

Implement a computer program following the system specifications given below.

- **Applicable Sets**

$\mathbf{N} : \{1, 2, 3, 4\}$

$\mathbf{P} : \mathbf{N} \times \mathbf{N}$

$\mathbf{B} : \{\text{true}, \text{false}\}$

$\mathbf{A} : \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

$\mathbf{B} : \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

$\mathbf{C} : \{(3, 1), (3, 2), (4, 1), (4, 2)\}$

$\mathbf{D} : \{(3, 3), (3, 4), (4, 3), (4, 4)\}$

$\mathbf{H}_n : \{(x, y) \mid n \in \mathbf{N} \wedge y \in \mathbf{N} \wedge x = n\}$

$\mathbf{V}_n : \{(x, y) \mid n \in \mathbf{N} \wedge x \in \mathbf{N} \wedge y = n\}$

- **System Variables**

$\text{uno}, \text{dos}, \text{tres}, \text{quatro}, \text{free} \subseteq \mathbf{P}$

$\text{over} \in \mathbf{B}$

$\text{result} \in \{\text{good}, \text{bad}\}$

$\text{turn} \in \mathbf{N}$

- **System Facts**

$\text{free} = \mathbf{P} - (\text{uno} \cup \text{dos} \cup \text{tres} \cup \text{quatro})$

$\text{over} \leftrightarrow (\text{free} = \emptyset)$

- **System Initialization**

$\text{uno} = \emptyset$

$\text{dos} = \emptyset$

$\text{tres} = \emptyset$

$\text{quatro} = \emptyset$

$\text{over} = \text{false}$

$\text{turn} = 1$

$\text{result} = \emptyset$

- **System States and Behavior**

NextMove($\text{posn} \in \mathbf{P}$) :

$(\text{posn} \in \text{free} \wedge \text{turn} = 1) \rightarrow \text{uno} = \text{uno} \cup \{\text{posn}\}$

$(\text{posn} \in \text{free} \wedge \text{turn} = 2) \rightarrow \text{dos} = \text{dos} \cup \{\text{posn}\}$

$(\text{posn} \in \text{free} \wedge \text{turn} = 3) \rightarrow \text{tres} = \text{tres} \cup \{\text{posn}\}$

$(\text{posn} \in \text{free} \wedge \text{turn} = 4) \rightarrow \text{quatro} = \text{quatro} \cup \{\text{posn}\}$

$(\text{posn} \in \text{free}) \rightarrow \text{turn} = \text{turn} + 1$

$(\text{turn} = 5) \rightarrow \text{turn} = 1$

$\text{Match}(\text{posns} \subseteq \mathbf{P}, \text{patn} \subseteq \mathbf{P}) = |\text{posns} \cap \text{patn}|$

$\text{Check}(\text{posns} \subseteq \mathbf{P}) = \text{Match}(\text{posns}, \mathbf{A}) = 1 \wedge \text{Match}(\text{posns}, \mathbf{B}) = 1 \wedge \text{Match}(\text{posns}, \mathbf{C}) = 1 \wedge \text{Match}(\text{posns}, \mathbf{D}) = 1 \wedge \forall n (\text{Match}(\text{posns}, \mathbf{H}_n) = 1) \wedge \forall n (\text{Match}(\text{posns}, \mathbf{V}_n) = 1)$

EndGame(over) :

$(\text{Check}(\text{uno}) = \text{true} \wedge \text{Check}(\text{dos}) = \text{true} \wedge \text{Check}(\text{tres}) = \text{true} \wedge \text{Check}(\text{quatro}) = \text{true}) \rightarrow \text{result} = \{\text{good}\}$

$(\text{over} \wedge \text{result} = \emptyset) \rightarrow \text{result} = \{\text{bad}\}$