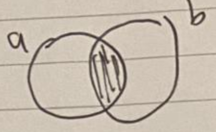
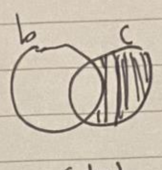
CL tutorial 5

CL Test

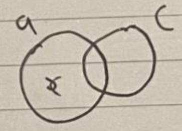
Q1)



$a \vDash \neg b$



$c \vDash b$



$a \nvDash c$

Saying

$$\frac{a \vDash \neg b \qquad c \vDash b}{a \nvDash c}$$

is unsound

The existential assumption assumes that no circle is actually empty, if a non shaded area exists, there is something in it.

some ~~a~~ a is not c would be sound under the existential assumption, as no unshaded circle is empty, meaning some a is not c

Q2)

Derive $\dfrac{b \vDash \neg c \quad c \nvDash \neg a}{a \nvDash b}$ from barbara

Barbara

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

contrapone rules

$$\frac{a \vDash b \quad a \nvDash c}{b \nvDash c}$$

contrapone rules

$$\frac{b \vDash c \quad a \nvDash c}{a \nvDash b}$$

replace c with ¬ c

$$\frac{b \vDash \neg c \quad a \nvDash \neg c}{a \nvDash b}$$ contrapone sequent

$$\frac{b \vDash \neg c \quad \neg\neg c \nvDash \neg a}{a \nvDash b}$$

double negation

$$\frac{b \vDash \neg c \quad c \nvDash \neg a}{a \nvDash b}$$ ∴ derived from Barbara

Q3) prove $(\neg p \wedge \neg q) \vee (p \vee q)$ is a tautology

as every branch ~~becomes~~ derives into an I rule, it's universally valid.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p \models p, q}\,I}{\models \neg p, p, q}\,\neg R \qquad \dfrac{\dfrac{}{q \models p, q}\,I}{\models \neg q, p, q}\,\neg R}{\models (\neg p \wedge \neg q), p, q}\,\wedge R}{\models (\neg p \wedge \neg q), (p \vee q)}\,\vee R}{\models (\neg p \wedge \neg q) \vee (p \vee q)}\,\vee R$$

as the left ~~hand~~ side of the sequent is empty and it's universally valid, we see that $(\neg p \wedge \neg q) \vee (p \vee q)$ is a tautology.

Q4) $p \to q$ is defined to be $q \lor \neg p$

$(\to L)$

$$\dfrac{\dfrac{\Gamma, q \models \Delta \qquad \dfrac{\Gamma \models p, \Delta}{\Gamma, \neg p \models \Delta}\,\neg R}{\Gamma, q \lor \neg p \models \Delta}\,\lor L}{\Gamma, p \to q \models \Delta}\,(\to L)$$

thus

$$\dfrac{\Gamma, q \models \Delta \qquad \Gamma \models p, \Delta}{\Gamma, p \to q \models \Delta}\,(\to L)$$

$(\to R)$

$$\dfrac{\dfrac{\dfrac{\Gamma, p \models q, \Delta}{\Gamma \models q, \neg p, \Delta}\,\neg R}{\Gamma \models (q \lor \neg p), \Delta}\,\lor R}{\Gamma \models p \to q, \Delta}\,(\to R)$$

thus

$$\dfrac{\Gamma, p \models q, \Delta}{\Gamma \models p \to q, \Delta}\,(\to R)$$