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CS 273A - Machine Learning: Fall 2013

Homework 2

## Problem 1: Bayes Classifiers

(a) Probabilities for Naive Bayes Classifier:
$p(y = 1) = 0.4$

| $p(Variable = 1\|y = 1)$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $\prod$ |
|---|---|---|---|---|---|---|
| Probability | 0.75 | 0.0 | 0.75 | 0.5 | 0.25 | 0 |
| $p(Variable = 1\|y = -1)$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $\prod$ |
| Probability | 0.5 | 0.833 | 0.667 | 0.833 | 0.33 | .0756 |

(b) For $x = (00000), p(y = 1) \prod_i p(x_i = 0) = 0.4 * (0.25 * 1 * 0.25 * 0.5 * 0.75) = 0.009375$ and $p(y = -1) \prod_i p(x_i = 0) = 0.6 * (0.5 * 0.167 * 0.33 * 0.167 * 0.67) = 0.00185$. So the predicted class would be $y = 1$

For $x = (11010), p(y = 1) \prod_i p(X_i = x_i) = 0.4 * (0.75 * 1 * 0.25 * 0.5 * 0.75) = 0.028$ and $p(y = -1) \prod_i p(x_i = 0) = 0.6 * (0.5 * 0.833 * 0.333 * 0.833 * 0.667) = 0.046236114$. So the predicted class would be $y = -1$

(c) $p(y = 1|x = (11010)) = \frac{p(y=1)p(x=(11010)|y=1)}{p(x=(11010))} = \frac{0.4*(0.75*1*0.25*0.5*0.75)}{(0.75*1*0.25*0.5*0.75)+(0.5*0.833*0.333*0.833*0.667)} = 0.0087$

(d) Because then our probability table will have $O(F^2)$, rather than $O(F)$, entries, where $F$ is the number of features we are training on, in order to account for dependence among the feature variables.

(e) We do not need to re-train the model. Because $x_1$ is independent of all other $x_i$, we can safely ignore $x_1$ entirely and make our predictions based on the classifier: $p(y) \prod_{i \neq 1} x_i$

## Problem 2: Decision Trees

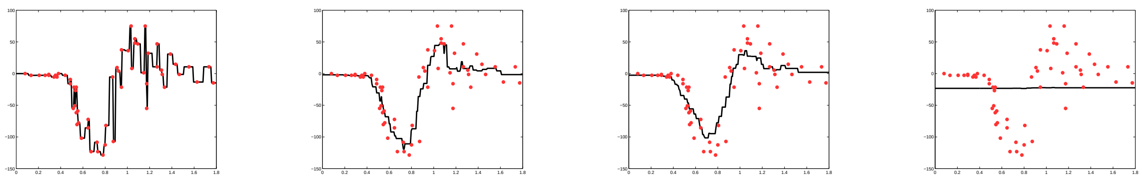
(a) $H(y) = -0.4 * log(0.4) - 0.6 * log(0.6) = 0.971$

(b) Using the formula $H(y|x_i = 0) = p(y = 1|x_i = 0) \log p(y = 1|x_i = 0) + p(y = 0|x_i = 0) \log p(y = 0|x_i = 0)$

| $Variable$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|
| Entropy $H(y\|x_i = 0)$ | 1 | 0.4312 | 1.03 | 0.931 | 0.887 |
| Entropy $H(y\|x_i = 1)$ | 0.811 | 0.22 | 0.701 | 0.72 | 1.028 |
| Info Gain | | | | | |

Therefore, we should split on feature $x_?$ first.

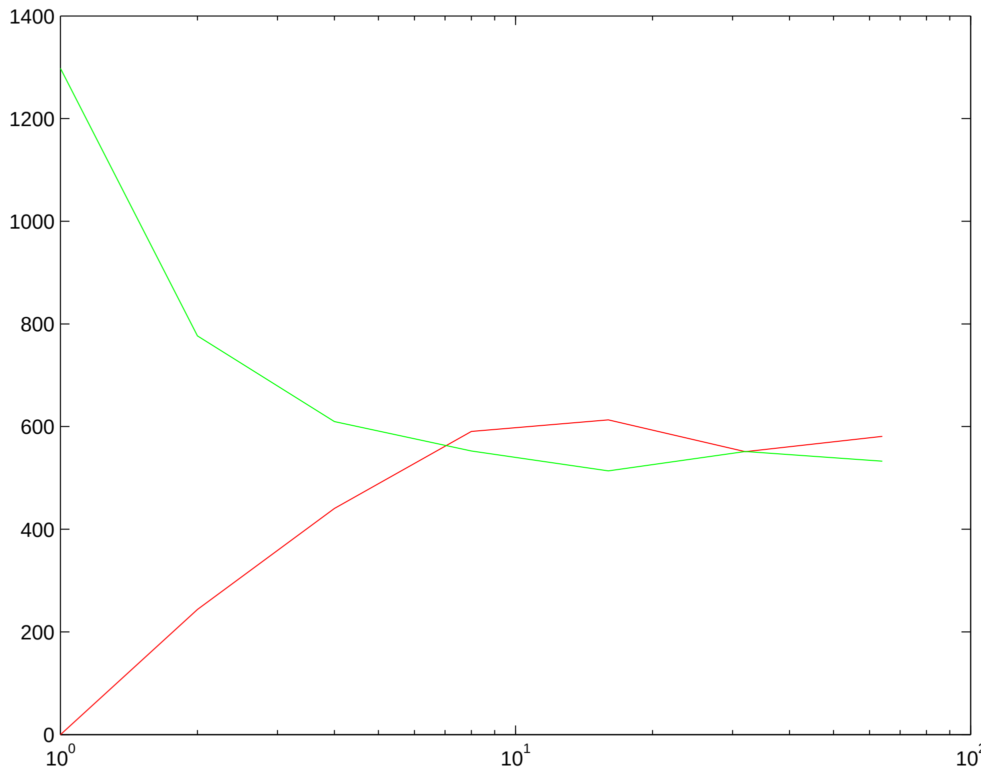
## Problem 2: K-Nearest Neighbors and Validation



$K = 1$ $K = 5$ $K = 10$ $K = 50$

(a)

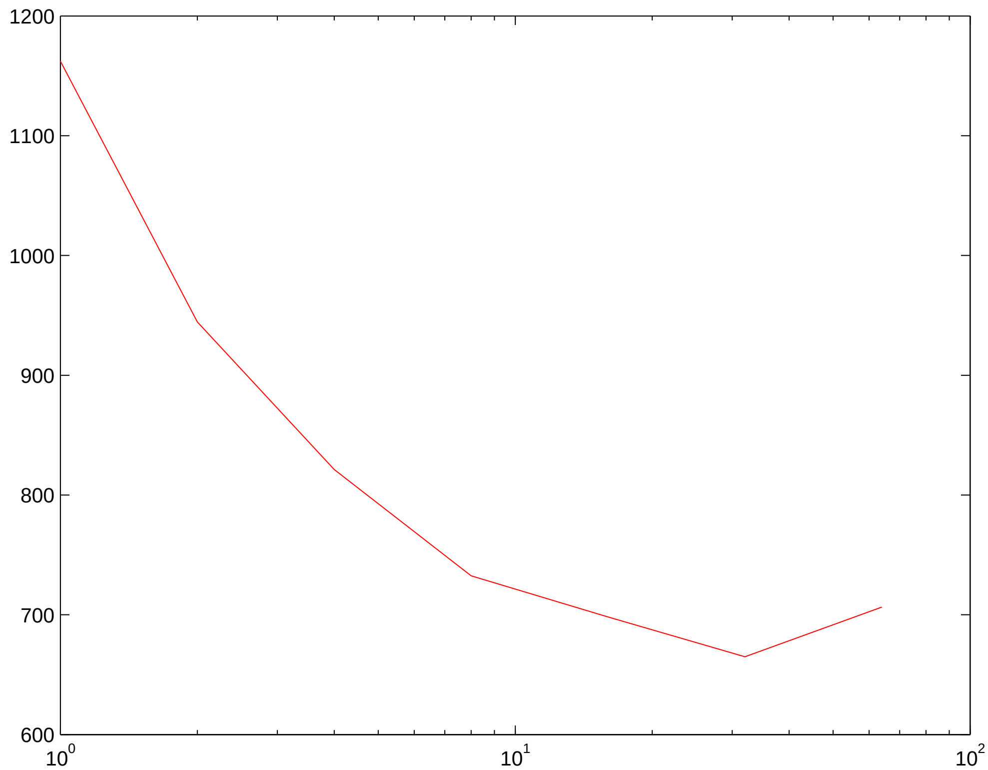
Given the following MSEs:




I would recommend choosing $K = 16$ since the MSE appears to increase for the test set after this value.

(b)

Given the following MSEs:



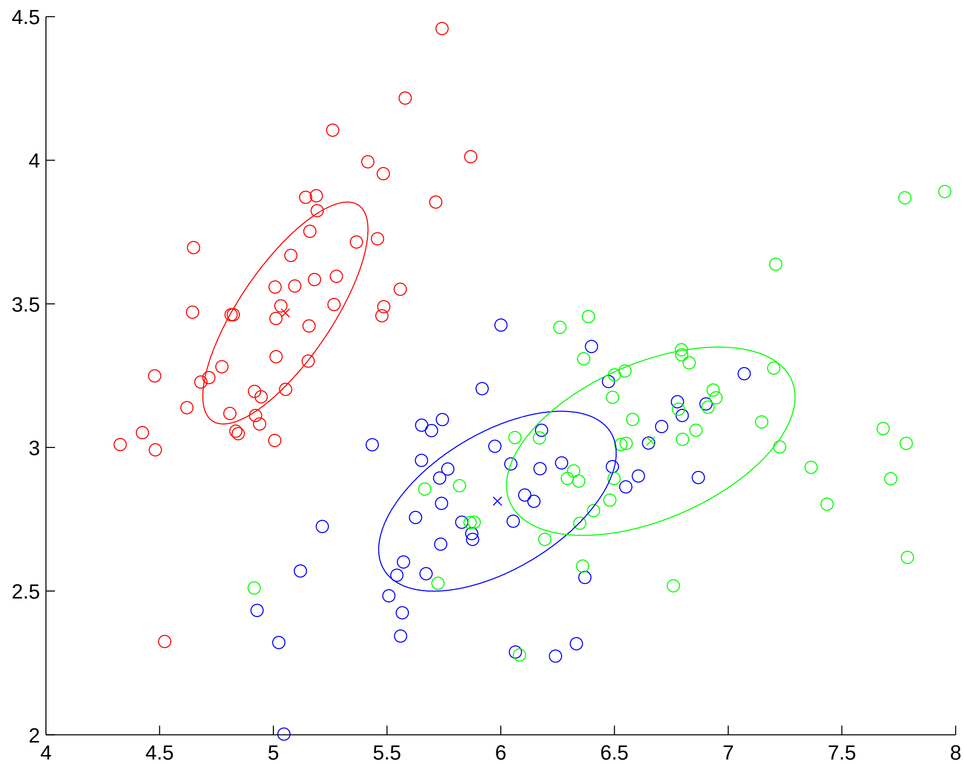

I would recommend choosing $K = 32$ since the MSE appears to increase for the test set after this value.

(c)

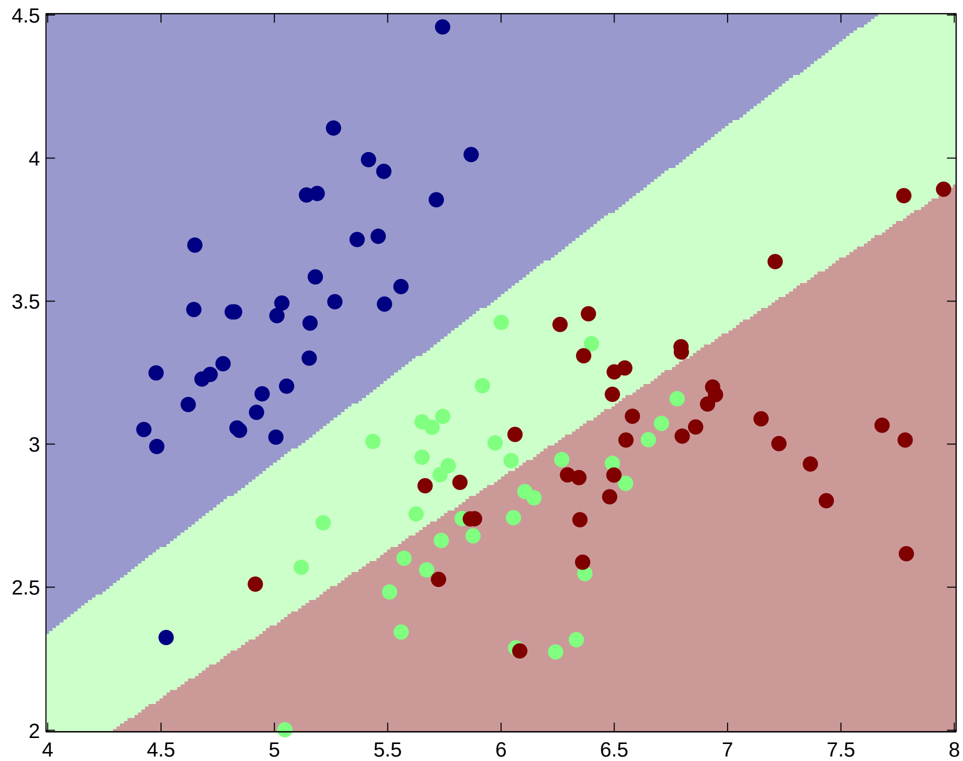
## Problem 3: Bayes Classifiers

(a) See plot in part c

(b) See plot in part c




(c)




(d)

(e) training error rate = 0.2252
test error rate = 0.5676

(f) training error rate = 0.0360
test error rate = 0.0541

I used the following code to store the normalized data in `d.norm_data`:

```
d.data = iris(:,1:end-1);     % extract data
d.mean = mean(d.data)
d.stdev = std(d.data)
d.norm_data = bsxfun(@minus, d.data, d.mean) % subtract mean
d.norm_data = bsxfun(@rdivide, d.norm_data, d.stdev) % divide by stdev
```