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I.1 When we regress age on married and schooling on married using a linear model we receive the following outputs:

Age Married Linear: Coefficient: 0.557 Standard Errors: 0.016

S Married Linear: Coefficient: 0.130 Standard Errors: 0.008

I.2 When we regress age on married and schooling on married using a probit model we receive the following outputs:

Age Married Probit: Coefficient: 0.144 Standard Errors: 0.043

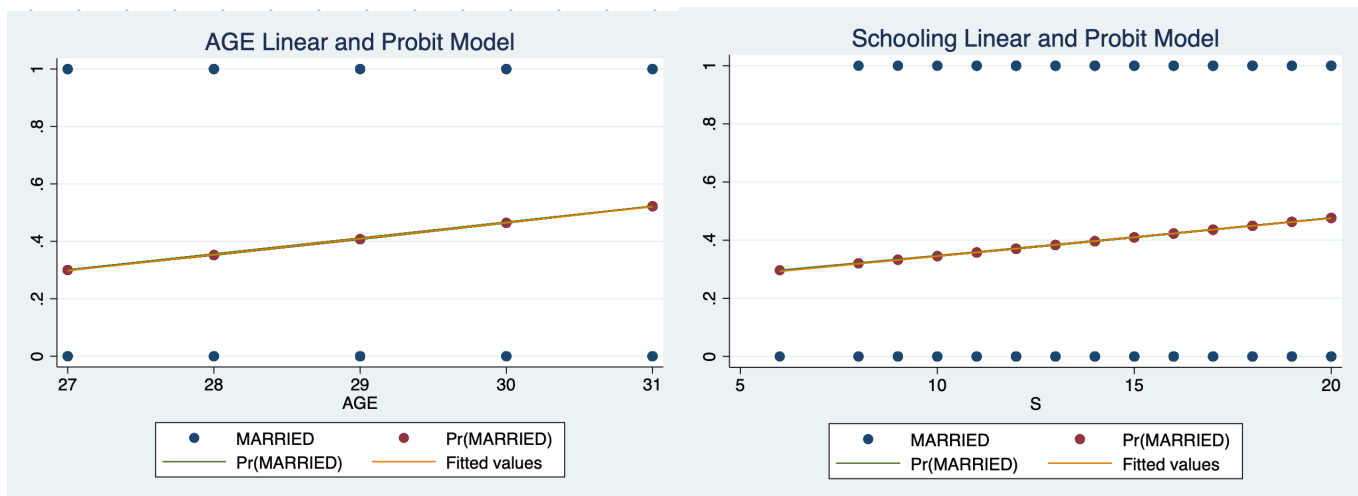
S Married Probit: Coefficient: 0.034 Standard Errors: 0.020

I.3 Linear Objective function: $\sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i)^2$

Probit Objective function: $\sum_{i=1}^n (Y_i \ln \Phi(\beta_0 + \beta_1 X_i) + (1 - Y_i) \ln(1 - \Phi(\beta_0 + \beta_1 X_i)))$

The linear model estimates the linear expectation of Y given X . Probit is the probability of Y given X ranging from 0 to 1 which is a nonlinear model. Probit is better if the Y is a binary outcome.

I.4



I.5 Based on our MSE for Age Probit is the better model with a MSE of 0.43808903 versus linear with an RMSE of 0.43814897 for our subsample

For schooling Probit is a better model with a MSE of 0.47444433 versus linear with an RMSE of 0.47470787 for our subsample

I6 According to our RSME for our entire model, probit is better for estimating age with a value of 0.48485471 versus linear with 0.48491913. for schooling our linear model yields a better fit with a RMSE of 0.4893582 versus the probit model with a fit of 0.48936767

I7 The only coefficients that can be interpreted as an effect of a unit change of the regressor are the slope coefficients of our OLS model.

I8 Probability of being married by age

Linear

Probit

AGE	Mean
27	.2976796
28	.3534031
29	.4091265
30	.46485
31	.5205736

AGE	Mean
27	.3001313
28	.3523128
29	.4074364
30	.4644593
31	.5222231

I9 The marginal probability of getting married after 27, 29, and 31 is the following for each model

```
Expression: Linear prediction, predict()
dy/dx wrt: AGE
1._at: AGE = 27
2._at: AGE = 29
3._at: AGE = 31
```

		Delta-method				[95% conf. interval]	
		dy/dx	std. err.	t	P> t		
AGE		Marginal Effect					
	_at						
	1	.0557235	.016204	3.44	0.001	.0238869	.0875601
	2	.0557235	.016204	3.44	0.001	.0238869	.0875601
	3	.0557235	.016204	3.44	0.001	.0238869	.0875601

```
Expression: Pr(MARRIED), predict()
dy/dx wrt: AGE
1._at: AGE = 27
2._at: AGE = 29
3._at: AGE = 31
```

		Delta-method				[95% conf. interval]	
		dy/dx	std. err.	z	P> z		
AGE		Marginal Effect					
	_at						
	1	.0504043	.0128045	3.94	0.000	.0253079	.0755006
	2	.0562589	.0167136	3.37	0.001	.0235008	.089017
	3	.0577326	.0168295	3.43	0.001	.0247474	.0907179

II 1

MARRIED	Coefficient	Robust std. err.	z	P> z	[95% conf. interval]	
AGE	.2290208	.0710142	3.22	0.001	.0898355	.3682061
S	.0573351	.0359714	1.59	0.111	-.0131675	.1278377
HEIGHT	.0257718	.0235736	1.09	0.274	-.0204316	.0719753
EARNINGS	.0133591	.0092017	1.45	0.147	-.004676	.0313941
URBAN	-.6720038	.2165565	-3.10	0.002	-1.096447	-.2475609
_cons	-9.357966	2.773062	-3.37	0.001	-14.79307	-3.922864

$$\text{II 2 } \sum_{i=1}^n \left[y_i \ln \Lambda(\beta_0 + \beta_1 \text{Age}_i + \beta_2 S_i + \beta_3 \text{HEIGHT}_i + \beta_4 \text{Earnings}_i + \beta_5 \text{URBAN}_i) + (1 - y_i) \ln(1 - \Lambda(\beta_0 + \beta_1 \text{Age}_i + \beta_2 S_i + \beta_3 \text{HEIGHT}_i + \beta_4 \text{Earnings}_i + \beta_5 \text{URBAN}_i)) \right]$$

II 3 For our logit model we have a RMSE of 0.47667035 which is lower than our probit or linear.

II 4 The marginal effect of age is $\sum_{i=1}^n \frac{\partial E[\text{Married}_i | \text{Age}_i]}{\partial \text{Age}_i} / n$ which does depend on our other regressors. This is unlike linear where our marginal effect would be $\frac{dy}{dx}$ and would be constant regardless of the value of X .

To show that it depends on other variables we will calculate the the coefficients at an amount and then change one variable to show the other coefficients change which can be seen below

```
. mfx compute, dydx at (27,16,66,18,0)
```

Marginal effects after logit
y = Pr(MARRIED) (predict)
= .42173904

variable	dy/dx	Std. err.	z	P> z	[95% C.I.]	X
AGE	.0558525	.01616	3.46	0.001	.024185	.08752		27
S	.0139826	.00896	1.56	0.118	-.003572	.031537		16
HEIGHT	.0062851	.00571	1.10	0.271	-.004899	.017469		66
EARNINGS	.0032579	.00223	1.46	0.145	-.001119	.007635		18
URBAN*	-.150361	.0512	-2.94	0.003	-.250714	-.050008		0

(*) dy/dx is for discrete change of dummy variable from 0 to 1

```
. mfx compute, dydx at (27,16,66,18,1)
```

Marginal effects after logit
y = Pr(MARRIED) (predict)
= .27137801

variable	dy/dx	Std. err.	z	P> z	[95% C.I.]	X
AGE	.0452847	.01131	4.00	0.000	.023112	.067458		27
S	.011337	.00736	1.54	0.123	-.003084	.025758		16
HEIGHT	.0050959	.00455	1.12	0.263	-.003824	.014016		66
EARNINGS	.0026415	.00182	1.45	0.147	-.000925	.006208		18
URBAN*	-.150361	.0512	-2.94	0.003	-.250714	-.050008		1

(*) dy/dx is for discrete change of dummy variable from 0 to 1

$$\text{III } \sum_{i=1}^n \log \left[\left(1 - \Phi \left(\frac{\beta_0 + \beta_1 \text{Schooling}_i + \beta_2 (\text{Schooling}_i \cdot \text{Urban}_i) + \beta_3 \text{experience}_i + \beta_4 \text{experience}_i^2 - \ln(2)}{\sigma} \right) \right)^{I(X_i \leq \ln 2)} \cdot \left(\frac{1}{\sigma} \Phi \left(\frac{\beta_0 + \beta_1 \text{Schooling}_i + \beta_2 (\text{Schooling}_i \cdot \text{Urban}_i) + \beta_3 \text{experience}_i + \beta_4 \text{experience}_i^2 - 2}{\sigma} \right) \right)^{I(X_i > \ln 2)} \right]$$

lny	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
S	.0964689	.0107491	8.97	0.000	.0753496	.1175882
S_URB	-.0026287	.0037242	-0.71	0.481	-.0099459	.0046885
EXP	.0022166	.0343286	0.06	0.949	-.0652308	.069664
EXP2	.003259	.0023837	1.37	0.172	-.0014244	.0079424
_cons	1.202117	.2170589	5.54	0.000	.7756492	1.628586
var(e.lny)	.2617688	.0165557			.2311806	.2964041

III 2 β_1 is the change in probability of y based on x above the limit of earnings being greater than 2. The model may be biased and inconsistent because we are not observing the earnings of people that are in school or unemployed

$$\text{IV 1 Likelihood}(\lambda) = P(X=2) \cdot P(X=5) \cdot P(X=2) = e^{-\lambda} \cdot \frac{\lambda^2}{2!} \cdot e^{-\lambda} \cdot \frac{\lambda^5}{5!} \cdot e^{-\lambda} \cdot \frac{\lambda^2}{2!}$$

$$\begin{aligned} 2 \ln(\text{likelihood}(\lambda)) &= \ln \left(e^{-\lambda} \cdot \frac{\lambda^2}{2!} \cdot e^{-\lambda} \cdot \frac{\lambda^5}{5!} \cdot e^{-\lambda} \cdot \frac{\lambda^2}{2!} \right) \\ &= \ln(e^{-3\lambda} + 2\ln\lambda - \ln(2!) + 5\ln(\lambda) - \ln(5!) + 2\ln(\lambda) - \ln(2!)) \end{aligned}$$

$$\ln(\text{likelihood}(\lambda)) = -3\lambda + 2\ln(\lambda) + 5\ln(\lambda) + 2\ln(\lambda)$$

$$\frac{\partial \ln(\text{likelihood}(\lambda))}{\partial \lambda} = -3 + \frac{2}{\lambda} + \frac{5}{\lambda} + \frac{2}{\lambda} = 0$$

$$\lambda = \frac{2+5+2}{3} = 3 = \text{MLE}$$