

Complex Numbers - Practice Exam 3

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Question 1. Let $z_1 = 1 + 3i$ and $z_2 = i$. Determine the value of

$$\operatorname{Re} \left(\frac{z_1 - \bar{z}_2}{\bar{z}_1 + z_2} \right).$$

Question 2. (Dr. Lloyd Gunatilake). Express

$$z = \frac{\sqrt{3} - i}{-\sqrt{2} + i\sqrt{6}}$$

in polar form.

Question 3. Let $p(z) = z^4 - (7 - i)z^3 + (16 - 3i)z^2 - (28 - 4i)z + (48 - 12i)$ be a polynomial.

- Show that $z = 4 - i$ is a root of $p(z)$.
- Determine whether $z = 4 + i$ is a root of $p(z)$.
- Does the result in part (b) contradict the conjugate root theorem?
- Show that $z = 3$ is a root of $p(z)$.
- Determine the remaining roots of $p(z)$ and write $p(z)$ as a product of linear factors.

Question 4. Let $z_1 = 1 + i\sqrt{3}$ and $z_2 = \sqrt{2} + i\sqrt{2}$.

- Write z_1 and z_2 in polar form.
- Write z_1/z_2 in polar form.
- Write z_1/z_2 in cartesian form.
- Hence, find the exact value of

$$\tan \left(\frac{\pi}{12} \right).$$