

Assignment

In this assignment, you will analyse the distribution of returns and a number of trading strategies. As part of the preliminary assignment, you have generated a daily stock file for your stock and a daily file "spy.csv" for the S&P-500 (using the symbol "spy"). For both files, you have data for 5 years.

In this assignment, you will investigate some questions concerning daily and monthly returns:

1. If you buy a stock for just one day, what is the best day of the week to do so. Specifically you will analyze the daily returns for each day of the week. A "Monday" return is the percent change in (adjusted) closing price from previous Friday to Monday. A "Tuesday" return is the percent change in price from Monday to Tuesday and so on.
2. what is the best and worst month to be invested in your stock and in S&P-500? Should you take a break from investing in the summer months?
3. comparison of returns if you have an "oracle" that can statistically predict the future behavior of your stock price for next day

We start with introducing some notation. Let $R = \{r_1, \dots, r_n\}$ denote your daily returns for your stock for n days. The mean of the daily returns

$$\mu(R) = \frac{r_1 + \dots + r_n}{n}$$

To compute the standard deviation $\sigma(R)$ we can use

$$\sigma^2(R) = \frac{r_1^2 + \dots + r_n^2}{n} - \mu^2(R)$$

Let us split the daily returns R into two sets

1. R^- : all negative returns
2. R^+ : all non-negative returns

Finally, let $|R^-|$ denote the number of days with negative returns, and $|R^+|$ denote the number of days with non-negative returns.

Question 1:

1. for each of the 5 years, compute the mean and standard deviation for the sets R , R^- and R^+ of daily returns for your stock for each day of the week
2. summarize your results in the table as shown below (5 tables total).

Day	$\mu(R)$	$\sigma(R)$	$ R^- $	$\mu(R^-)$	$\sigma(R^-)$	$ R^+ $	$\mu(R^+)$	$\sigma(R^+)$
Monday								
Tuesday								
Wednesday								
Thursday								
Friday								

3. are there more days with negative or non-negative returns?
4. does your stock lose more on a "down" day than it gains on an "up" days.
5. are these results the same across days of the week?

Question 2: Examine your 5 tables.

1. are there any patterns across days of the week?
2. are there any patterns across different years for the same day of the week?
3. what are the best and worst days of the week to be invested for each year?
4. do these days change from year to year for your stock?

Question 3: Compute the aggregate table across all 5 years, one table for both your stock and one table for S&P-500 (using data for "spy").

1. what is the best and worst days of the week for each?
2. are these days the same for your stock as they are for S&P-500?
3. in 5 years you have about 250 trading days. If daily returns were normally distributed with mean μ and standard deviation σ , then 5% of these days will have returns r with $r < \mu - 2\sigma$ or $r > \mu + 2\sigma$. For how many trading days do you have daily returns outside of $\mu \pm 2\sigma$ range (for your stock and for S&P-500)? Is it consistent with normality of returns?

For the next questions, suppose that you have an **"oracle"**. This oracle knows all "True" labels for any stock for any day in the future. In particular, this oracle could tell you (statistically, of course) whether price of any stock would increase or decrease the next day. Let p be the probability that your oracle predicts the "True" label correctly for each day. Assume that you have no transaction costs.

To implement such an oracle, we do the following: given probability p , generate a random number r in interval $[0, 1]$. If $r \leq p$ then oracle gives the "True" label and if $r > p$ then oracle gives the opposite of "True" label.

If $p = 1$, then oracle gives you the correct "True" label for each day. Therefore, if you "day trade" according to these labels, you will never lose. On the other extreme, if $p = 0$ then oracle would tell you the opposite of "True" labels. In this case, you will lose on every day that you are invested. For the general case $0 < p < 1$, the oracle can only predict "True" labels statistically with probability p of predicting the label correctly.

You start with \$100 on the first trading day of year 1 to trade your stock and another \$100 to trade "spy". We ignore transaction costs and any taxes.

Question 4: Let $p = 1$ and you listen to the oracle and follow its advice. How much money will you have on the last trading day of year 5:

1. your stock?
2. S&P-500 stock?
3. take the last two digits of your BU id and add this number to \$100. Assume that this is the desired amount. For example, if your BU id ends on 47, then your desired amount is $\$100 + \$47 = \$147$. If $p = 1$, then how many days would it take for your stock and for S&P-500 to get to that amount?

Question 5: Consider "buy-and-hold" strategy: you buy on the first trading day and sell on the last day. So you do not

listen to your oracle at all. As before, assume that you start with \$100 for both your stock and "spy".

1. how much money will you have on the last trading day of year 5?
2. how do these results compare with results obtained in question 4?
3. Summer Vacation for your stock? You implement "buy-and-hold" except that you sell on the last trading day in May and buy on the last trading day in August. You do this every year. In other words, your stock takes a long 3-month summer vacation in each of 5 years. What are the final amounts for your stock and for S&P-500? Is this a good strategy? Summarize your findings in the table:

Strategy	Your Stock	S&P-500
Buy-and-hold		
Buy-and-hold with Summer Vacation		

4. Monthly Vacation for your stock? For each month, you implement "buy-and-hold" except that you sell on the first trading day of that month and buy on the last trading day of that month. You do this for that month every year. In other words, your stock takes a montly vacation. What are

the final amounts for your stock and for S&P-500 for such a strategy? Summarize your findings in the table:

Strategy	Your Stock	S&P-500
Buy-and-hold (B&H)		
B&H without January		
B&H without February		
.....
.....
B&H without December		

What is the best and worst month to take a vacation for your stock and for S&P-500?

Question 6: Compute and compare the final value of your portfolio for buy-and-hold strategy with oracle for different values of $p = \{0, 0.1, 0.2, \dots, 1\}$. Summarize the results in the table:

Strategy	Your Stock	S&P-500
Buy-and-hold		
$p = 0$		
$p = 0.1$		
$p = 0.2$		
.....
$p = 1.0$		

As before, assume that you start with \$100 for both your stock and "spy".

1. for what value of p , the results are comparable to buy-and-hold strategy. Is this value the same for your stock as it is for SP=500?
2. examine your table. Any patterns?

Question 7: You are given a choice of an oracle. You choose the perfect oracle with $p = 1$. On the way home, you got so excited that you dropped your oracle (clumsy you!!!). The oracle is now very angry with you. It decides to take revenge by giving you wrong advice from time to time. Unlike the general statistical oracle, this "angry" oracle decided to lie to you on the best and worst trading days. Specifically, let us consider the following three scenarios:

- (a) the angry oracle gave you wrong results for the best 10 trading days. In other words, you missed the best 10 days and your overall profit will be lower.
- (b) the angry oracle gave you wrong results for worst 10 trading days. In other words, you traded on the the worst 10 days and your overall profit will be lower.
- (c) the angry oracle gave you wrong results for best 5 days and

for the worst 5 days. In other words, you missed the best 5 trading days and you were invested during the worst 5 days.

Please answer the following questions:

1. for each of the scenarios above (a,b and c), compute the final amount that you will have for both your stock and "spy". Summarize your results in the following table:

Strategy	Your Stock	S&P-500
Buy-and-hold (B&H)		
$p = 1$ (never lose)		
missed 10 best days		
invested during 10 worst days		
missed 5 best days and invested during 5 worst days		

2. do you lose more by missing the best days or by being invested on the worst days?
3. for each part (a),(b) and (c) what are your estimates for the probability p if you used statistical oracle instead of an angry one.