Instructions for this exam:

- You have 1 hour to write solutions to the three problems below. Each problem is worth 8 points.
- You must write in complete sentences and justify all of your work.
- You are NOT allowed to use outside resources, such as notes, the textbook, the internet, or communications with other people.
- Before you begin, copy the following statement and sign your name:

  "All solutions will be my own and I will not consult outside resources. I understand that doing otherwise would be unfair to my classmates and a violation of the U's honor code."
  -Your Signature
- At the end of 1 hour, stop writing and begin scanning your solutions and uploading them to Gradescope, as you would do with a homework assignment.
  - Create a PDF scan of your work and save it to your computer.
  - Go to `https://www.gradescope.com/courses/79451`, click "Exam 3" to upload your PDF.
- If technical difficulties arise, private-message me in the chat, or send an email to moss@math.utah.edu.

Exam 3, Math 3210
November 13, 2020

1. (8 pts) Prove that the function

$$f(x) = \begin{cases} \sqrt{x}\cos\left(\dfrac{1+x}{x}\right) & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$$

   is continuous on the domain $[0, \infty)$.

2. (8 pts) Show that the following functions are uniformly continuous on the given domain, or prove that they are not:
   (a) $f(x) = x^2$ on the domain $(0, 1)$.
   (b) $f(x) = x^2$ on the domain $[1, \infty)$.

3. (8 pts) Consider the sequence of functions $\{f_n\}$ on $[0, \infty)$ defined by $f_n(x) = \dfrac{nx}{1 + nx}$.
   (a) Show that $\{f_n\}$ converges pointwise to the function

$$f(x) = \begin{cases} 1 & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x = 0 \end{cases}.$$

   (b) Show that $\{f_n\}$ does not converge uniformly to $f(x)$.

4. (8 pts) Let $f(x) = \dfrac{x}{1+x}$. Use the definition of the derivative to show that $f'(0) = 1$. Give a thorough justification for every step in your proof.