Complex Numbers - Practice Exam 6

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Question 1. Solve the equation
$$4z^2 + 17 = 4z.$$

Question 2.

    a. Determine the 3 roots of $\sqrt{2} + i\sqrt{6}$.

    b. How many degrees apart are two consecutive roots of $\sqrt{2} + i\sqrt{6}$ on the unit circle?

Question 3. Let $p(z) = z^3 + i\lambda z^2 + \mu z - 4i$ and suppose that $z = 3$ and $z = 2i$ are roots of p. Determine the values of λ and μ.

Question 4. (Dr. Lloyd Gunatilake).

    a. Show that
$$\frac{e^{i\vartheta} - 1}{e^{i\vartheta} + 1} = i \tan \frac{\vartheta}{2}.$$

    b. Find all rots of the equation $z^5 - 1 = 0$ in polar form.

    c. Hence, or otherwise, show that solutions to the equation
$$(1 + iz)^5 - (1 - iz)^5 = 0$$

    are of the form $z = \tan \phi$, where $-\pi < \phi \le \pi$.