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Math 3210-001
12/9/20

Exam 4

All solutions will be my own and I will not consult outside resources. I understand that doing otherwise would be unfair to my classmates and a violation of the U's honor code. –

(1) The statement is false.
Counterexample:

$f(x) = (-1)^x$. $|f(x)| = 1$ which, of course, is integrable.
However, $f(x)$ is not integrable on some interval $[a,b]$ because

$$\underline{\int_a^b} f(x)\,dx \neq \overline{\int_a^b} f(x)\,dx$$

where the lower & upper integrals are defined the same as in the book. Thus,
$|f(x)|$ integrable on $[a,b]$ $\not\Rightarrow$ $f(x)$ integrable on $[a,b]$. ∎

$$(2)\ \sum_{k=0}^{\infty} \int_{1/2}^{3/4} x^k \, dx = \sum_{k=0}^{\infty} \left(\frac{\left(\frac{3}{4}\right)^{k+1}}{k+1} - \frac{\left(\frac{1}{2}\right)^{k+1}}{k+1} \right)$$

by the FTC.

So we have $\displaystyle\sum_{k=1}^{\infty} \frac{\left(\frac{1}{4}\right)^k}{k} = \sum_{k=1}^{\infty} \frac{1}{k4^k} = \ln(2)$

From what weve talked about in class regarding series of this form.

Thus, $\displaystyle\sum_{k=0}^{\infty} \int_{1/2}^{3/4} x^k \, dx = \ln(2)$ $\blacksquare$

(3) The absolute value of the terms gives

$$\sum_{k=1}^{\infty} \frac{\sqrt{k}}{2k^2 - 1}.$$ This is similar to $\frac{1}{2k^{3/2} - 1}$,

so we'll use the comparison test.

$$\frac{\sqrt{k}}{2k^2 - 1} \leq \frac{1}{2k^{3/2} - 1}$$ for k sufficiently large.

$\sum_{k=1}^{\infty} \frac{1}{2k^{3/2} - 1}$ is a p-series, which we know

converges for $p > 1$.

Thus, since $\frac{\sqrt{k}}{2k^2 - 1} \leq \frac{1}{2k^{3/2} - 1}$ for k sufficiently large, the series $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{2k^2 - 1}$ must also converge by the comparison test.

Hence,

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sqrt{k}}{2k^2 - 1}$$ converges

absolutely because the series of absolute value of its terms converges. $\blacksquare$

(4) $R = \frac{1}{\limsup |3^k|^{1/k}} = \frac{1}{3}$ from what we showed in class. So the radius of convergence is $(a - \frac{1}{3}, a + \frac{1}{3})$ or $(-\frac{1}{3}, \frac{1}{3})$ since $a = 0$. We still need to check the end points.

For $x = -\frac{1}{3}$: $\sum_{k=1}^{\infty} 3^k (-\frac{1}{3})^{2k} = \sum_{k=1}^{\infty} (\frac{1}{3})^k$ which converges because it's a geometric series with $r < 1$.

For $x = \frac{1}{3}$: $\sum_{k=1}^{\infty} 3^k (\frac{1}{3})^{2k} = \sum_{k=1}^{\infty} (\frac{1}{3})^k$

which converges by the same logic as $x = \frac{-1}{3}$. Thus, the power series does converge at the end points of the interval. So the radius of convergence is $[\frac{-1}{3}, \frac{1}{3}]$ ∎

Note: $R = \frac{1}{\limsup |3^k|^{1/k}} = \frac{1}{\lim_{k \to \infty} (3^k)^{1/k}}$, which is what we've talked about in class.