Complex Numbers - Practice Exam 5

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Question 1. Let $z_1 = 4 - i$ and $z_2 = \sqrt{6} + i\sqrt{5}$. Calculate the modulus of w, where

$$w = \frac{3(z_1 + z_2)}{iz_2 + \overline{z_1}}.$$

Question 2. Let $p(z) = z^3 + \lambda z^2 + \mu z + \xi$, where $\lambda, \mu, \xi \in \mathbb{R}$ are nonzero. Suppose z = 3 and z = 1 + 2i are roots of p(z). Determine the values of λ, μ and ξ .

Question 3. (Dr. Lloyd Gunatilake). Determine the value of $k \in \mathbb{R}$ such that p(ki) = 0, where $p(z) = z^3 + 7z - 6i = 0$.

Question 4. Suppose that

$$z^{8} - 2z^{4}\cos\vartheta + 1 = (z^{4} - e^{i\vartheta})(z^{4} - e^{-i\vartheta}).$$

Determine the solutions of

$$z^8 - z^4 = -1$$

in polar form.