

Board Problems for Lesson 27: Binomial, Geometric, and Normal Distributions

1. Michael Jordan, Hall of Fame basketball player, made about 85% of his free throws during his prime years with the Chicago Bulls. During a free throw contest, he will attempt 20 shots. Assume all of his free throws are independent of one another.

a. What is the probability that Jordan makes exactly 15 of his attempts?

let $X = \#$ free throws made

$X \sim \text{Bin}(n = 20, \pi = 0.85)$

find $P(X = 15) = \text{dbinom}(15, 20, 0.85) = 0.1028$

b. What is the probability that Jordan makes more than 18 free throws?

find $P(X > 18) = 1 - P(X \leq 18) = 1 - \text{pbinom}(18, 20, 0.85) = 0.1756$

c. What is the probability that Jordan makes at least 14 of his attempts?

find $P(X \geq 14) = 1 - P(X \leq 13) = 1 - \text{pbinom}(13, 20, 0.85) = 0.9781$

2. Historically, a Soldier hits a target at 50 meters 80% of the time. Assume shots are independent of one another.

a. What is the probability that the Soldier misses on the first two shots?

let $Y = \#$ shots fired until 1st hit

find $P(\text{miss 1st and 2nd}) = P(\text{miss}) * P(\text{miss}) = 0.20 * 0.20 = 0.04$

b. What is the probability that the Soldier misses a few in a row, and then hits the target on the 4th shot?

find $P(\text{miss 3 and hit on 4th}) = P(Y = 4) = \text{dgeom}(k-1, \text{prob}) = \text{dgeom}(3, 0.80) = 0.0064$

c. What is the probability that the Soldier requires at least 4 shots to hit the target?

find $P(\text{at least 3 misses}) = P(\text{hits on 4th or later shot}) = P(Y \geq 4)$

$P(Y \geq 4) = 1 - \text{pgeom}(k-2, \text{prob}) = 1 - \text{pgeom}(2, 0.80) = 0.008$

3. The weekly amount of diesel fuel your unit uses was observed, over a long period of time, to be approximately normally distributed with a mean of 400 gallons and a variance of 400 gallons.

a. If 420 gallons are budgeted for next week, what is the probability that the actual amount consumed will be within the budget?

Since $\text{Var}(X) = \sigma^2 = 400$, so $\text{SD}(X) = \sigma = 20$

$$P(X \leq 420) = \text{pnorm}(420, \text{mean} = 400, \text{sd} = 20) = 0.841$$

b. If 450 gallons are budgeted for next week, what is the probability that the actual amount consumed will exceed the budgeted amount?

Since $\text{Var}(X) = \sigma^2 = 400$, so $\text{SD}(X) = \sigma = 20$

$$P(X > 450) = 1 - P(X \leq 450) = 1 - \text{pnorm}(450, \text{mean} = 400, \text{sd} = 20) = 0.00621$$

c. What is the probability your unit uses between 390 and 425 gallons of fuel?

$$P(390 \leq X \leq 425) = P(X \leq 425) - P(X \leq 390)$$

$$= \text{pnorm}(425, \text{mean} = 400, \text{sd} = 20) - \text{pnorm}(390, \text{mean} = 400, \text{sd} = 20) = 0.586$$