Homework 3

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Homework Description

Regular Expressions

Course Details

- **Course** - CS435

- **Instructor** - Dr. Chi-Cheng Lin

Homework Results

Problem Set 1

Exercises from Chapter 6.

- Provide answer.

- Explanation of how regular expression was constructed.

  Problem 2b

  $\{w \in \{a, b\}^{\star} : \text{w does not end in ba}\}$

  Answer

  $\varepsilon \cup a \cup (a \cup b)^{\star}(b \cup aa)$

  Explanation

  Strings Accepted: ε, a, b, **aa**, a**b**, b**b**, ...

  General Rule: Anything that ends with **aa** or **b** $\rightarrow (b \cup aa)$

  Other considerations: ε, and a string can end with a single a when a is by itself.

  Problem 2d

  $\{w \in \{0, 1\}^{\star} : \text{w corresponds to the binary encoding, without leading 0's, of natural numbers that are evenly divisible}$

  Answer

  $1(1 \cup 0)^{\star}00$

  Explanation

  Strings Accepted: 100 (4), 1000 (8), 1100 (12), 10000 (16), 10100 (20), 11000 (24), 11100 (28)...

  General Rule: Anything that ends with 00

  Other considerations: Cannot start with 0, and 0 is divisible by 4 but 0 is not in the set of natural numbers.

  Problem 2e

  $\{w \in \{0, 1\}^{\star} : \text{w corresponds to the binary encoding, without leading 0's, of natural numbers that are powers of 4}\}$

  Answer

  $1(00)^{\star}$

  Explanation

  Strings Accepted: 1 (1), 100 (4), 10000 (16), 1000000 (64)

  General Rule: Starts with 1 and is followed by pairs of zeros * Other Considerations: 1 is accepted since $4^0 = 1$ and 1 is a natural number.

Problem 2k

$\{w \in \{a, b\}^* : \text{w has both aa and aba as substrings}\}$

Answer

$\{(a \cup b)^* aaba(a \cup b)^*\} \cup \{(a \cup b)^* abaa(a \cup b)^*\} \cup \{(a \cup b)^* aa(a \cup b)^* aba(a \cup b)^*\} \cup \{(a \cup b)^* aba(a \cup b)^* aa(a \cup b)^*\}$

Explanation

Strings Accepted: aaba, abaa, aaaba, abaaa, ...

Considerations: There are 4 different accepting states here, where aa and aba are combined (front or back), or they happen in sequence (aa before or after) with anything else in between them.

Problem 2o

$\{w \in \{a, b\}^* : \#_a(w) \equiv_3 0\}$

Answer

$\varepsilon \cup (b^* ab^* ab^* ab^*)^* b^*$

Explanation

Strings Accepted: aaa, baaa, aaab, abaa, aaba, aaabaaa, ...

General Rule: Any a can be preceded by a b. Any a can be followed by a b. If there is an a, it must be accompanied by at least 2 more a's.

Other considerations: ε will be accepted since $\#_a(\varepsilon) \equiv_3 0$, and string with all b's will also be accepted.

Problem 2

Exercise 7a from textbook. Follow Kline Theory algorithm in textbook step by step to construct FSM.
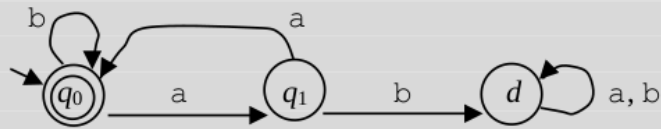
Regular Expression

$(b(b \cup \varepsilon)b)^*$

Result
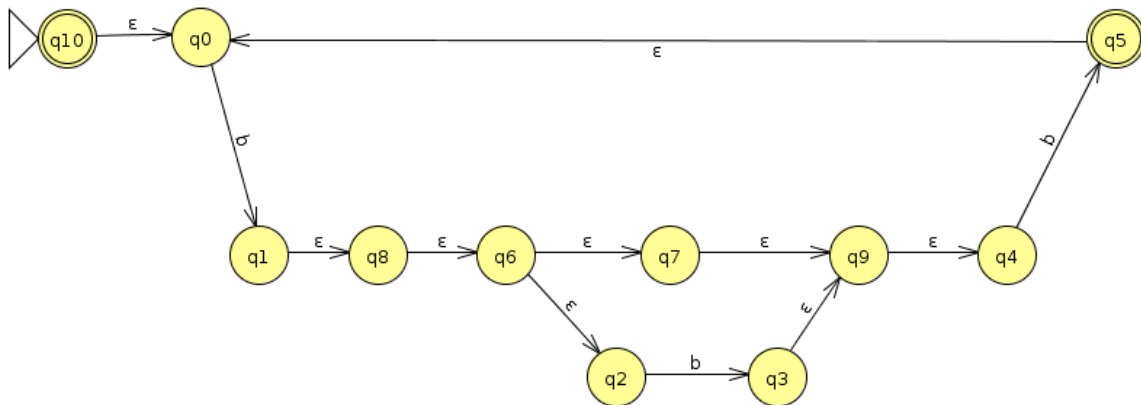
Problem 3

Exercise 10 from text book. Use *fsmtoregexheuristic* to construct regular expression that describes L(M) from example 5.3.

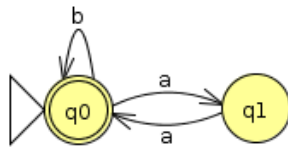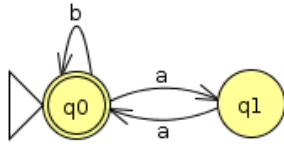### DFSM

Example 5.3 Even Length Regions of a's

Let $L = \{w \in \{a, b\}^* : \text{every a region in } w \text{ is of even length}\}$. L can be accepted by the DFSM M:
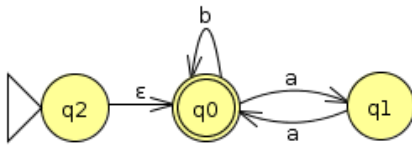

Steps

Step 1

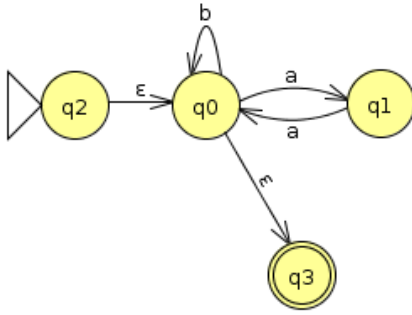Remove any states that are unreachable from the start state.

Step 2

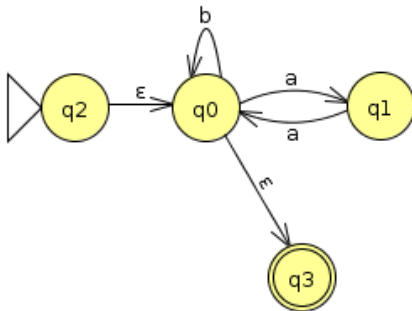If M has no accepting states then halt and return $\emptyset$

Step 3

If the state state is part of a loop, add in an extra state

Step 4

If there is more than 1 accepting state, or there are transitions to the accepting state. Create a new accepting state.
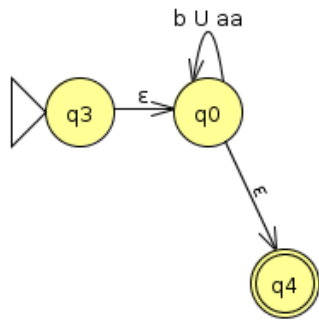
Step 5

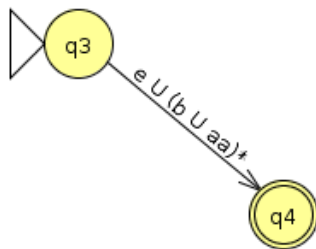If at this point there is only one state then return ε.

Step 6

Rip out states until only a starting state and final state remain.
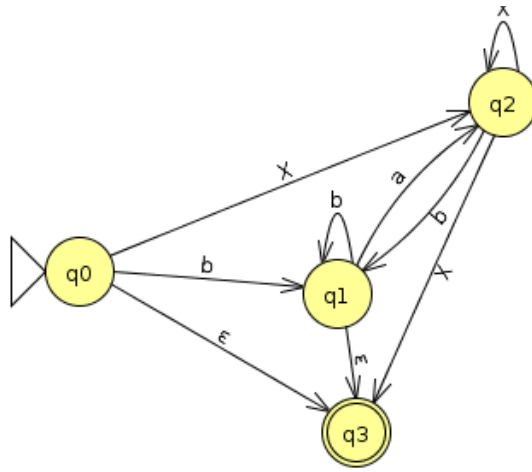
RIP q1:
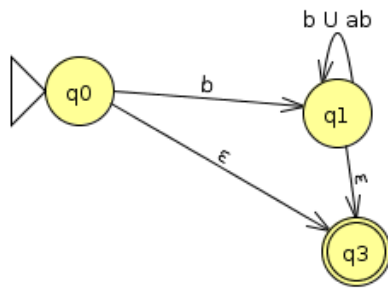
RIP q0:

Step 7
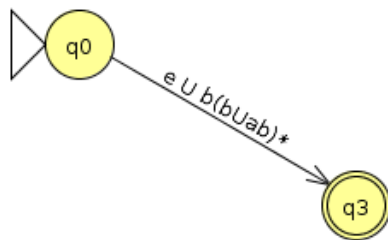
Return regular expression

$$\varepsilon \cup (b \cup aa)^{\star}$$

Problem 4

Standardized State

RIP Q2

RIP Q1

Final Answer

$$\varepsilon \cup b(b \cup ab)^{\star}$$

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