Homework 03: Midterm Review

Due date: Monday, 7/23 at 11:59 pm

Instructions:

Submit a typed or neatly handwritten scan of your responses on Canvas in PDF format.

Note: you will need to submit a separate PDF per each section.

1. Big-O Notation

Let $f(n) = 10^6 \cdot n^2 + 10^8 \cdot n^{1.5} + n^3 + 10^{10} \cdot n^{2.99}$. Show that $f(n) = O(n^3)$ by finding a constant c and an integer n_0 and applying the Big-O definition.

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2. Solving Recurrences

(a) For the following recurrence relations, state which case of the Master Theorem applies and give the Big-Θ runtime bound.

(i)

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 7 \cdot T(n/2) + n^2 & \text{otherwise} \end{cases}$$

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(ii)

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 4 \cdot T(n/2) + n^2 & \text{otherwise} \end{cases}$$

(iii)

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2 \cdot T(n/2) + \sqrt{n} & \text{otherwise} \end{cases}$$

(iv)

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 4 \cdot T(n/2) + n^3 & \text{otherwise} \end{cases}$$

(v)

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 3 \cdot T(n/2) + n & \text{otherwise} \end{cases}$$

1 Big-O Notation

$f(n) = 10^6 \cdot n^2 + 10^8 \cdot n^{1.5} + n^3 + 10^{10} \cdot n^{2.99}$

$= 10^6 \cdot n^2 + 10^8 \cdot n^{\frac{3}{2}} + n^3 + 10^{10} \cdot n^{3-0.01} \leq \underline{10^6} \cdot n^3 + \underline{10^8} \cdot n^3 + \underline{n^3} + \underline{10^{10}} \cdot n^3$

$\leq \underbrace{(10^6 + 10^8 + 1 + 10^{10})}_{c} \cdot n^3$

$\therefore f(n) \leq 10101000001 \cdot n^3$

when $n_0 = 1$ $c = 10101000001$

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2 (i) $T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 7 \cdot T(n/2) + n^2 & \text{otherwise} \end{cases}$

Apply the master Theorem

Let $a = 7$. $b = 2$. $c = 2$

if $\log_b a = \log_2 7 > 2$

Then $T(n)$ is $\boxed{\Theta(n^{\log_2 7})}$

(ii) $T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 4T(n/2) + n^2 & \text{otherwise} \end{cases}$

Apply the master theorem

let $a = 4$. $b = 2$. $c = 2$

if $\log_b a = \log_2 4 = 2$

Then $T(n)$ is $\boxed{\Theta(n^2 \log n)}$

(iii) $T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T(n/2) + \sqrt{n} = 2T(n/2) + n^{\frac{1}{2}} & \text{otherwise} \end{cases}$

Apply the master theorem. let $a = 2$ $b = 2$ $c = \frac{1}{2}$

if $\log_2 2 = 1 > \frac{1}{2}$ Then $T(n)$ is $\boxed{\Theta(n)}$

(iv) $T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 4T(n/2) + n^3 & \text{otherwise} \end{cases}$

Apply the master theorem

let $a = 4$, $b = 2$ $c = 3$

if $\log_2 4 = 2 < 3$ Then $T(n)$ is $\boxed{\Theta(n^3)}$

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v) $T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 3T(n/2) + n & \text{otherwise} \end{cases}$

Apply the master theorem

Let $a = 3$. $b = 2$ $c = 1$

if $\log_2 3 > 1$ Then $T(n)$ is $\Theta(n^{\log_2 3})$