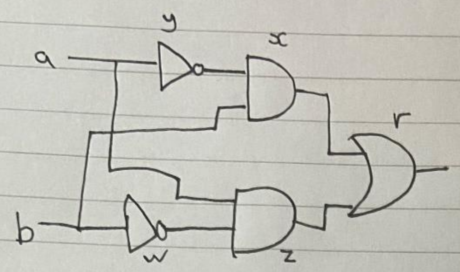
CL Tutorial 8

Ex 1

Ex 1



$$(\neg a \wedge b) \vee (a \wedge \neg b)$$

$y \leftrightarrow \neg a$: $(a \vee y) \wedge (\neg y \vee \neg a)$

$w \leftrightarrow \neg b$: $(b \vee w) \wedge (\neg w \vee \neg b)$

$z \leftrightarrow a \wedge w$: $(\neg z \vee a) \wedge (\neg z \vee w) \wedge (\neg a \vee \neg w \vee z)$

$x \leftrightarrow y \wedge b$: $(\neg x \vee y) \wedge (\neg x \vee b) \wedge (\neg y \vee \neg b \vee x)$

$r \leftrightarrow x \vee z$: $(\neg x \vee r) \wedge (\neg z \vee r) \wedge (\neg r \vee x \vee z)$

Take conjunction of these:

$$(a \vee y) \wedge (\neg y \vee \neg a) \wedge (b \vee w) \wedge (\neg w \vee \neg b) \wedge (\neg z \vee a) \wedge (\neg z \vee w) \wedge (\neg a \vee \neg w \vee z) \wedge (\neg x \vee y) \wedge (\neg x \vee b) \wedge (\neg y \vee \neg b \vee x) \wedge (\neg x \vee r) \wedge (\neg z \vee r) \wedge (\neg r \vee x \vee z)$$

require output r to be 1:

$$(a \vee y) \wedge (\neg y \vee \neg a) \wedge (b \vee w) \wedge (\neg w \vee \neg b) \wedge (\neg z \vee a) \wedge (\neg z \vee w) \wedge (\neg a \vee \neg w \vee z) \wedge (\neg x \vee y) \wedge (\neg x \vee b) \wedge (\neg y \vee \neg b \vee x) \wedge \underbrace{(\neg x \vee 1)}_{1} \wedge \underbrace{(\neg z \vee 1)}_{1} \wedge (0 \vee x \vee z)$$

Simplify:

$$(a \vee y) \wedge (\neg y \vee \neg a) \wedge (b \vee w) \wedge (\neg w \vee \neg b) \wedge (\neg z \vee a) \wedge (\neg z \vee w) \wedge (\neg a \vee \neg w \vee z) \wedge (\neg x \vee y) \wedge (\neg x \vee b) \wedge (\neg y \vee \neg b \vee x) \wedge (x \vee z)$$

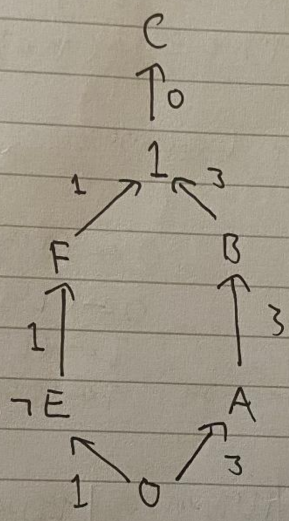
equisatisfiable CNF expression.

Ex 2

Ex2

$$(E \vee F) \wedge (\neg A \vee B) \wedge C$$

$$(\neg E \to F) \wedge (A \to B) \wedge (1 \to C)$$



Adding the numbers on the right hand side there are $3+3+3=9$ cuts, i.e. 9 combinations that satisfy the CNF expression.

Ex 3

Ex3

$B = \{0, 1\}$

$B \times B = \{ (0,0), (1,1), (0,1), (1,0) \}$

Set $B \times B \to B$

Let $B \times B$ be the domain, then the function is equal to 1 for all points in $B \times B$ which belong to $\{0,1\}$ and is equal to 0 for all points which do not belong to $\{1,0\}$.

This definition gives us

$(0,0) \to 1$ where $(0,0) = 0$ and $(1,1) = 1$ (definition from pdf)

$(1,1) \to 1$ as 0 and $1 \in \{0,1\}$ they are 1 and

$(0,1) \to 0$ the others aren't.

$(1,0) \to 0$

General operator truth table:

$\wedge =$

a	b	a∧b
0	0	0
0	1	0
1	0	0
1	1	1

$\vee =$

a	b	a∨b
0	0	0
0	1	1
1	0	1
1	1	1

$\neg =$

a	b
1	0
0	1

Ex 3 part 2

defining $\wedge$, $\vee$ and $\neg$ on set $B \times B \to B$ elements.

$\wedge =$

a	b	$a \wedge b$
(0,1) or (1,0)	(0,1)or(1,0)	(0,1)or(1,0)
(0,1) or (1,0)	(0,0)or(1,1)	(0,1)or(1,0)
(0,0) or (1,1)	(0,1)or(1,0)	(0,1)or(1,0)
(0,0) or (1,1)	(0,0) or (1,1)	(0,0)or(1,1

(0,1) or(1,0) is 0
(0,0)or(1,1) is 1

$\vee =$

a	b	$a \vee b$
(0,1)or(1,0)	(0,1)or(1,0)	(0,1)or(1,0)
(0,1)or(1,0)	(0,0)or(1,1)	(0,0)or(1,1)
(0,0)or(1,1)	(0,1)or(1,0)	(0,0)or(1,1)
(0,0)or(1,1	(0,0)or(1,1)	(0,0)or(1,1)

$\neg =$

a	b
(0,0)or(1,1)	(0,1)or(1,0)
(0,1)or(1,0)	(0,0)or(1,1)

Let $a = (0,0) \to 1$ be elements of the set $B \times B \to B$
$b = (1,1) \to 1$
$c = (0,1) \to 0$
$d = (1,0) \to 0$

$\therefore$ Show as Associativity boolean algebra:

$$(a \vee b) \vee c = a \vee (b \vee c)$$
$$(1 \vee 1) \vee 0 = 1 \vee (1 \vee 0)$$
$$1 = 1 \quad \therefore \text{ valid}$$

Another example (Distributivity):

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$
$$1 \vee (1 \wedge 0) = (1 \vee 1) \wedge (1 \vee 0)$$
$$1 \vee 0 = 1 \wedge 1$$
$$1 = 1 \quad \therefore \text{ valid}$$

Thus 0 and 1 elements have been identified, And,Or,Not operators have been defined and it has been shown as Boolean algebra (twice).