

Lesson 14 - Two Groups - Two Proportions

Lesson Objectives: 1. Simulation Based Two proportions

Parameter - association with direction

The difference in true proportions

$$\pi_1 - \pi_2$$

Statistic - Difference in observed proportions

$$\hat{p}_1 - \hat{p}_2$$

$$H_0: \pi_1 - \pi_2 = 0 \quad \text{*Difference*}$$

$$H_a: \pi_1 - \pi_2 \begin{matrix} > 0 \\ \neq 0 \\ < 0 \end{matrix}$$

Simulation Based

$$Z = \frac{\text{observed} - \text{null}}{SD} = \frac{(\hat{p}_1 - \hat{p}_2) - (\pi_1 - \pi_2)}{SD}$$

*Where SD of null dist comes from simulation

$$CI = \text{observed} \pm \text{multiplier} * SD \rightarrow \hat{p}_1 - \hat{p}_2 \pm M * SD$$

We are comparing conditional proportions. $\rightarrow \frac{\text{Group A}}{\text{cant success}} \quad \frac{\text{Group B}}{\text{cant take}}$

Relative Risk \rightarrow ratio of conditional proportions
Tells you how many times more likely an outcome is in one group compared to the other.

Overall Idea - assess whether two sample proportions differ enough to conclude that something other than random chance is responsible for the observed difference in groups.

$H_0: \pi_1 - \pi_2 = 0$ literal translation: The proportion of success in Group 1 (π_1) minus the proportion of success in group 2 (π_2) is 0

There is no association between variables

$$H_a: \pi_1 - \pi_2 \neq 0$$

There is an association

Dolphin Therapy - Card Simulation

$$H_0: \pi_{\text{dolphin}} - \pi_{\text{control}} = 0$$

There is no association between swimming w/ dolphins and improvement in depression

$$H_a: \pi_{\text{dolphin}} - \pi_{\text{control}} > 0$$

There is an association. Swimming w/ dolphins increases probability of improving depression symptoms.

Results of The experiment

	Dolphin	Control	
Improve	10	3	13
No Improve	5	12	17
Total	15	15	30

$$\hat{p}_{\text{dolphin}} = \frac{10}{15}$$

$$\hat{p}_{\text{control}} = \frac{3}{15}$$

difference in conditional proportions

$$\hat{p}_{\text{dolphin}} - \hat{p}_{\text{control}} = \frac{10}{15} - \frac{3}{15} = \frac{7}{15} = .4667 \text{ difference of } 46.67 \text{ percentage points}$$

This seems like a lot but perhaps the 13 participants were going to show improvement regardless of group and we just happened by random chance, assigned more improvers to the dolphin group

Simulation: 13x Red → improvement
17x Blue → no-improvement

Assume these outcomes were going to happen no matter which group

(2) Shuffle and simulate random assignment of subjects by putting in 2x groups (15 each)

→ Pile on Left = dolphin
R = control

(3) Calculate difference in conditional proportions

$$\hat{p}_d - \hat{p}_c = \frac{\# \text{ red cards in each group}}{15}$$

(4) Build distribution (centred on 0)

(5) See if our statistic is likely to happen by random chance alone

