

## Elementary Function Theory – Lecture 3

In this lecture we focus exclusively on cubic functions and quartic functions.

**Q1.** Expand the expression  $(x - 3)(x + 1)(x - 5)$ .

**Q2.** Expand the expression  $(x - \frac{1}{3})(x + \frac{1}{4})(x - \frac{1}{8})$ .

**Q3.** Expand the expression  $(x + \sqrt{3})(x - \sqrt{2})(x - 1)$ .

**Q4.** Factorise the following expressions.

a.  $x^3 + 3x^2 + 3x + 1$ .

b.  $x^3 - 6x^2 + 5x + 12$ .

c.  $2x^3 + 8x^2 - 38x + 28$ .

**Q5.** Solve the following equations.

a.  $x^4 - 2x^2 - 3 = 0$ .

b.  $x^4 - 5x^2 - 6 = 0$ .

c.  $x^4 - x^2 - 2 = 0$ .

**Q6.** Sketch the following curves, stating all relevant features.

a.  $f(x) = (x + 1)(x - 2)(x - 3)$ .

b.  $f(x) = (x - 3)(x - 4)(x - 2).$

c.  $f(x) = (x + 1)(x - 8)(x - 1).$



d.  $f(x) = (x - 3)(x + 5)(x + 9)$ .

**Q7.** Sketch the following curves, stating all relevant features.

a.  $f(x) = (x - 1)^2(x - 4)$ .

b.  $f(x) = (x - 3)(x + 1)(x - 6)^2$ .

c.  $f(x) = (x - 3)(x - 6)$ .

d.  $f(x) = x(x - 4)(x - 4)(x + 6)$ .

**Q8.** Sketch the following curves, stating all relevant features.

a.  $f(x) = (x - 3)^3 + 1$ .

b.  $f(x) = (x - 4)^3 - 5$ .

c.  $f(x) = (x + 1)^3 + 2.$

d.  $f(x) = x^3 + 2.$

**Q9.** State the transformations necessary to map  $f(x) = x^3$  to  $\tilde{f}(x) = \frac{1}{2}(x+1)^3 - 4$ .

**Q10.** State the transformations necessary to map  $f(x) = x^3$  to  $\tilde{f}(x) = (1-2x)^3 + 16$ .



**Q11.** State the transformations necessary to map  $f(x) = x^3$  to

$$\tilde{f}(x) = -\frac{1}{\sqrt{3}}(6x - 9)^3 + \frac{1}{\pi}.$$