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Student Name:

Exploration 11.5 Skee-Ball

In the game of Skee-Ball, a player rolls a small ball up a ramp, hoping to land the ball in the holes with the highest point values (see the diagram of point values). The game of Skee-Ball has gained in popularity in recent years, with many cities running Skee-Ball leagues. There is even a Skee-Ball national championship!


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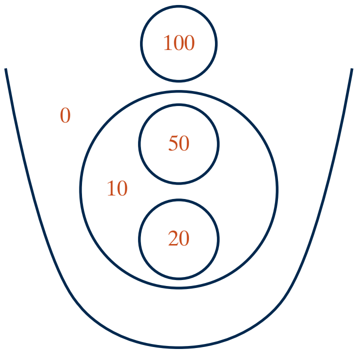
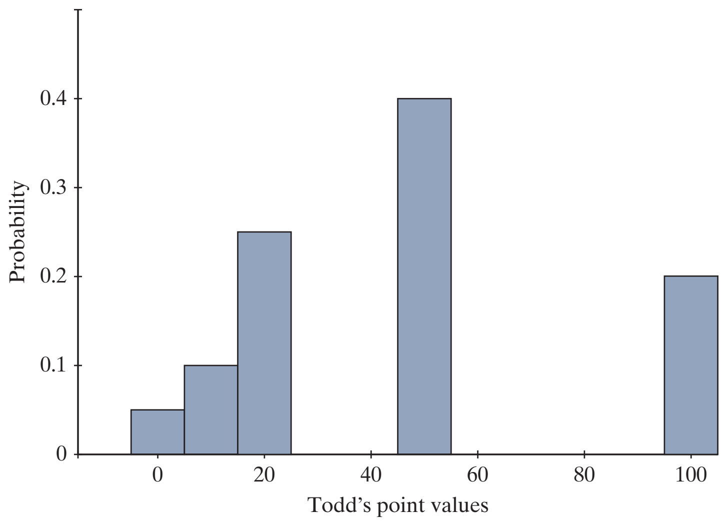


Todd and Jill, both experienced Skee-Ball players, are considering entering a partner Skee-Ball competition hosted by their local league.

Let T be the score associated with the ball landing in a particular region (0, 10, 20, 50, or 100) on a single roll when Todd plays the game. Based on his long history of playing this game, the probability distribution for Todd's score is

t (points)	0	10	20	50	100
$p(t)$	0.05	0.10	0.25	0.40	0.20



1. Determine the expected value of Todd's score from a single roll, $E(T)$.

2. Write a sentence interpreting what this expected value means in this context.

3. Determine the variance and standard deviation of Todd's score from a single roll. (i.e., What are $Var(T)$ and $SD(T)$?)

What if Todd gets paid 1 penny per point each time he rolls? Let D be the dollars earned by Todd on a roll.

4. Make a probability distribution table for D showing the possible values of D and the probability of each value. (*Hint*: The outcome of D depends on the outcome of T, and you know the probability for each outcome of T.) Sketch the probability distribution.

5. What is the expected value of D? (i.e., What is $E(D)$?)

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6. What is the numeric relationship between $E(D)$ and $E(T)$? Why does this relationship make sense?

7. Use the probability distribution for D to calculate the variance and standard deviation of D (Todd's earnings) from a single roll. (i.e., What are $Var(D)$ and $SD(D)$?)

8. What is the numeric relationship between $SD(T)$ and $SD(D)$? Why does this relationship make sense?

9. Now suppose Todd has to pay 25 cents each time he rolls the ball. Let the random variable R represent Todd's net profit from a single roll, where a net loss means that Todd's profit is negative (e.g., if he rolls a 10, his profit is −\$0.15). Determine the probability distribution of R. (*Hint*: Start by thinking about how many different values there will be for R. How does that number relate to the number of possible values of T? How will the probabilities for the different outcomes of T and R relate? Sketch a graph of the probability distribution of R.)

10. Determine the expected value of Todd's net profit from a single roll.

11. How does $E(R)$ compare to $E(D)$? Explain why this makes sense.

Key Idea

Subtracting (or adding) a constant a to the random variable subtracts (or adds) a from the expected value of the random variable. Thus,

$$\mu_Y = \mu_{a+X} = a + \mu_X$$

12. Use the probability distribution to calculate the variance and standard deviation of Todd's net profit from a single roll.

13. How does $SD(R)$ compare to $SD(D)$? Explain why this makes sense.

Key Idea

Subtracting (or adding) a constant to the random variable shifts the distribution of the random variable but does not change the relative distances between values. Thus,

$$\sigma_Y^2 = \sigma_{a+X}^2 = \sigma_X^2$$
$$\sigma_Y = \sigma_{a+X} = \sigma_X$$

When we add (or subtract) a constant a and/or multiply (or divide) a random variable by a constant b, we call it a *linear transformation*. In fact, we can combine together the rules you just learned into one set.

Key Idea

If random variable Y is related to X as follows: $Y = a + bX$, then Y is a linear transformation of X and the following statements are true:

$$\mu_Y = \mu_{a+bX} = a + b_{\mu_X}$$
$$\sigma_Y^2 = \sigma_{a+bX}^2 = b^2\sigma_X^2$$
$$\sigma_Y = \sigma_{a+bX} = |b|\sigma_X$$

Notably, although linear transformations can change the mean (shifting) and variability (stretching) of a random variable, linear transformations of a random variable do not change the general shape of the probability distribution.

14. Given that $R = T/100 - 0.25$, using the shortcut equations from the Key Idea box find Todd's average earnings from this game ($E(R)$). Does it match what you calculated from the probability distribution?

15. Using the shortcut equations from the Key Idea box find the standard deviation of Todd's average earnings from this game ($SD(R)$). Does it match what you calculated from the probability distribution?

Jill's Skee-Ball Performance

Now let J be the score associated with the ball landing in a particular region on a single roll when Jill plays the game. The probability distribution for J is

j (point value)	0	10	20	50	100
$p(j)$	0.05	0.05	0.15	0.60	0.15

16. Produce graphs of Todd's and Jill's point-value probability distributions, using the same scales so they can be compared.

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17. Summarize (in words, without doing calculations) how the probability distributions of Todd's points and Jill's points compare. Address who you think would have the larger average score in the long run, and who you think would have more variability in scores in the long run.

18. Determine the expected value and standard deviation of Jill's score on a single roll.

19. Who has the larger expected value: Todd or Jill? Who has the larger standard deviation? Were your earlier predictions correct?

Todd's and Jill's Combined Performance in a Partner League

20. Now suppose that Todd's and Jill's points (from a single roll) are to be added. Let the random variable $C = T + J$ represent their combined (added) scores. Make a prediction of the expected value of C.

21. Do you think that the standard deviation of C will be larger, smaller, or the same as the standard deviations of T and J? Explain your thinking, without performing any calculations.

22. To find these values exactly, you first need to find the probability distribution for C. To do this, first list three possible values for C.

23. If you assume that Todd's roll is independent of Jill's roll, calculate the probability for each of the three possible values for C you provided in #22.

To find the expected value and standard deviation for C, you could fill in the entire probability distribution for C. But this will be a fairly tedious process. We could use simulation to give us an idea of the behavior of C. **Figure 11.5.3** shows the point values of 10,000 rolls by Todd added to the point values of 10,000 rolls by Jill.

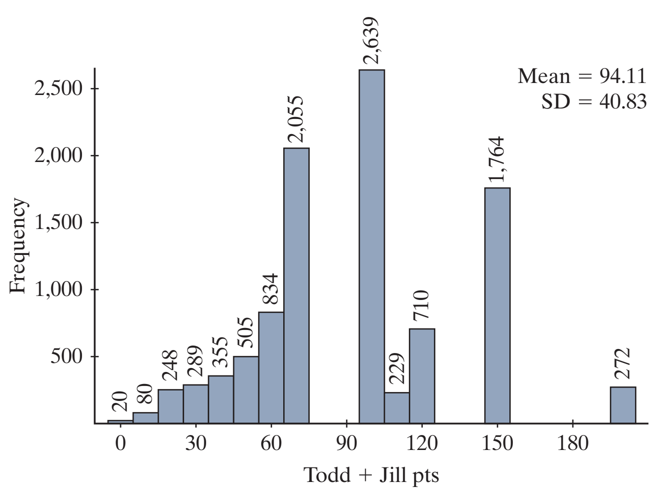

FIGURE 11.5.3 Point totals summing Todd's and Jill's scores together for 10,000 rolls.

24. What is the least likely outcome (but possible) for the sum? Why? What is the largest possible sum? Roughly how often do they score a point total equaling this largest possible sum?

25. How do the mean and standard deviation of theses sums compare to your predictions in #20 and #21?

Again, there are also shortcut formulas that tell us how certain types of combinations of random variables will behave without carrying out the simulation.

Key Idea

In general, when summing two or more random variables

- The expected value of the sum of the random variables is the sum of the expected values.
  - For example, $E(X + Y) = E(X) + E(Y)$
- If the random variables are independent, the variance of their sum is the sum of their variances.
  - For example, $Var(X + Y) = Var(X) + Var(Y)$
- The standard deviation of the sum of random variables is not necessarily equal to the sum of the standard deviations of the random variables, even if the random variables are independent.

26. Use the rule for expected values to find the expected combined score. Is it similar to the simulation results?

27. Use the rule for the variance of the combined score.

28. Take the square root of your answer to #27 to find the standard deviation of their combined score. Show that this is not equal to the sum of Todd and Jill's individual standard deviations. Is it similar to the simulation results?

The Difference in Todd's and Jill's Performance

Suppose instead of adding Todd's and Jill's scores together, you subtracted them (Todd – Jill). This would allow you to determine how often Todd scores higher than Jill in the long run.

29. Briefly describe how you could use simulation to explore the distribution of these differences.

30. Make predictions for the mean and the standard deviation of this distribution.

Figure 11.5.4 shows the differences in the scores (Todd – Jill) for 10,000 rolls.

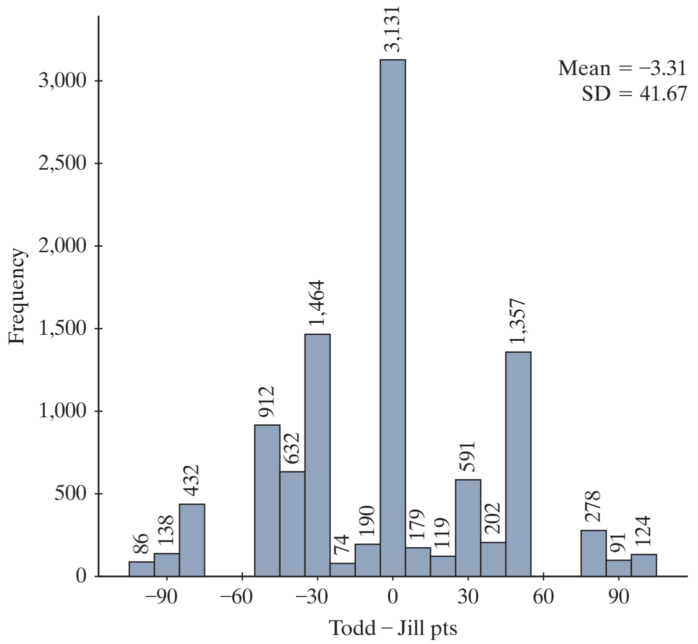

FIGURE 11.5.4 Differences in scores (Todd – Jill) for 10,000 rolls.

31. Which difference occurred the most frequently? Does this make sense? What is the largest possible difference? How likely is this outcome? Does that make sense?

32. How do the mean and standard deviation compare to your predictions? In particular, why does it make sense that the standard deviation of the differences is *larger* (in fact, similar to the standard deviation of the sums), rather than being the difference in the standard deviations?

Key Idea

When combining random variables X and Y, $\mu_{X+Y} = \mu_X + \mu_Y$ and $\mu_{X-Y} = \mu_X - \mu_Y$. This is true regardless of whether X and Y are independent. If the random variables are independent, then it is also true that $\sigma^2_{X\pm Y} = \sigma^2_X + \sigma^2_Y$; however, that relationship is not true if X and Y are not independent.

33. Use the rules shown in the Key Idea box to determine the expected value and standard deviation of the *difference* between Todd's and Jill's scores ($S = T - J$). How does these values compare to the simulation results? How does $SD(T - J)$ compare to $SD(T + J)$?

34. Is $E(S)$ positive, negative, or zero? Interpret what this means about Todd's and Jill's scores in the long run.

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Summing Several Random Variables

A typical game of Skee-Ball consists of 9 rolls. Let's assume that the rolls are independent.

35. Determine the expected value and standard deviation of the sum of Todd's scores over 9 rolls. Also mention what assumption you need to make for one of these calculations.

Now suppose that Todd was given the option of: (A) adding his scores on 9 independent rolls, or (B) multiplying his score on one roll by 9.

36. Calculate and compare the expected values with option (A) and option (B). Which option (if either) would Todd prefer? Why?

37. Calculate and compare the standard deviations with option (A) and option (B). Which option (if either) would Todd prefer? Why?