**Idaho Redistricting**
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**Operations Research IEM 4013**
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## Executive Summary Letter

Congressional redistricting in the United States is an important part of the country's democratic process. The shape, population, minority concentration, and compactness of congressional districts are all important factors to give American citizens an equal vote. There are many ways to go about creating congressional districts, and in this report, the project group will display a few different ways to do this using integer programming.

The project group chose to create congressional districts for the state of Idaho. Here is a brief summary of the following report:

To better understand the problem, federal and state redistricting criteria was collected. These requirements and suggestions helped form a solid foundation for the modelling constraints.

The project group decided to test three different integer programming models, each at several population deviation values (population deviation being the percent difference in population between Idaho's two congressional districts). We used a least-cut-edges model that minimizes the number of edges split between districts, a minimum-perimeter model that minimizes the perimeter between districts, and a moment-of-inertia model that minimizes the distance of counties from the center of each district. Each model's objective is to make the districts as compact as possible. After this, a summary of the results was included in this report.

The results were then analyzed based on the districting criteria, minority populations, and city contiguity. Based on this, the project group found a model that stood out from the others. The minimum-perimeter model (at a 0.1% population deviation) gave a population difference of only 292 people and provided two well-rounded districts. This map generally kept minority populations together, large city populations together, and gave compact-looking districts. Citizens of the state of Idaho may have a better understanding of the geographical characteristics of the state as well as where different ethnic areas are located, but the project group thinks this specific output fits all the criteria the best.

## Introduction

Every ten years, the census gives new population numbers for each state in the United States. From these numbers, the US government allocates a certain number of representatives to serve in the House of Representatives. Each state receives a number of representatives based on their population.

It is then up to each state to determine what portions of the population vote on each representative (called districts). Each district within a state votes together on one representative. In an attempt to give equal representation to all of the citizens of the state, these districts are subject to many federal and state-level criteria.

For this specific project, the team chose to create districts for Idaho. The federal and Idaho-specific criteria are presented in the next section.

## Federal and State Criteria

**Federal Criteria:**
The following criteria was taken from the NCSL's front page on congressional redistricting:

- All districts within a state must be nearly equal in population, with a goal of achieving perfectly equal districts. At the federal level, there is no legal definition for the maximum population deviation.
- District plans must not intentionally or inadvertently discriminate on the basis of race or attempt to dilute the minority vote.
- The states are allowed to adopt their own redistricting criteria in addition to the mandatory ones outlined in the US constitution and Voting Rights Act.

Citation: https://www.ncsl.org/research/redistricting/redistricting-criteria.aspx

**Idaho Criteria:**
The following criteria was taken from the official website of Idaho Legislature. The two sections in their law that mention congressional redistricting are below:

- Districts must be composed of contiguous counties unless one county makes up an entire district. Districts must follow the criteria in the US constitution.

Citation:(Section 5 – Idaho State Legislature )

- “The total state population as reported by the U.S. census bureau, and the population of subunits determined therefrom, shall be exclusive permissible data.”
- “To the maximum extent possible, districts shall preserve traditional neighborhoods and local communities of interest.”
- “Districts shall be substantially equal in population and should seek to comply with all applicable federal standards and statutes.”
- “To the maximum extent possible, the plan should avoid drawing districts that are oddly shaped.”
- “Division of counties shall be avoided whenever possible. In the event that a county must be divided, the number of such divisions, per county, should be kept to a minimum.”
- “To the extent that counties must be divided to create districts, such districts shall be composed of contiguous counties.”
- “Counties shall not be divided to protect a particular political party or a particular incumbent.”

(Citation: Section 72-1506 – Idaho State Legislature )

There are two criteria that the project group took special consideration for. The project group felt that not splitting counties into two different districts and preserving traditional neighborhoods and communities of interest were especially important for this project. These two criteria served as a major decision point for choosing the best congressional district map.

## Problem

The goal is to redistrict Idaho into its two congressional districts. The state can be redistricted many different ways, but the objective is to find a plan that retains the integrity of historical neighborhoods and ethnic groups while also minimizing the difference in population.

# Integer Programming Model

There are several different models the team used to create redistricting plans for Idaho. Each method (along with its objective and constraints) are presented below:

**Minimum Cut Edges Model**

Sets:
*V is the set of nodes*
*E is the set of edges*
*N(i) is the set of neighbors of node/county i*

Indices:
*i is a node/county in the map (each node gets an index, 1 to n)*
*j is a district within the map (each district gets an index, 1 to k)*
*e is an edge connecting two nodes/counties on the map (one edge for each connecting county)*

Variables:
$x_{ij} = \{0,\ 1\}\ 1\ --\ \ node\ i\ \in\ V is\ assigned\ to\ district\ j\ \in\ \{1,\ 2,\ ...,\ k\};\ 0\ --\ \ otherwise$
$y_e = \{0,\ 1\} 1\ --\ \ edge\ e\ \in\ E\ is\ cut;\ 0\ --\ otherwise$
$r_{ij} = \{0,\ 1\}\ 1\ --\ node\ i\ is\ the\ root\ of\ district\ j;\ 0\ --\ otherwise$
$f_{ij} =\ flow\ on\ edge\{i,\ j\}$
$f_{ji} =\ flow\ on\ edge\{j,\ i\}$

Parameters:
*U = upper population bound*
*L = lower population bound*
$p_i$ *= population of county/node i*
*k = number of districts*
*n = number of nodes/counties*
*M = n - k + 1*

$$\textit{minimize} \quad \sum_{e \in E} y_e$$

Cut edge constraints

$$\textit{subject to} \quad \sum_{j=1}^{k} x_{ij} = 1 \qquad \forall i \in V \qquad \text{(1a)}$$

$$L \leq \sum_{i \in V} p_i x_{ij} \leq U \qquad \forall j \in \{1, 2, \ldots, k\} \qquad \text{(1b)}$$

$$x_{uj} - x_{vj} \leq y_e \qquad \forall e = \{u, v\} \in E, \forall j \in \{1, 2, \ldots, k\} \qquad \text{(1c)}$$

$$x_{ij} \in \{1, 0\} \qquad \forall i \in V, \forall j \in \{1, 2, \ldots, k\} \qquad \text{(1d)}$$

$$y_e \in \{1, 0\} \qquad \forall e \in E \qquad \text{(1e)}$$

The objective is to minimize the number of cut edges. Cut edges are edges in the graph that cross a district boundary. (1a) Each node i is assigned to only one district j. (1b) The population of each district lies between the lower and upper population bounds. (1c) Edge e is cut if node u is assigned to district j, but node v is not assigned to district j. (1d) All assignment variables are binary. (1e) All edge variables are binary.

Contiguity constraints

$$\sum_{i \in V} r_{ij} = 1 \qquad \forall j \in \{1, 2, \ldots, k\} \qquad \text{(2a)}$$

$$r_{ij} \leq x_{ij} \qquad \forall i \in V, \forall j \in \{1, 2, \ldots, k\} \qquad \text{(2b)}$$

$$\sum_{u \in N(i)} (f_{ui} - f_{iu}) \geq 1 - M \sum_{j=1}^{k} r_{ij} \qquad \forall i \in V \qquad \text{(2c)}$$

$$f_{ij} + f_{ji} \leq M(1 - y_e) \qquad \forall e = \{i, j\} \in E \qquad \text{(2d)}$$

$$f_{ij}, f_{ji} \geq 0 \qquad \forall \{i, j\} \in E \qquad \text{(2e)}$$

$$r_{ij} \in \{1, 0\} \qquad \forall i \in V, \forall j \in \{1, 2, \ldots, k\} \qquad \text{(2f)}$$

(2a) Each district j only has one root node. (2b) A node i can not be the root of a district that it is not assigned to. (2c) Node i must consume flow if it is not the root of its district. (2d) Flow is not allowed to be present along cut edges. (2e) All flow variables are nonnegative. (2f) All root variables are binary.

**<u>Minimum District Perimeter Model</u>**

Sets:
*E is the set of edges*
*V is the set of nodes*
*N(i) is the set of neighbors of node i*

Indices:
*i is a node/county in the map (each node gets an index, 1 to n)*
*j is a district within the map (each district gets an index, 1 to k)*
*e is an edge connecting two nodes/counties on the map (one edge for each connecting county)*

Variables:
$x_{ij} = \{0,\ 1\}\ 1\ --\ \ vertex\ i\ \in\ V is\ assigned\ to\ district\ j\ \in\ \{1,\ 2,\ ...,\ k\};\ 0\ --\ \ otherwise$
$y_{ij} = \{0,\ 1\} 1\ --\ \ edge\ \{i,\ j\}\ \in\ E\ is\ cut;\ 0\ --\ otherwise$
$r_{ij} = \{0,\ 1\} 1\ --\ node\ i\ is\ the\ root\ of\ district\ j;\ 0\ --\ otherwise$
$f_{ij} = flow\ on\ edge\{i,\ j\}$
$f_{ji} = flow\ on\ edge\{j,\ i\}$

Parameters:
$t_e = \ the\ shared\ perimeter\ between\ the\ two\ nodes\ in\ edge\ e\ \in\ E$
*k = the number of districts*
*M = n - k + 1*

$$minimize \quad \sum_{e \in E} t_e y_e$$

Perimeter constraints

$$such\ that \quad \sum_{j=1}^{k} x_{ij} = 1 \qquad \forall i \in V \qquad (1a)$$

$$L \leq \sum_{i \in V} p_i x_{ij} \leq U \qquad \forall j \in \{1, 2, ..., k\} \qquad (1b)$$

$$x_{iv} - x_{jv} \leq y_e \qquad \forall \{i, j\} \in E, \forall v \in \{1, 2, ..., k\} \qquad (1c)$$

$$x_{ij} \in \{1, 0\} \qquad \forall i \in V, \forall j \in \{1, 2, ..., k\} \qquad (1d)$$

$$y_{ij} \in \{1, 0\} \qquad \forall \{i, j\} \in E \qquad (1e)$$

The objective is to minimize the perimeter along cut edges. The perimeter is the distance along a county boundary that is split between two districts (1a) Each node i is assigned to one district j. (1b) The population of each district must lie between the upper and lower bounds. (1c) An edge is cut if node i is assigned to district v, and node j is not assigned to district v. (1d) All assignment variables are binary. (1e) All edge variables are binary.

Contiguity constraints

$$\sum_{i \in V} r_{ij} = 1 \qquad \forall j \in \{1, 2, ..., k\} \qquad (2a)$$

$$r_{ij} \leq x_{ij} \qquad \forall i \in V, \forall j \in \{1, 2, ..., k\} \qquad (2b)$$

$$\sum_{u \in N(i)} (f_{ui} - f_{iu}) \geq 1 - M \sum_{j=1}^{k} r_{ij} \qquad \forall i \in V \qquad (2c)$$

$$f_{ij} + f_{ji} \leq M(1 - y_e) \qquad \forall e = \{i, j\} \in E \qquad (2d)$$

$$f_{ij}, f_{ji} \geq 0 \qquad \forall \{i, j\} \in E \qquad (2e)$$

$$r_{ij} \in \{1, 0\} \qquad \forall i \in V, \forall j \in \{1, 2, ..., k\} \qquad (2f)$$

(2a) Each district j has only one root. (2b) A node i can not be the root of a district it is not assigned to. (2c) Node i must consume flow if it is not the root of its district. (2d) Flow is not allowed to be present along cut edges. (2e) All flow variables are nonnegative. (2f) All root variables are binary.

**<u>Minimum Moment of Inertia Model</u>**

Sets:
*V is the set of nodes*
*N(i) is the set of neighbors of node i*

Variables:
$x_{ij} = \{1,\ 0\}\ 1\ --\ vertex\ i\ is\ assigned\ to\ the\ district\ centered\ at\ j;\ 0\ --\ otherwise$
$f^{v}_{ij} = the\ flow\ from\ district\ center\ v,\ sent\ across\ directed\ edge\ (i,\ j)$

Indices:
*i is a node/county in the map (each node gets an index, 1 to n)*
*j is a district within the map (each district gets an index, 1 to k)*

Parameters:
$w_{ij} = p_i d^2_{ij} = the\ penalty\ for\ moment\ of\ inertia\ objective$
$p_i = the\ population\ of\ node\ i$
$d_{ij} = the\ distance\ from\ node\ i\ to\ node\ j$
*k = the number of districts*
*L = lower population bound*
*U = upper population bound*

$$minimize \quad \sum_{i \in V} \sum_{j \in V} w_{ij} x_{ij}$$

Moment of inertia constraints

$$such\ that \quad \sum_{j \in V} x_{ij} = 1 \qquad \forall i \in V \qquad (1a)$$

$$\sum_{j \in V} x_{jj} = k \qquad (1b)$$

$$L x_{jj} \leq \sum_{i \in V} p_i x_{ij} \leq U x_{jj} \qquad \forall j \in V \qquad (1c)$$

$$x_{ij} \leq x_{jj} \qquad \forall i, j \in V \qquad (1d)$$

$$x_{ij} \in \{1,\ 0\} \qquad \forall i, j \in V \qquad (1e)$$

The objective is to minimize the moment of inertia summed up over each node in all districts. The moment of inertia is defined by the penalty equation defined above. Theoretically, this model will minimize the distance of counties from their assigned district's center. (1a) Each node is assigned to only one district. (1b) k number of districts are made in the map. (1c) Each district's population is within the lower and upper bounds. (1d) A node is not assigned to a district without a center. (1e) Ensures the assignment variables are binary.

Contiguity constraints

$$\sum_{u \in N(i)} (f^j_{ui} - f^j_{iu}) = x_{ij} \qquad \forall i \in V \backslash \{j\},\ \forall j \in V \qquad (2a)$$

$$\sum_{u \in N(i)} f^j_{ui} \leq x_{ij}(n - 1) \qquad \forall i \in V \backslash \{j\},\ \forall j \in V \qquad (2b)$$

$$\sum_{u \in N(i)} f^j_{uj} = 0 \qquad \forall j \in V \qquad (2c)$$

$$f^v_{ij}, f^v_{ji} \geq 0 \qquad \forall \{i,\ j\} \in E,\ \forall v \in V \qquad (2d)$$

(2a) If node i is assigned to district j, node i consumes one unit of flow of type j (otherwise using no flow). (2b) Node i can only receive type j flow if it is already assigned to district j. (2c) Prevents flow circulations. (2d) Ensures that flow variables are nonnegative.

## Code

The code is submitted separately. There should be three additional files of code submitted. The code was adapted from that of Dr. Austin Buchanan for Idaho. Here is a link to the source: https://github.com/AustinLBuchanan/Districting-Examples

The population data (json file) was taken from Daryl DeFord's 2010 Graphs website. Here is a link:
http://people.csail.mit.edu/ddeford/COUNTY/16.html

The shape files used to create the map images in Python were taken from Eugene Lykhovyd's 2010 data website. Here is a link: https://lykhovyd.com/files/public/districting/ID/counties/maps/

https://github.com/KyleHumphreys/IEM-4013-Idaho-Districting.git

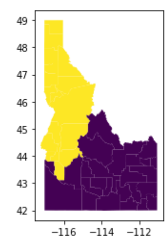
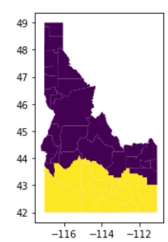
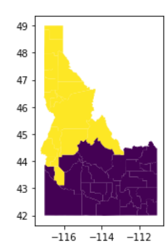
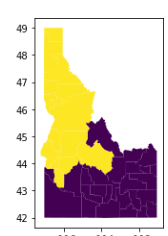
## Experiments

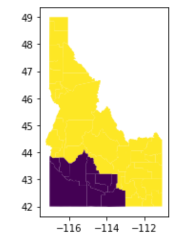
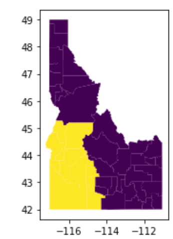
Given that Idaho only has two congressional districts, the optimization models run with Gurobi 9.1.1 can be completed in under one or two seconds for most population deviations. While the actual objective value of each respective model is not included, the tables in the next section display the districts' population differences which are vital to choosing the appropriate map. The project group optimized the minimum moment of inertia maps using a laptop computer with an i5 processor and a 12.0 GB RAM. The project group optimized the minimum perimeter maps and the minimum cut edges maps on a laptop with an i7 processor and a 12.0 GB RAM.

The project group's models all solved to optimality.

## Districting Plan

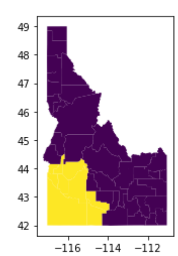
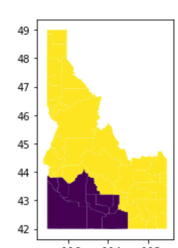
The following table (Table 1) displays redistricting information using the minimum cut edges model.

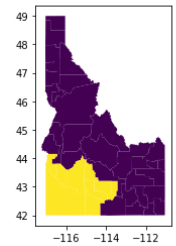
| Deviation Used | Population Difference | Map |
|---|---|---|
| 1%, 0.9% | 6,538 |  |
| 0.8% | 1,576 |  |
| 0.7% | 4,722 |  |
| 0.6%, 0.5% | 2,198 |  |

| | | |
|---|---|---|
| 0.2% | 950 |  |
| 0.1% | 556 |  |

The project team did not accept the first or third map in Table 1 because it splits the Boise metropolitan area into two districts. The project team did not accept the second map because it has a large population difference. The map also separates the Native Americans in Fort Hall from those that are in the panhandle (see Native American map below). The fourth map was rejected because the districts were not as compact as they could be. The project team rejected the fifth map because other results shown later in the report give better results. The project team did not accept the sixth map in Table 1 because the purple district is barely connected.

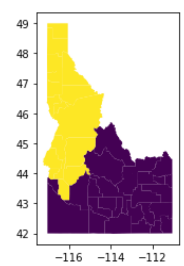
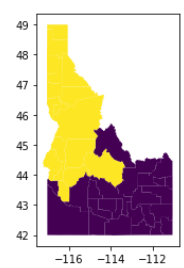
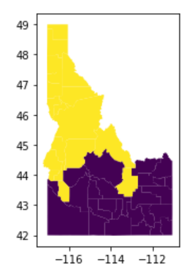
The following table (Table 2) displays redistricting information using the minimum district perimeter model.

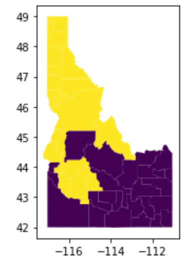
| Deviation Used | Population Difference | Map |
|---|---|---|
| 1%, 0.9%, 0.8%, 0.7% | 5,002 |  |
| 0.6%, 0.5%, 0.4%, 0.3%, 0.2% | 950 |  |

| | | |
|---|---|---|
| 0.1% | 292 |  |

The project team did not accept the first map in Table 2 because its population difference is large. The project team did not accept the second map because it has a larger population difference than the third map below it. The third map in Table 2 is the map the project team is proposing.

The following table (Table 3) displays redistricting information using the minimum moment of inertia model.

| Deviation Used | Population Difference | Map |
|---|---|---|
| 1%, 0.9% | 6,538 |  |
| 0.8%, 0.7%, 0.6%, 0.5%, 0.4%, 0.3% | 2,198 |  |
| 0.2% | 1,060 |  |

| | | |
|---|---|---|
| 0.1% | 674 |  |

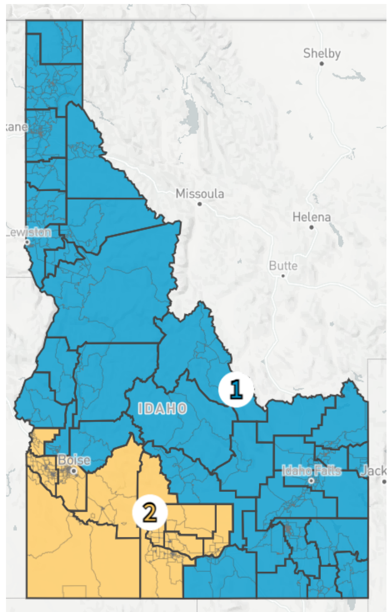
The project team did not accept the first map in Table 3 because it splits the Boise metropolitan area into two districts. The project team did not accept all of the other maps in Table 3 because the districts were not as compact as they could be.

## Evaluation of Plan

When evaluating the possible district maps for Idaho, population difference, district shapes, and district locations have all been taken into consideration. Looking at Table 2, the map created using 0.1% population deviation yielded the best results in terms of population difference for the given deviations. This particular map also created a compact district in the southwest corner of the state which encompasses the majority of Boise and the surrounding suburbs. Along with meeting the objectives of low population difference and compactness, the districts mostly maintain ethnic communities. District 1, as labeled in a below map, retains the majority of the Native American population. District 2 manages to retain a significant portion of the Hispanic population within the state. For these reasons, the project group proposes this particular district map be used. The map could potentially be improved by somebody who is native to the area and understands the population demographics considering none of the team members have spent much time in the state.

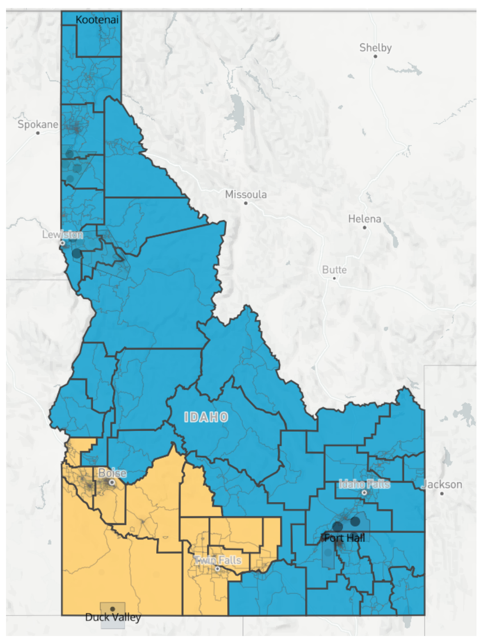
Refer to the figures below for a visualization of the proposed map and the criteria discussed above.

This map displays the districts in the proposed map.



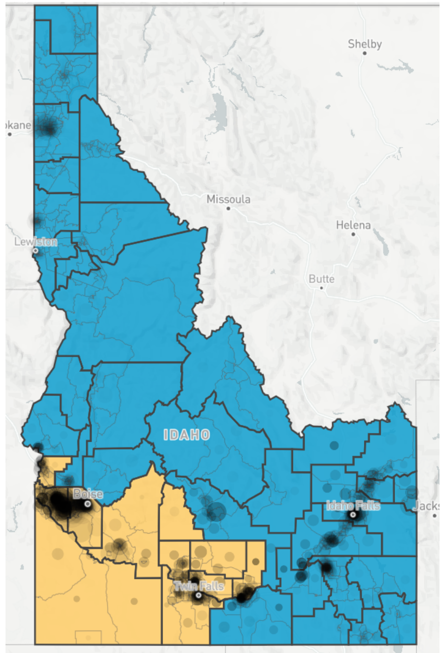

https://districtr.org/plan/16778

Native American Population



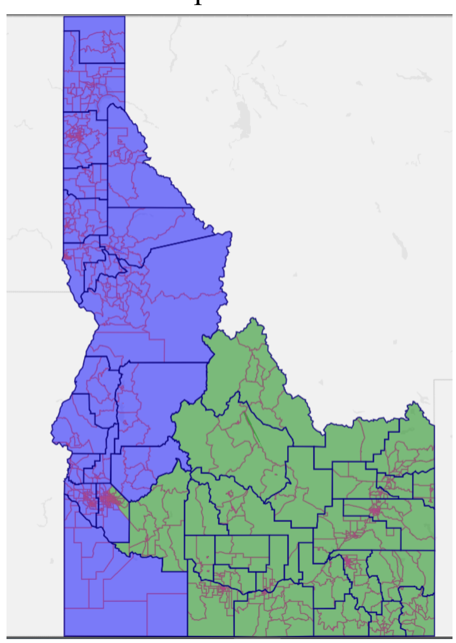

https://districtr.org/plan/16778

Hispanic Population




https://districtr.org/plan/16778

This map displays the current districts of Idaho including the precincts.
The way Idaho's districts are now Boise is separated between the two counties, but the proposed plan does not separate the Boise area.



https://davesredistricting.org/maps#viewmap::14e046b2-98a2-4b59-bfe3-6c8afee85f43

## Conclusions

After reviewing the results for each integer programming model, one districting map stood out from the others. As shown in the results section above, using the minimum-perimeter model with a 0.1% population deviation gives a well-rounded map for Idaho's two congressional districts. This map has the smallest population difference of all the results (with a difference of 292 people) while also keeping the capital city, Boise, contained in one district. Marginalized communities were taken into consideration as one district in the map aims to let several Native American populations vote together, and the other district focuses on the Hispanic population. Considering all of these facts, this particular map has been proposed for the new districts of Idaho.