1.

(a) $\frac{1}{4} + \frac{1}{3} = \boxed{\frac{7}{12}} = P(X=1)$

(b) $\frac{1/3}{1/6 + 1/3} = \frac{1/3}{1/2} = \boxed{\frac{2}{3}} = P(x=1 | y=1)$

(c) $Var(X) = \sum_{i=1}^{n} (x_i - \mu)^2 P(x_i)$

$P(x=1) = 7/12 \qquad P(x=0) = 5/12$

$\mu = (7/12)(1) + (5/12)(0) = (7/12)$

$Var(x) = (1 - 7/12)^2 (7/12) + (0 - 7/12)^2 (5/12)$

$= \boxed{0.243}$

(d) $P(x=1 | y=1) = \frac{2}{3} \qquad P(x=0 | y=1) = 1/3$

$\mu = (\frac{2}{3})(1) + (1/3)(0) = (2/3)$

$Var(X | Y=1) = (1 - \frac{2}{3})^2 (\frac{2}{3}) + (0 - \frac{2}{3})^2 (\frac{1}{3})$

$\boxed{= 0.111}$

(e) $F = X^3 + X^2 + 3Y^7 \mid Y=1$

$F \mid X=1 = 1 + 1 + 3 = 5$

$F \mid X=0 = 0 + 0 + 3 = 3$

$P_F(X=1) = P(X=1 \mid Y=1) = \frac{2}{3}$

$P_F(X=0) = P(X=0 \mid Y=1) = 1/3$

$E[F] = (2/3)(5) + (1/3)(3) = \boxed{4\,1/3}$

2.

$V_1 = [1,1,1] \quad V_2 = [1,0,0]$

$p = AX = A(A^TA)^{-1}A^Tb$

b = vector to projected

A = [V₁ V₂]

x = the number multiplied with A to get projection

Used numpy to get answers

$P1\_projection = [42, 30, 30]$

$P2\_projection = [26, 19, 19]$

$P3\_projection = [4, 3, 3]$

3. Used the Normal approximation of a binomial distribution

np and n(p-1) both greater than 5

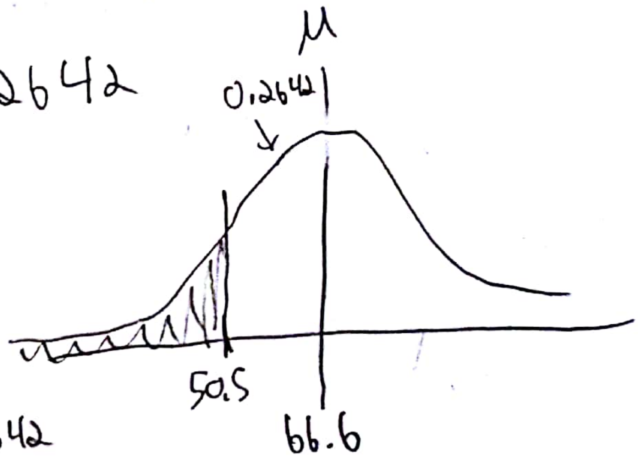
$\mu = np = 100(2/3) = 66.6$

$\sigma = np(1-p) = (100)(2/3)(1/3) = 22.22$

added continuity correction factor

$$Z = \frac{x-\mu}{\sigma} = \frac{50.5 - 66.6}{22.22} = -0.725$$

Z-table lookup gives 0.2642



$P(X \leq 50) = 0.5 - 0.2642$

$= \boxed{0.2358}$