability of tails $T = 0.5$.

(b) Let $f(p)$ represent the likelihood function $P(p|X_i) = \frac{P(X_i|p)*P(p)}{P(X_i)}$. Then $f(p) = \frac{p(1-p)*p}{0.5} = \frac{p^2-p^3}{0.5}$

Solve for the value of p that gives the most likely model:

$$\frac{df}{dp} = \frac{2p-3p^2}{0.5}$$

Set the derivative equal to 0 and solve for p:

$$\frac{2p-3p^2}{0.5} = 0$$
$$p = 0, \frac{2}{3}$$

Therefore, the value of p that gives the most likely model is $\boxed{\text{p} = \frac{2}{3}}$

Question 4

Problem

Use python to plot the pdf and cdf of the Rayleigh distribution ($f(x) = xe^{-x^2/2}$) for values of x in the range $[-2, 4]$. The function scipy.stats.rayleigh may be useful.

Solution

```
from scipy import stats
import matplotlib.pyplot as plt
%matplotlib inline

mean, var, skew, kurt = rayleigh.stats(moments='mvsk')
x = np.linspace(rayleigh.ppf(0.01),
              rayleigh.ppf(0.99), 100)

plt.plot(sp.stats.rayleigh().pdf(x), label = "PDF")
plt.plot(sp.stats.rayleigh.cdf(x), label = "CDF")
plt.xlim(-2, 10)
plt.legend()
plt.show()

# Reference link:
# https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.rayleigh.html
```