Solutions for Chapter 11 Explorations

Section 11.5: Random Variable Rules

Exploration 11.5: Skee-Ball

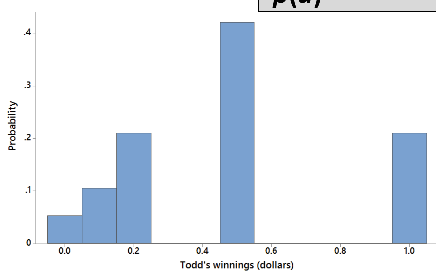
1. Expected score E(T) = 0(0.05) + 10(0.10) + 20(0.25) + 50(0.40) + 100(0.20) = 46 points.

2. If he continues to roll many, many times (indefinitely), his average score across all of the rolls will eventually approach 46 points.

3. Variance Var(T) = $(0 – 46)^2(0.05) + (10 – 46)^2(0.10) + (20 – 46)^2(0.25) + (50 – 46)^2(0.40) +$
$(100 – 46)^2(0.20)$ = 994
   SD(T) = $\sqrt{994} \approx$ 31.53 points

4.

***d* (dollar amount)**	0	.10	.20	.50	1.00
***p*(*d*)**	0.05	0.10	0.25	0.40	0.20



5. E(D) = 0(0.05) + .10(0.10) + .20(0.25) + .50(0.40) + 1.00(0.20) = $0.46

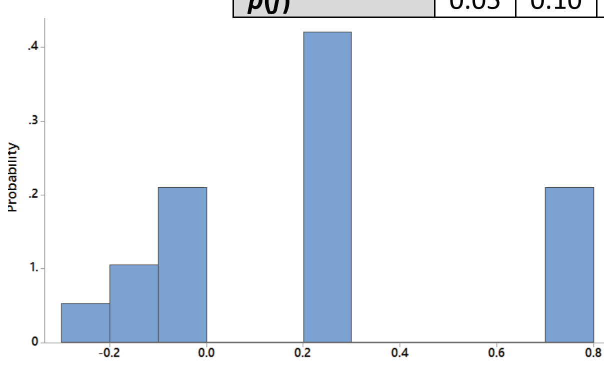
6. E(D) = E(T)/100 as we convert from points to cents.

7. Var(D) = $(0 – .46)^2(0.05) + (.10 – .46)^2(0.10) + (.20 – .46)^2(0.25) + (.50 – .46)^2(0.40) +$
   $(1.00 – .46)^2(0.20)$ = 0.0994
   SD(D) = 0.315

8. SD(D) = SD(T)/100 , we can factor out 1/100 squared from each term in the variance calculation

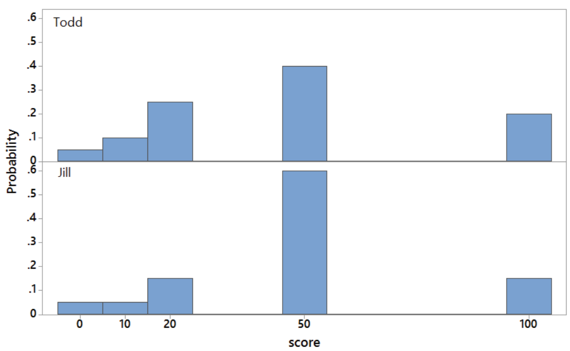
9.

***t* (point value)**	0	10	20	50	100
***r* (profit)**	-.25	-.15	-.05	.25	.75
***p*(*f*)**	0.05	0.10	0.25	0.40	0.20



(Note: Just relabels x-axis values)

10. $E(R) = -.25(0.05) + -.15(0.10) + -.05(0.25) + .25(0.40) + .75(0.20) = \$0.21$
11. $E(R) = E(D) - .25$
12. $Var(R) = (-.25 - .21)^2(0.05) + (-.15 - .21)^2(0.10) + (-.05 - .21)^2(0.25) + (.25 - .21)^2(0.40) + (.75 - .21)^2(0.20) = 0.0994$; $SD(R) = \sqrt{0.0994} = \$0.315$
13. SD(R) = SD(D) (shifting the distribution over does not change the spread of the distribution)
14. $E(R) = E(T/100 - .25) = 1/100\ E(T) - .25 = 46/100 - .25 = \$0.21$ (agrees)
15. $Var(R) = Var(T/100 - .25) = 1/100^2\ V(T) = 994/100^2 = 0.0994$
    SD(T) = \$0.315 (agrees)
16.



17. There is more probability around the lower scores for Todd compared to Jill. This will lower the expected score for Todd. He also has more 100 point rolls and Jill scores 50 points pretty consistently so we expect Todd to have more variability.
18. $E(J) = 0(0.05) + 10(0.05) + 20(0.15) + 50(0.60) + 100(0.15) = 48.5$ points
    $Var(J) = (0 - 48.5)^2(0.05) + (10 - 48.5)^2(0.05) + (20 - 48.5)^2(0.15) + (50 - 48.5)^2(0.60) + (100 - 48.5)^2(0.15) = 712.75$
    $SD(J) = \sqrt{712.75} \approx 26.7$ points.
19. Jill's expected value is larger than Todd's (48.5 > 46)
    Jill's standard deviation (26.7) is smaller than Todd's (31.5).
20. Predictions will vary.
21. Predictions will vary but now we have two random things happening so that might be harder to predict.
22. Possible values for *C* include 0 + 0 = 0, 0 + 100 = 100, 20 + 50 = 70
23. For the sum to be zero, both players need to roll a zero. Because of the independence assumption, we can multiply the two probabilities: $P(0 + 0) = 0.05(0.05) = 0.0025$
    For the sum to be 100, one player needs to roll a zero and the other 100: $P(0 + 100) = 0.05(0.15) + (0.20)(0.05) = 0.0175$
    Also two ways for the sum to be 70: $(0.40)(0.15) + (0.25)(0.60) = 0.21$
24. The least likely sum is zero, this would mean both players had to roll a zero, each player's least likely roll, which has a probability of $(0.05)(0.05) = 0.0025$. The largest possible sum is 200. This

happened about 0.0272 or 2.7% of the time in the simulation (both players score 100, probability 0.15(0.20) = 0.03).

25. Comparisons will vary
26. E(C) = E(T) + E(J) = 46 + 48.5 = 94.5 points; close to the simulation
27. Var(C) = Var(T) + Var(J) = 994 + 712.75 = 1706.75
28. SD(C) = $\sqrt{1706.75}$= 41.3 points; close to the simulation but not equal to 26.7 + 31.53 = 58.23
29. We would randomly generate a score for each player using the given probability distribution and then subtract (rather than add) those two scores from each other (Todd – Jill).
30. Predictions will vary
31. A difference of zero (both players rolling the same score) was the most common. It makes sense that the two players will score the same as they had similar probability distributions. The largest differences of -100 and 100, one player gets zero points and the other player gets 100 points, but these outcomes are more rare.
32. Comparisons will vary. It makes sense for the difference to also have a large standard deviation as either player can score up to 100 points more than the other player.
33. E(S) = E(T) – E(J) = 46 – 48.5 = -2.5 points; similar to simulation results

    Var(S) = Var(T) + Var(J) = 994 + 712.75 = 1706.8
    SD(S) = $\sqrt{1706.8}$≈ 41.3; similar to simulation results and the same as the standard deviation of the sum
34. The expected value is negative as Jill will average slightly more points than Todd.
    In the long run, we expect Jill's average score to be 2.5 higher than Todd's average score.
35. E(sum of 9 rolls) = E($T_1 + \cdots + T_9$) = E(T_1) + ⋯ + E(T_9) = 9E(T) = 9(46) = 414 points
    Var(sum of 9 rolls) = Var($T_1 + \cdots + T_9$) = Var(T_1) + ⋯ + Var(T_9); because we are assuming the 9 rolls are independent
    = 9Var(T) = 9(994) = 8946;
    SD(9T) = $\sqrt{8946}$= 94.58
36. E(option B) = E(9T) = 9E(T) = 414 points, the same as option A
37. SD(option B) = SD(9T) = 9SD(T) = 9(31.53) = 283.77, much larger than option A
    With 9 rolls, he is more likely to get a total score close to his expected value than when just rolling once and multiplying by 9.