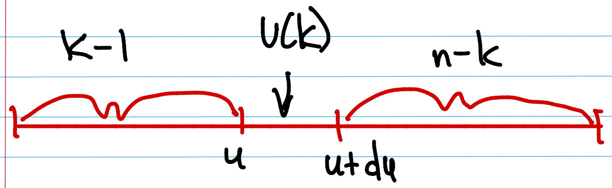
HW Problem 1)

$\{U_1, \ldots, U_n\} \to U_{(1)} \leq U_{(2)} \leq \ldots \leq U_{(n)}$



$\Rightarrow Pr(u \leq U_{(k)} \leq u+du)$

$\Downarrow$

$\binom{n}{k-1} \cdot u^{k-1}(1-u)^{n-k}$

$$F(k) = \int_0^1 \binom{n}{k-1} u^{k-1}(1-u)^{n-k}\,du \leftarrow \text{Use Integration by Parts}$$

Let, $w = (1-u)^{n-k}$ $\quad dv = u^{k-1}$

$dw = -(n-k)(1-u)^{n-k-1}\,du$ $\quad v = \frac{1}{k} \cdot u^k$

$$= \binom{n}{k-1} \cdot \left[\cancel{\frac{1}{k} u^k \cdot (1-u)^{n-k} \Big|_0^1}^{\,0} + \frac{(n-k)}{k} \cdot \int_0^1 u^k (1-u)^{n-k-1}\,du \right] \leftarrow \text{1st time Integration by Parts}$$

$$= \binom{n}{k-1} \cdot \left[\frac{(n-k)}{k} \cdot \frac{(n-k-1)}{k+1} \cdot \int_0^1 u^{k+1}(1+u)^{n-k-2}\,du \right] \leftarrow \text{2nd time Integration by Parts}$$

$\vdots$ Integration by Parts $(n-k)$ times

$$= \binom{n}{k-1} \cdot \left[\frac{(n-k)!(k-1)!}{n!} \cdot \int_0^1 u^n\,du \right] = \frac{\cancel{n!}}{(n-k+1)!\cancel{(k-1)!}} \cdot \frac{\cancel{(k-1)!}}{\cancel{n!}} \cdot \left[\frac{1}{(n+1)} \cdot u^{n+1} \Big|_0^1 \right]$$

$$= \frac{1}{(n+1)(n-k+1)!}$$

HW Problem 2)

Let $Y = F(X)$ where X is a RV w/ distribution F.

Then $F_Y(x) = \mathbb{P}(Y \leq x) = \mathbb{P}(F(X) \leq x)$

$= \mathbb{P}(X \leq F^{-1}(x)) = F(F^{-1}(x)) = x \leftarrow$ This corresponds the the CDF of a Uniform RV