Instructions for this exam:

- You have 1 hour to write solutions to the three problems below. Each problem is worth 10 points.
- You must write in complete sentences and justify all of your work.
- You are NOT allowed to use outside resources, such as notes, the textbook, the internet, or people.
- Before you begin, copy the following statement and sign your name:

  "All solutions will be my own and I will not consult outside resources. I understand that doing otherwise would be unfair to my classmates and a violation of the U's honor code."
  -Your Signature

- At the end of 1 hour, stop writing and begin scanning your solutions and uploading them to Gradescope, as you would do with a homework assignment.
  - Create a PDF scan of your work and save it to your computer.
  - Go to `https://www.gradescope.com/courses/79451`, click "Exam 3" to upload your PDF.
- If technical difficulties arise, private-message me in the chat, or send an email to moss@math.utah.edu.

Exam 1, Math 3210
September 18, 2020

1. (10pts) Use induction to prove that $\sum_{k=1}^{n}(-1)^k k^2 = \frac{(-1)^n n(n+1)}{2}$.
2. (10pts) Prove the statements in parts (a) and (b)
   (a) (5pts) Suppose F is a field and x and y are elements of F. Prove that $(-1) \cdot x = -x$.
   (b) (5pts) Suppose F is an ordered field and x, y are elements of F. Prove that $x > 0$ and $y > 0$ implies $xy > 0$.

   Your proofs may only use the axioms of a field (or an ordered field) and the following fact: if x is an element of a field, then $x \cdot 0 = 0$.
3. (10pts) In the following problem you may use basic properties of $+$, $\cdot$, and $<$ on $\mathbb{Q}$.
   (a) (5pts) Let x and y be rational numbers. Prove that $y < x$ implies $y^3 < x^3$.
   (b) (5pts) Prove that the set $L = \{x \in \mathbb{Q} : x^3 < 5\}$ satisfies properties (1) and (3) of the definition of a Dedekind cut.

Challenge (0pts) Show that L satisfies property (2) of the definition of a Dedekind cut.

Important axioms and definitions

N1. There is an element 1 in $\mathbb{N}$.

N2. For each $n \in \mathbb{N}$, there is a successor element $s(n) \in \mathbb{N}$.

N3. 1 is not the successor of any element of $\mathbb{N}$.

N4. If two elements of $\mathbb{N}$ have the same successor, they are equal.

N5. If a subset A of $\mathbb{N}$ contains 1 and is closed under succession, then $A = \mathbb{N}$.

A1. $x + y = y + x$ for all x and y.

A2. $x + (y + z) = (x + y) + z$ for all x, y, z.

A3. There is an element 0 such that $0 + x = x$ for all x.

A4. For each x there is an element, denoted $-x$, such that $x + (-x) = 0$.

M1. $xy = yx$ for all x, y.

M2. $x(yz) = (xy)z$ for all x, y, z.

M3. There is an element 1 such that $1 \neq 0$ and $1 \cdot x = x$ for all x.

M4. For each nonzero element x there is an element, denoted x^{-1}, such that $x^{-1} \cdot x = 1$.

D. $x \cdot (y + z) = x \cdot y + x \cdot z$ for all x, y, and z.

O1. Either $x \leq y$ or $y \leq x$.

O2. If $x \leq y$ and $y \leq x$, then $x = y$.

O3. If $x \leq y$ and $y \leq z$, then $x \leq z$.

O4. If $x \leq y$ then $x + z \leq y + z$.

O5. If $x \leq y$ and $0 \leq z$, then $xz \leq yz$.

C. Every nonempty subset that is bounded above has a least upper bound.