

L28 - Named Distributions (Binomial, Geometric, Normal Distributions)

Binomial RV - # of successes for a fixed # of trials $X \sim B(n, \pi)$

$$P(X=K) = \binom{n}{K} \pi^K (1-\pi)^{n-K}$$

K : # Successes in n : Trials

1. Binary (Success/fail)

2. Independent trials

3. fixed # of trials (n)

4. Constant prob of success (π)

To find # of arrangements of K successes in n trials

$$\binom{n}{K} = \frac{n!}{K!(n-K)!}$$

To find probabilities:

$$P(X=K) \quad \text{dbinom}(K, n, \pi)$$

$$E(X) = \mu_x = n \cdot \pi$$

$$P(X \leq K) \quad \text{pbinom}(K, n, \pi)$$

$$SD(X) = \sigma_x = \sqrt{n \cdot \pi (1-\pi)}$$

$$P(X < K) \quad \text{pbinom}(K-1, n, \pi)$$

$$P(X > K) \quad 1 - \text{pbinom}(K, n, \pi)$$

$$P(X \geq K) \quad 1 - \text{pbinom}(K-1, n, \pi)$$

Geometric RV - # of attempts until 1st success

$$Y \sim G(\pi)$$

$$P(Y=K) = (1-\pi)^{K-1} \pi$$

1. binary

2. independent trials

3. Constant prob of success

To find probabilities

$$P(Y=K) \quad \text{dgeom}(K-1, \pi)$$

$$P(Y \leq K) \quad \text{pgeom}(K-1, \pi)$$

$$P(Y < K) \quad \text{pgeom}(K-2, \pi)$$

$$P(Y > K) \quad 1 - \text{pgeom}(K-1, \pi)$$

$$P(Y \geq K) \quad 1 - \text{pgeom}(K-2, \pi)$$

* Probability that the first success occurs on the K^{th} trial.

$$E(Y) = \frac{1}{\pi}$$

$$SD(Y) = \frac{\sqrt{1-\pi}}{\pi}$$

Normal RV - $X \sim N(\mu, \sigma)$

- bell shaped curve

- centered at μ

- Standard dev of σ



$$Z = \frac{X - \mu}{\sigma}$$

$$P(X < a) \quad \text{pnorm}(a, \mu, \sigma)$$

$$P(X > a) \quad 1 - \text{pnorm}(a, \mu, \sigma)$$

$$P(a < X < b) = \text{pnorm}(b, \mu, \sigma) - \text{pnorm}(a, \mu, \sigma)$$

Example. Probability of exactly 1 head out of 3 flips

$$X \sim B(n, \pi) = B(3, 0.5)$$

Sample space

3 ways to solve.

H H H
H H T
H T T
T T T
T T H
T H H
H T H
T H T

1. $P(X=1) = 3/8 \leftarrow \text{Sample space}$

2. $P(X=1) = \binom{3}{1} \cdot 0.5^1 (1-0.5)^{3-1} = 3/8$

$$\binom{3}{1} = \frac{3!}{1!(3-1)!} = 3$$

3. $P(X=1) = \text{dbinom}(K, n, \pi) = \text{dbinom}(1, 3, 0.5)$

Prob that 1st head is on the 5th flip

TTTTTH

$$P(Y=5) \rightarrow (1-0.5)^{5-1} \cdot 0.5 = .0312$$

$$P(Y=5) \rightarrow \text{dgeom}(K-1, \pi)$$