CL   Tutorial 3

Ex1)

universe = students
a = dilligent
b = ignorant
c = successful

$$\frac{a \models c \quad b \models \neg c}{a \not\models \neg b}$$

some a is b
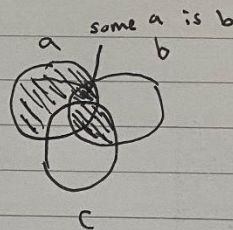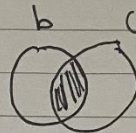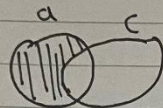
Therefore

$$\frac{a \models c \quad b \models \neg c}{a \not\models \neg b}$$

is   unsound.

Counter!

Suppose Jim is a successful dilligent student but not an ignorant student. As nothing else exists the syllogism is disproved.
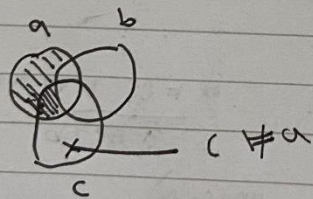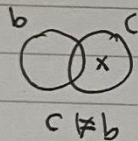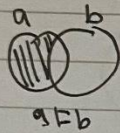
Ex 1b)

Universe of animals

a = eagle

b = can fly

c = pig

$$\frac{a \not\models b \quad c \not\models b}{c \not\models a}$$

a ⊨ b

c ⊭ b

c ⊭ a

Deriving from
Barbara

$$\frac{a \models b \quad b \models c}{a \models c}$$

re written:

$$\frac{c \models a \quad a \models b}{c \models b}$$

denying conclusion 2nd premise:

$$\frac{a \models b \quad c \not\models b}{c \not\models a}$$

thus it is sound as
it can be derived.

Hence the conclusion c ⊭ a or some pigs
are not eagles is sound via venn
diagram.

Ex 2

For any sound syllogism there has to be an even number of $\not\models$ symbols

For any sound syllogism there must also be an even number of $\neg$ symbols.

No animals are unicorns ~~$\neg a \models \neg b$~~ $\neg a \models b$

^All unicorns are horses $b \models c$

some horses are not animals $c \not\models a$

$$\frac{\neg a \models \neg b \quad b \models c}{c \not\models a}$$

odd number of $\not\models$ and odd number of $\neg$ tell us the syllogism is not sound.

Ex 3

a)

a   b

c   a

a   b

→ some c is b

c
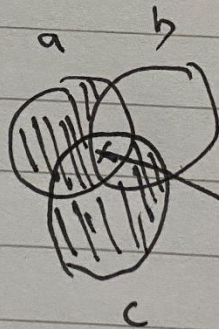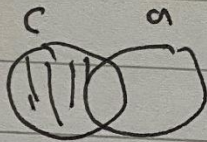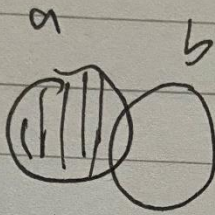
∴ is sound

3b)

From the sound syllogism

$$\frac{a \vDash b \qquad c \nvDash \neg a}{c \nvDash \neg b}$$

1)

we get an equivalent syllogism

$$\frac{c \vDash b \qquad c \nvDash \neg c}{c \nvDash \neg b}$$

where $c \nvDash \neg c$

assumes the existence
of c from
the existential assumption

$$\overline{a \nvDash \neg a} \text{ where}$$
$\qquad\qquad a \text{ is } c.$

Another sound syllogism

$$\frac{b \vDash c \qquad a \nvDash c}{a \nvDash b}$$

denying the conclusion
gives

$$\frac{b \vDash c \qquad a \vDash b}{a \vDash c}$$

Subbing $a = c, b = a, c = b$

$$\frac{a \vDash b \qquad c \vDash a}{c \vDash b}$$

$\Big($

$\longrightarrow$ From this sound syllogism, we see $c \vDash b$ where $a \vDash b$ and $c \vDash a$

As $c \vDash b$ where $a \vDash b$ $c \vDash a$, and the existential assumption implies c
exists, 1) implies $c \nvDash \neg b$.

Therefore $a \vDash b$ $c \vDash a$ implies $c \nvDash \neg b$ ∴

$$\frac{a \vDash b \qquad c \vDash a}{c \nvDash \neg b}$$

Aristotle's syllogism derived.

E5)

1) $(\text{isBig} \land \text{isAmber}) \not\vDash \text{hasThickBorder}$

2) $(\text{isSmall} \not\vDash \neg\text{isDisc})$

3) $(\text{isSmall} \land \text{isSquare}) \vDash \neg\text{isAmber}$