

Name:

Math 340, Spring 2021, Homework 8

1. A parking lot has 10 parking spaces arranged in a row. There are 7 cars parked. Assume that each car owner has picked a random a parking place among the spaces available. What is the probability that the three empty places are adjacent to one another?
2. If the letters in the word MISSISSIPPI are mixed up and randomly arranged so each letter is equally likely to be in any spot, what's the probability they will spell MISSISSIPPI?
3. Of 10 people at a protest march, 7 wear red face masks and 3 wear white face masks. If two of the people are selected at random to interview, what is the probability that they're both wearing red face masks?
4. A fair die is tossed five times. What is the probability of exactly two 3's and three 1's?
5. In a state where license plates contain six digits (repeats allowed), what is the probability that the license number of a randomly selected car has exactly two 9's? Assume that each digit of the license number is randomly selected from $\{0, 1, 2, \dots, 9\}$.

6. Suppose you have a bookshelf where you randomly place 3 math books, 5 books about teaching, and 4 music books. What's the probability that:

a. the math books are together and the music books are together?

b. the books about teaching are together?

7. Suppose 7 men and 6 women attended a concert and their tickets were together in one row. If they are randomly placed in seats, what's the probability that:

a. the men and women alternate so that men sat by women and women sat by men?

b. all men sat together, and all women sat together?

8. A dancing contest has 7 competitors, of whom 2 are American, 2 are Ecuadorian, and 3 are Mexican. Suppose the contest result lists *only* the nationality of the dancers. If all outcomes are equally likely, what is the probability that the Mexican dancers place 1-3, Ecuadorians 4-5, and Americans 6-7?

9. A six-card hand is dealt from an ordinary deck of 52 cards. Find the probability that

a. All six cards are diamonds.

b. There are three cards of one suit and three of another suit.

10a. Suppose a student takes a 10-question true-false test. Find the probability that the student gets all problems correct if she guesses the answer to each question.

b. Suppose a student takes a 10-question multiple choice test where the first 5 questions are true-false and the last 5 are multiple choice, each with 4 possible answers. Find the probability that the student gets all problems correct if she guesses the answer to each question.

c. Suppose a student takes a 10-question multiple choice test where 5 questions are true-false and 5 are multiple choice, each with 4 possible answers. Find the probability that the student gets all problems correct if she guesses the answer to each question.

11. Suppose you purchase one Big Bucks lottery ticket. Here's how to play the Big Bucks lottery: Pick three numbers from 1 to 39 (no repeats allowed) and then pick a letter in the set $\{A, B, C, D, E, F\}$. Find the probability of the event that exactly 2 of your numbers match the winning numbers, but your letter doesn't match the winning one.

12a. Suppose 35 people are on Mars, where they were born. What's the probability at least two of them have the same birthday? (On Mars, a year is about 687 days; you may assume a person is equally likely to have been born on any day in the Mars year.)

b. Now suppose n people are on Mars, where they were born. Find the **smallest** n such that the probability that at least two of the people have the same birthday is greater than $\frac{1}{2}$.