L26 Discrete Random Variables

Probability Distribution - distribution of possible outcomes and their probabilities

Expected Value - The long run average of a random variable

$$E(X) = x_1 P(x_1) + x_2 P(x_2) + \ldots$$

Variance

$$Var(X) = \sigma_X^2 = (x_1 - \mu_x)^2 \cdot p_1 + (x_1 - \mu_x)^2 \cdot p_2 + \ldots$$

Standard Deviation

$$SD(x) = \sigma_x = \sqrt{Var(x)}$$

Linear Transformations

If $Y = a + b \cdot X$

$$E(Y) = a + b \cdot E(x)$$

$$Var(Y) = b^2 \cdot Var(x)$$

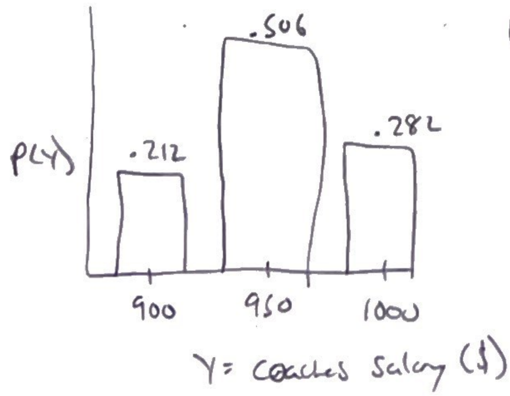
$$SD(Y) = |b| \cdot SD(x)$$

Addition Rule

$$E(x+Y) = E(x) + E(Y)$$

if x and y are independent,
$Var(x+Y) = Var(x) + Var(Y)$

Army/Navy Problem continued...



Why do linear transformations work?

- Same shape as the original distribution, we are only changing the scale

L26- Continued - Example Problem

Army/Navy wins.

A : beat AF
B : beat Navy
] -assume A and B are independent

$P(A) = .47$ $P(A') = 1 - P(A) = .53$
$P(B) = .60$ $P(B') = 1 - P(B) = .40$

<u>0 wins case</u>

$P(A' \cap B') = P(A') \cdot P(B') = (.53) \cdot (.40) = .212$

<u>1 win</u>

$P(A \cap B') = .47(.40) = .188$

$P(A' \cap B) = .318$

Total $= .188 + .318 = .506$

<u>2 wins</u>

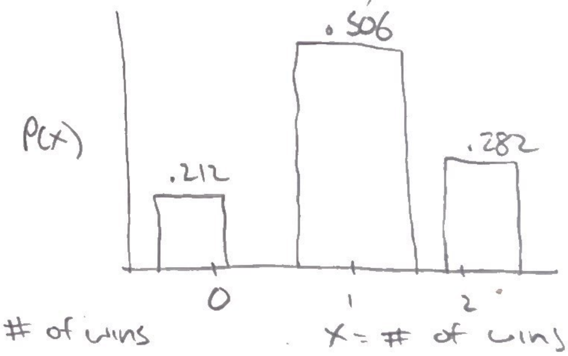
$P(A \cap B) = .282$

Probability Table

X	0	1	2
P(x)	.212	.506	.282

Let x = # of wins

Probability Distribution



Calculate E(x), Var(x), SD(x)

long run avg # of wins

$E(x) = 0(.212) + 1(.506) + 2(.282) = \underline{\underline{1.07}}$

$Var(x) = (0-1.07)^2 * (.212) + (1-1.07)^2 * (.506) + (2-1.07)^2 * (.282)$
$= \underline{\underline{.4891}}$

$SD(x) = \sqrt{.4891} = .699$

How much will coach get paid?

- Base Salary : \$900K
- Bonus for each win: \$50K

This is a linear Transformation

$Y = \underset{a}{900} + \underset{b}{50} \cdot X$ (in \$1,000's)

Calculate E(Y), Var(Y), SD(Y)

$E(Y) = a + b \cdot E(x) = 900 + 50(1.07) = \$953.5$

$Var(Y) = b^2 \cdot Var(x) = 50^2 \cdot .4891 = 1222.75$

$SD(Y) = |b| \cdot SD(x) = 50 \times .699 = \$34.95$

Var(Y) hard to interpret

SD(Y) → easier. It is back in the same units as E(Y)

SD(Y) - the avg distance from the mean for a individual observation