Homework 4

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Homework Description

Regular Languages and Pumping Theorem

Course Details

- **Course** - CS435
- **Instructor** - Dr. Chi-Cheng Lin

Homework Results

Problem Set 1

Show that the following languages are regular, and provide a proof.

Exercise 1a

Language: $\{a^i b^j : i, j \geq 0 \text{ and } i + j = 5\}$

Proof Approach: Show that the language is finite

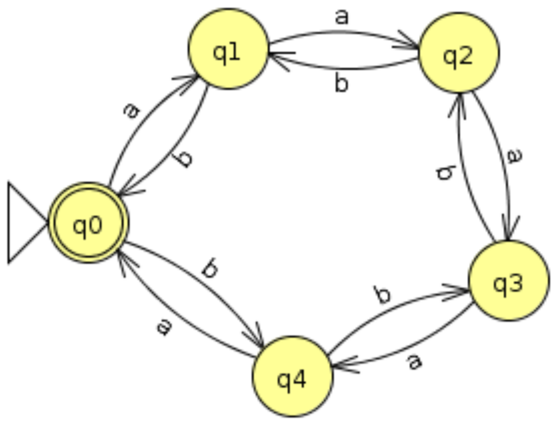
Proof by exhaustion: Assume $i, j \in \mathbb{N}$, then the set of all values $(i, j) : i, j \geq 0 \text{ and } i + j = 5$ are $\{(0, 5)(1, 4)(2, 3)(3, 2)(4, 1)(5, 0)\}$. This set is finite, therefore this language is also finite.

Exercise 1c

Language: $\{a^i b^j : i, j \geq 0 \text{ and } |i - j| \equiv_5 0\}$

Proof Approach: Show a FSM that accepts the language correctly

Proof by asset:



This finite state machine correctly accepts this language by incrementing (mod 5) for each a, then decrements (mod 5) for each b.

Exercise 1e

Language: $\left\{a^i b^j : 0 \leq i < j < 2000\right\}$

Proof Approach: Show that the language is finite

Proof by quantification:

Assume $i, j \in \mathbb{N}$. Let S be the set of all values $(i, j) : 0 \leq i < j < 2000$. To show that the language is finite we must prove that S is finite. In order to that we can calculate the size of S.

By mathematical principles we can safely say the range of i is (0, 1998) and the range of j is (1,1999). Therefore, when i = 0, j can be anything between 1 - 1999. Below is a table of the values of i, and how many options there are for j:

i	#(j)
0	(1999 - 1) - 0 = 1998
1	(1999 - 1) - 1 = 1997
2	(1999 - 1) - 2 = 1996 ...

From this we can write an equation for the size of S. $|S| = \sum_{x=0}^{1998} 1998 - x = 1997001$

Exercise 1p

Language: $\left\{w : w \in \{a - z\}^{\star} \text{and the letters of w appear in reserve alphabetical order}\right\}$

Proof Approach: Show a regular expression that describes language

Proof by RegEx: Since this language has a strict ordering we can develope a regular expression to represent accepted expressions. For this we can accept zero or more of any letter as long as they are in reverse order so the regular expression would be: $z^\star y^\star x^\star w^\star v^\star u^\star t^\star s^\star r^\star q^\star p^\star o^\star n^\star m^\star l^\star k^\star j^\star i^\star h^\star g^\star f^\star e^\star d^\star c^\star b^\star a^\star$

Problem Set 2

Show that the following languages are not regular, by using the pumping theorem.

Exercise 1b

Language: $L = \left\{a^i b^j : i, j \geq 0 \text{ and } i - j = 5\right\}$

1. Let $w = a^{k+5} b^k$, w is in L since $(k+5) - k = 5$ and $|w| = 2k + 5 \geq k$
2. Let $w = xyz: |xy| \leq k$. Therefore, y contains all a's. Let $y = a^p, p \geq 1$.
3. Let $w' = wy^2 z = a^{k+5+p} b^k$
4. We have, $i - j = (k + 5 + p) - (k) = 5 + p \neq 5$. Therefore w' is not in L and L is not regular.

Exercise 1n

Language: $L = \left\{w \in \{a, b\}^\star : \text{w contains exactly two more b's than a's}\right\}$

1. Let $w = a^{k+2} b^{k+4}$, w is in L since $(k+4) - (k+2) = 2$ and $|w| = 2k + 6 > k$
2. Let $w = xyz: |xy| \leq k$. Therefore, y contains all a's. Let $y = a^p, p \geq 1$.
3. Let $w' = wy^2 z = a^{k+2+p} b^{k+4}$
4. We have, $j - i = (k + 4) - (k + 2 + p) = 2 - p \neq 2$ Therefore w' is not in L and L is not regular.

Exercise 3

Language: $\{aba^n b^n : n \geq 0\}$

1. By the Closure Theorem for Concatenation we know that for any regular language $L = L_1 L_2$. If L is regular then L_1 and L_2 must also be regular.
2. Let $L_1 = \{ab\}$ and $L_2 = \{a^n b^n : n \geq 0\}$
3. Show that L_2 is not regular using the pumping theorem.
4. Let $w = a^k b^k$, w is in L_2 since $k \geq 0$ and $|w| = 2k > k$
5. Let $w = xyz: |xy| \leq k$. Therefore, y contains all a's. Let $y = a^p, p \geq 1$.
6. Let $w' = wy^2 z = a^{k+p} b^k$
7. We have, $k + p \neq k$. Therefore, w' is not in L_2 and L_2 is not regular. Therefore, L is not regular.

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