1 $Y_t = \mu + \rho Y_{t-1} + \varepsilon_t$

1a $|\rho|<1$ $E[Y_t] = E[\mu + \rho Y_{t-1} + \varepsilon_t]$

$= E(\mu) + E(\rho Y_{t-1}) + E(\varepsilon_t)$

$= \mu + \rho E(Y_{t-1})$

since $|\rho|<1$ $E(Y_t) = \mu + \rho E(Y_t)$

$\mu = E(Y_t) - \rho E(Y_t)$

$\mu = E(Y_t)(1-\rho)$

$E(Y_t) = \dfrac{\mu}{(1-\rho)}$ = Unconditional mean

1b Show $E[Y_{t+h}|Y_t] = \rho^h Y_t + \mu \sum_{l=0}^{h-1} \rho^l$

assume $h=1$ $E[Y_{t+1}|Y_t] = \rho Y_t + \mu + \underbrace{E(\varepsilon_{t+1}|Y_t)}_{=0} = \rho Y_t + \mu$

assuming the formula works for $h=k$ we will show it works for $h=k+1$

$E[Y_{t+k+1}|Y_t] = \rho E[Y_{t+k}|Y_t] + \mu + 0$

$= \rho(\rho^k Y_t + \mu \sum_{l=0}^{k-1} \rho^l) + \mu$

$= \rho^{k+1} Y_t + \mu \sum_{l=0}^{k} \rho^l$

thus $E[Y_{t+h}|Y_t] = \rho^h Y_t + \mu \sum_{l=0}^{h-1} \rho^l = \rho^h Y_t + \mu/(1-\rho)$

1c given $|\rho|<1$

$\lim_{h\to\infty} E[Y_{t+h}|Y_t] = \lim_{h\to\infty} \rho^h Y_t + \lim_{h\to\infty} \mu/(1-\rho) = \mu/(1-\rho)$

$\lim_{h\to\infty} E[Y_{t+h}|Y_t]$ does not depend on Y_t

1d $E[Y_{t+h}|Y_t] = \rho^h Y_t + \mu \sum_{l=0}^{h-1} \rho^l$ given $|\rho|=1$ $E[Y_{t+h}|Y_t] = Y_t + h\mu$

$$\lim_{h\to\infty} E[Y_{t+h}|Y_t] = \lim_{h\to\infty} Y_t + \lim_{h\to\infty} h\mu = Y_t + \mu \lim_{h\to\infty} h \begin{cases} \text{if } \mu < 0 \to = -\infty \\ \text{if } \mu = 0 \to = 0 \\ \text{if } \mu > 0 \to = \infty \end{cases}$$

No it does not depend on Y_t it depends on μ.

3 $Y_t = \beta_1 + \delta t + \varepsilon_t, t = 1, \dots, T$

Based on the FOC's of the OLS

$$\hat{\delta} = \frac{\sum_{t=1}^T (t-\bar{t})(Y_t - \hat{Y}_t)}{\sum_{t=1}^T (t-\bar{t})}$$

$$= \frac{\sum_{t=1}^T (t-\bar{t})(\beta + \delta t + \varepsilon_t - \hat{Y}_t)}{\sum_{t=1}^T (t-\bar{t})}$$

$$= \frac{\sum_{t=1}^T [(t-\bar{t})\delta t + \beta_1 (t-\bar{t}) + (t-\bar{t})\varepsilon_t - (t-\bar{t})\hat{Y}_t]}{\sum_{t=1}^T (t-\bar{t})^2}$$

$$= \frac{\sum_{t=1}^T (t-\bar{t})\delta t}{\sum_{t=1}^T (t-\bar{t})^2} + \frac{\sum_{t=1}^T (t-\bar{t})(\beta_1 - \hat{Y}_t)}{\sum_{t=1}^T (t-\bar{t})^2} + \frac{\sum_{t=1}^T (t-\bar{t})\varepsilon_t}{\sum_{t=1}^T (t-\bar{t})^2}$$

$$= \delta + \frac{\sum_{t=1}^T (\varepsilon_t - \bar{\varepsilon}_t)(t - \frac{T+1}{2})}{\sum_{t=1}^T (t - \frac{T+1}{2})^2}$$

$$var(\hat{\delta} - \delta)\sqrt{T} = var\left[\frac{\sqrt{T}\sum_{t=1}^T \varepsilon_t (t - \frac{T+1}{2})}{\sum_{t=1}^T (t - \frac{T+1}{2})^2} \quad \bar{\varepsilon}_t \quad \frac{\sqrt{T}\sum_{t=1}^T (t - \frac{T+1}{2})}{\sum_{t=1}^T (t - \frac{T+1}{2})^2}\right]$$

$$= var\left[\varepsilon_t \quad \frac{T\sum_{t=1}^T (t - \frac{T+1}{2})^2}{(\sum_{t=1}^T (t - \frac{T+1}{2})^2)^2}\right]$$

$$= var\left[\varepsilon_t \cdot \frac{T}{\sum_{t=1}^T (t - \frac{T+1}{2})^2}\right]$$

given $\sum_{t=1}^T t^2 = \frac{T(2T+1)(T+1)}{6}$

$$\sum_{t=1}^T t^2 - 2\sum_{t=1}^T (t \cdot \frac{T+1}{2}) + T(\frac{T+1}{2})^2 = \frac{T(2T+1)(T+1)}{6} - \frac{T(T+1)^2}{2} + \frac{T(T+1)^2}{4}$$

$$= T\left[\frac{(T+1)(2T+1)}{6} - \frac{(T+1)^2}{2} + \frac{(T+1)^2}{4}\right]$$

$$= \frac{T(T^2-1)}{12}$$

$$Var(\hat{\delta}-\delta) = \frac{\varepsilon_T \, T \, 12}{(T+1)(T-1)T}$$

$$= Var(\varepsilon_T) \cdot \frac{12}{(T+1)(T-1)}$$

$$= \sigma^2 \cdot 0 \text{ when } T \to \infty$$

$$= 0$$

δ is superconsistent if the following model holds

$$E(\hat{\delta}) = E(\delta) + E\left[\sum_{t=1}^{T} \frac{t-\bar{t}}{\sum_{t=1}^{T}(t-\frac{T+1}{2})^2}(\varepsilon_t - \bar{\varepsilon}_T)\right]$$

$$= \delta + \left(\sum_{t=1}^{T} \frac{t-\bar{t}}{\sum_{t=1}^{T}(t-\frac{T+1}{2})^2}\right) E(\varepsilon_t - \bar{\varepsilon}_T)$$

$$= \delta + 0$$

$$= \delta$$

In a finite sample it is unbiased

Kyle Nabors worked with Bibang and Ricardo

1.
2. Seasonal Patterns
    a. After generating our seasonal dummy variable and computing the regression of gcem on L(0/10) we have the following results seen in Figure 1. Using this we can then calculate the average value for gcem for every month to determine which has the largest and smallest average value seen in Figure 2. Using this we find that March has the largest average value with 0.2396536 and December has the smallest with a value of -0.3081064.
    b. Preforming the OLS of gcem on grres (x) and then with 1, 3, and 5 lags. The 3 regressions are seen in Figure 3. We can then find the three chi2 for each of these regressions in Figure 4. Using these we can calculate the nR2 for each of these regressions and compare them to the Chi2 to test for serial correlation which is seen in the following table.

	nR2	Chi2	Asymptotic Distribution
1 Lag	69.0844	69.237	X^2(1)
3 Lags	101.9592	103.322	X^2(3)
5 Lags	114.8512	117.311	X^2(5)

    Based on this we see that the Stata reported Chi2 is close to our nR2 for all lags 1, 3, and 5.
    c. Comparing Newey-West standard errors to our regular standard errors we see how the two models differ in Figure 5. As we can see the Newey west SE for grres is 0.408 and the normal regression SE is 0.376. Since there is serial correlation in the error terms the robust standard errors from our OLS regression are not consistent. This differs from our Newey-West SE which are consistent to order p since Newey-West standard errors account for the presence of serial correlation.
    d. Testing the null hypothesis that Beta1 equals Beta2 we find the T-test seen in Figure 6. Based on this we reject the null hypothesis in favour of the alternative hypothesis that Beta1 does not equal Beta2 with a 99% significance level.

Appendix:

Figure 1:

```
. reg gcem L(0/10).seasonal,robust

Linear regression                               Number of obs     =        300
                                                F(11, 288)        =     222.59
                                                Prob > F          =     0.0000
                                                R-squared         =     0.8579
                                                Root MSE          =     .07263

------------------------------------------------------------------------------
             |               Robust
        gcem | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    seasonal |
         --. |   .0615033   .0311696     1.97   0.049     .0001543    .1228524
         L1. |   .4572201   .0277411    16.48   0.000      .402619    .5118211
         L2. |     .54776   .0227141    24.12   0.000     .5030534    .5924667
         L3. |   .5216514   .0162686    32.06   0.000      .489631    .5536717
         L4. |   .3994491   .0145876    27.38   0.000     .3707372     .428161
         L5. |   .4189702   .0127077    32.97   0.000     .3939584     .443982
         L6. |   .2630278   .0138355    19.01   0.000     .2357962    .2902593
         L7. |   .3745389    .012687    29.52   0.000     .3495679    .3995098
         L8. |   .2636277   .0132052    19.96   0.000     .2376368    .2896186
         L9. |   .3240971   .0131053    24.73   0.000     .2983027    .3498914
        L10. |    .070113   .0160822     4.36   0.000     .0384595    .1017666
             |
       _cons |  -.3081064   .0114932   -26.81   0.000    -.3307278    -.285485
------------------------------------------------------------------------------
```

Figure 2:

```
. reg gcem L(0/11).seasonal, nocons robust

Linear regression                               Number of obs     =        299
                                                F(12, 287)        =     210.04
                                                Prob > F          =     0.0000
                                                R-squared         =     0.8572
                                                Root MSE          =      .0726

------------------------------------------------------------------------------
             |               Robust
        gcem | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    seasonal |
         --. |  -.2466031   .0289752    -8.51   0.000     -.303634   -.1895722
         L1. |   .1491136     .02525     5.91   0.000     .0994149    .1988124
         L2. |   .2396536   .0195931    12.23   0.000     .2010892     .278218
         L3. |   .2135449   .0115148    18.55   0.000     .1908808    .2362091
         L4. |   .0913427   .0089842    10.17   0.000     .0736594     .109026
         L5. |   .1108637   .0054218    20.45   0.000     .1001922    .1215353
         L6. |  -.0450786   .0077029    -5.85   0.000    -.0602401   -.0299172
         L7. |   .0664325    .005373    12.36   0.000      .055857    .0770079
         L8. |  -.0444787   .0065029    -6.84   0.000    -.0572782   -.0316792
         L9. |   .0159906   .0062976     2.54   0.012     .0035952     .028386
        L10. |  -.2346745    .011197   -20.96   0.000    -.2567131   -.2126358
        L11. |  -.3081064    .011494   -26.81   0.000    -.3307297   -.2854831
------------------------------------------------------------------------------
```

Figure 3:

```
. estout lag1 lag3 lag5, cells(b(star fmt(3)) se(par fmt(2)))   ///
>    legend label varlabels(_cons constant)                    ///
>    stats(r2 df_r bic, fmt(3 0 1) label(R-sqr dfres BIC))
```

	1 Lag b/se	3 Lag b/se	5 Lag b/se
x	0.091	-0.049	0.060
	(0.32)	(0.30)	(0.30)
L.Residuals	0.474***	0.495***	0.499***
	(0.05)	(0.05)	(0.06)
L2.Residuals		0.003	-0.085
		(0.06)	(0.06)
L3.Residuals		-0.335***	-0.344***
		(0.05)	(0.06)
L4.Residuals			0.149*
			(0.06)
L5.Residuals			-0.264***
			(0.06)
constant	-0.000	-0.001	-0.002
	(0.01)	(0.01)	(0.01)
R-sqr	0.224	0.333	0.378
dfres	305	301	297
BIC	-219.6	-253.8	-261.5

* p<0.05, ** p<0.01, *** p<0.001

Figure 4:

lags(*p*)	chi2	df	Prob > chi2
1	69.237	1	0.0000
3	103.322	3	0.0000
5	117.311	5	0.0000

Figure 5:

	Newey SE b/se	Normal SE b/se
x	0.512	0.512
	(0.408)	(0.376)
constant	0.003	0.003
	(0.012)	(0.011)
R-sqr		0.007
dfres	307	307
BIC	.	-148.2

* p<0.05, ** p<0.01, *** p<0.001

Figure 6:

Two-sample t test with equal variances

Group	Obs	Mean	Std. err.	Std. dev.	[95% conf.	interval]
1	25	-.2466031	.0289732	.1448661	-.3064009	-.1868053
2	26	.1477424	.0242965	.1238882	.0977028	.1977819
Combined	51	-.0455642	.0335445	.2395556	-.1129403	.0218119
diff		-.3943454	.037695		-.4700964	-.3185944

diff = mean(1) - mean(2) t = -10.4615

H0: diff = 0 Degrees of freedom = 49

Ha: diff < 0 Ha: diff != 0 Ha: diff > 0

Pr(T < t) = 0.0000 Pr(|T| > |t|) = 0.0000 Pr(T > t) = 1.0000