Complex Numbers - Practice Exam 8

Kyle Broder – ANU – MSI – 2017

Question 1. Let $z = \sin\vartheta + i\cos\vartheta$. Show that

$$|z+1| = \sqrt{2+2\sin\vartheta}.$$

Question 2. Determine the roots, in polar form, of

$$z^2 - 2z\sin\vartheta + 1 = 0.$$

Question 3. Show that $|z-2i| = 4 - |z|$ may be written as

$$\frac{x^2}{3} + \frac{(y-1)^2}{4} = 1.$$

Question 4. Determine, with justification, whether it is possible to have a degree three polynomial $p(z)$, with real coefficients, such that $z = 2, z = 3+i$ and $z = -3-i$ are the roots of $p(z)$.

Question 5. (Dr. Lloyd Gunatilake).

a. Solve the equation $z^5 - 1 = 0$.

b. Hence, show that the roots of the equation $(z+1)^5 = z^5$ are given by

$$z_k = -\frac{1}{2} - \frac{1}{2}i\cot\left(\frac{k\pi}{5}\right),$$

where $k = 1, 2, 3, 4$.