Analytic Stacks

People

January 2026

1.4 Replete and locally weakly contractible topoi

Throuought this section the word "topos" refers to the category of sheaves on some site

Definition 1.1 (Replete Topos)**.** A topos X is *replete* if epimorphisms are closed under sequential limits, that is for a functor $F : \mathbb{N} \to X$ with epimorphic transition maps $F_{n+1} \twoheadrightarrow F_n$, then the map $\lim F \to F_n$ is epic for each n

Definition 1.2. An object in a a topos is called

1. Compact if the "underlying geometric structure" is compact, ie if the geometric morphism $X/a \to \mathrm{Sh}(*) = \mathcal{S}et$ is proper
2. Stable if for all morphisms $Y \to X$ with Y compact, the domain of the kernel pair $R \rightrightarrows Y$ of f is also compact
3. Coherent if it is compact and stable

Definition 1.3 (Locally Weakly Contractible Topos)**.** An object F in a topos X is called weakly contractible if every epimorphism $G \twoheadrightarrow F$ has a section. We say that X is *locally weakly contractible* if each $a \in X$ admits an epimorphism $\bigsqcup Y_i \twoheadrightarrow X$ with Y_i coherent and weakly contractible