PMSM equation in the phase domain

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We assume that the magnet induced a sinusoidal back-EMF in the machine stator coils.

Let's start with a round-rotor PMSM, that is a BLDC. In this case, the inductance matrix is constant and we have

$$V_{abc} = RI_{abc} + L\frac{dI_{abc}}{dt} + \omega * \psi_{abc} \qquad (1)$$

where

$[L]$ is the inductance matrix, $L = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}$, ω is the electric frequency,

I_{abc} is the stator current inside the winding, $\omega * \psi_{abc}$ is the sinusoidal back-EMF generated by magnet flux linked into the stator windings, R is the stator resistance and V_{abc} is the voltage across the stator windings.

By reporting the phase values on d and q axis, one obtain the $dq <=> abc$ orthonormal transforms given the following equations

$$\begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} = TV_{abc}, \quad V_{abc} = T^{-1}\begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix}$$

$$T = \sqrt{2/3}\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}\begin{bmatrix} 1 & -0.5 & -0.5 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}$$

The torque equation is :(no 3/2 factor because of the orthonormal Park transformation used).

$$T_e = pp\left[\psi i_q + (L_d - L_q)i_d i_q\right]$$

with pp the number of pole pairs. The inductance matrix being constant means $L_d = L_q$ so the torque equation in this case is simply $T_e = pp\left[\psi i_q\right]$.

Now, let's derive the more general case, $L_d \mathrel{!}= L_q$ and therefore with time-varying L_{abc}. L_{abc} can be decomposed in two parts: a fixed and a variable (Saliency) part, corresponding in the d-q domain as:

$$L_{dq} = \begin{bmatrix} (L_d + L_q)/2 \\ (L_d + L_q)/2 \end{bmatrix} + \begin{bmatrix} (L_d - L_q)/2 \\ (-L_d + L_q)/2 \end{bmatrix} = L_{dqAVG} + L_{dqSAL}$$

$$V_{abc} = RI_{abc} + L_{abcAVG}\frac{dI_{abc}}{dt} + \frac{d(T^{-1}L_{dqSAL}I_{dq})}{dt} + \omega * \psi_{abc}$$

The first 2 terms of Eq. represent a mutual inductor with losses, and the last term is still the back-EMF induced by the magnet. We need to compute the 3rd term with a derivation. And since our objective is to have a constant admittance as seen by the DAE solver, this term has to be computed and inserted in the equation with a delay. The term is cancelled when $L_d = L_q$. With the derivative chain rule, we obtain

$$\frac{d(T^{-1}L_{dqSAL}I_{dq})}{dt} = \sqrt{2/3} * \omega * \begin{bmatrix} 1 & -0.5 & -0.5 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}^t \begin{bmatrix} -\sin(\theta) & -\cos(\theta) \\ \cos(\theta) & -\sin(\theta) \end{bmatrix} L_{dqSAL}I_{dq} + T^{-1}\frac{d(L_{dqSAL}I_{dq})}{dt}$$

This is how the MATE PMSM model is designed. The major part of the inductance is constant and implemented with a SimScape Electric Mutual Inductance block and is suitable for HDL Coder.