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ECN 140
Assignment 3

1 a. minimise $\sum_{i=1}^{n}(y_i - b_1 x_{1i} - b_2 x_{2i})^2$

$$\min_{b_1, b_2} f(x, y, b_1, b_2) \sum_{i=1}^{n}(y_i - b_1 x_{1i} - b_2 x_{2i})^2$$

b. $\frac{\partial f}{\partial b_1} = -2\sum_{i=1}^{n}(y_i - b_1 x_{1i} - b_2 x_{2i})x_{1i} = 0$

$$\frac{\partial R}{\partial b_2} = -2\sum_{i=1}^{n}(y_i - b_1 x_{1i} - b_2 x_{2i})x_{2i} = 0$$

c. $-2\sum_{i=1}^{n}(y_i - b_1 x_{1i} - b_2 x_{2i})x_{1i} = 0$

$$-\sum y_i x_{1i} + b_1 \sum x_{1i}^2 + b_2 \sum x_{1i} x_{2i} = 0$$

$$b_1 \sum x_{1i}^2 = \sum y_i x_{1i} - b_2 \sum x_{1i} x_{2i}$$

$$b_1 = \frac{\sum y_i x_{1i} - b_2 \sum x_{1i} x_{2i}}{\sum x_{1i}^2}$$

$$b_1 = \hat{\beta}_1 - \frac{b_2 \sum x_{1i} x_{2i}}{\sum x_{1i}^2} \qquad \sum x_{1i} x_{2i} = 0$$

$$b_1 = \hat{\beta}_1 - \cancel{\frac{b_2 \cdot 0}{\sum x_{1i}^2}}$$

$$b_1 = \hat{\beta}_1 = \frac{\sum y_i x_{1i}}{\sum x_{1i}^2}$$

d. $\sum x_{1i} x_{2i} \neq 0$

$$\hat{\beta}_1 = \frac{\sum Y_i X_{1i} - \hat{\beta}_2 \sum X_{1i} X_{2i}}{\sum X_{1i}^2}$$

$$\hat{\beta}_2 = \frac{\sum Y_i X_{2i} - \hat{\beta}_1 \sum X_{1i} X_{2i}}{\sum X_{2i}^2}$$

$$\hat{\beta}_1 = \frac{\sum Y_i X_{1i}}{\sum X_{1i}^2} - \hat{\beta}_2 \frac{\sum X_{1i} X_{2i}}{\sum X_{1i}^2}$$

$$\hat{\beta}_1 = \frac{\sum Y_i X_{1i}}{\sum X_{1i}^2} - \left(\frac{\sum Y_i X_{2i} - \hat{\beta}_1 \sum X_{1i} X_{2i}}{\sum X_{2i}^2} \right) \frac{\sum X_{1i} X_{2i}}{\sum X_{1i}^2}$$

$$\left(1 - \frac{(\sum X_{1i} X_{2i})^2}{\sum X_{2i}^2 \sum X_{1i}^2}\right) \hat{\beta}_1 = \frac{\sum Y_i X_{1i}}{\sum X_{1i}^2} - \frac{\sum Y_i X_{2i} \sum X_{2i}}{\sum X_{1i} \sum X_{2i}^2}$$

$$\hat{\beta}_1 = \frac{\dfrac{\sum Y_i X_{1i}}{\sum X_{1i}^2} - \dfrac{\sum Y_i X_{2i} \sum X_{2i}}{\sum X_{1i} \sum X_{2i}^2}}{\left(1 - \dfrac{(\sum X_{1i} X_{2i})^2}{\sum X_{2i}^2 \sum X_{1i}^2}\right)}$$

e. $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$

$$f = \sum_{i=1}^{n} (Y_i - b_0 - b_1 X_{1i} - b_2 X_{2i})^2$$

$$\frac{df}{db_0} = -2 \sum_{i=1}^{n} (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{1i} - \hat{\beta}_2 X_{2i})$$

$$0 = \sum (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{1i} - \hat{\beta}_2 X_{2i}) \rightarrow$$

$$\sum \hat{\beta}_0 = \sum Y_i - \hat{\beta}_1 X_{1i} - \hat{\beta}_2 X_{2i}$$

$$\frac{n}{n} \hat{\beta}_0 = \frac{1}{n} \sum Y_i - \hat{\beta}_1 \frac{1}{n} \sum X_{1i} - \hat{\beta}_2 \frac{1}{n} \sum X_{2i}$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}_1 - \hat{\beta}_2 \bar{X}_2$$

$$f = \sum \left(y_i - (\bar{y} - \hat{\beta}_1 \bar{x}_1 - \hat{\beta}_2 \bar{x}_2) - \hat{\beta}_1 x_{1i} - \hat{\beta}_2 x_{2i} \right)^2$$

$$= \sum \left((y_i - \bar{y}) - \hat{\beta}_1 (x_{1i} - \bar{x}_1) - \hat{\beta}_2 (x_{2i} - \bar{x}_2) \right)^2$$

$$= \sum \left(\tilde{y}_i - \hat{\beta}_1 \tilde{x}_{1i} - \hat{\beta}_2 \tilde{x}_{2i} \right)^2$$

$$\hat{\beta}_1 = \frac{\sum \tilde{y}_i \tilde{x}_{1i}}{\sum \tilde{x}_{1i}^2} = \frac{\sum (y_i - \bar{y})(x_{1i} - \bar{x}_1)}{\sum (x_{1i} - \bar{x}_1)^2}$$