Introduction to Matrices

Kyle Broder – ANU – MSI – 2017

A matrix is a rectangular array of numbers which forms the central object of mathematics, in particular, finite dimensional linear algebra.

Example 1. Let

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 4 & 3 \\ 1 & 7 \end{pmatrix}.$$

Compute $A + B$.

Proof. Addition of matrices is done component-wise. Therefore, we simply observe that

$$\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ 1 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ 3 & 10 \end{pmatrix}.$$

□

Exercise 1. Let

$$A = \begin{pmatrix} 1 & 4 \\ 0 & 10 \end{pmatrix} \text{ and } B = \begin{pmatrix} 13 & 1 \\ 0 & 0 \end{pmatrix}.$$

a. Compute $A + B$.
b. Compute $A - B$.
c. Compute $B - A$.

Exercise 2. Let

$$A = \begin{pmatrix} 1 & \frac{3}{2} \\ 1 & 5 \end{pmatrix} \text{ and } B = \begin{pmatrix} \frac{1}{3} & 1 \\ 6 & 17 \end{pmatrix}.$$

a. Compute $A + B$.
b. Compute $A - B$.
c. Compute $B - A$.

Exercise 3. Let

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 3 \\ 5 & 1 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 4 & 3 & 8 \\ 6 & 2 & 1 \\ 5 & 10 & 12 \end{pmatrix}.$$

a. Compute $A + B$.
b. Compute $A - B$.
c. Compute $B - A$.

Exercise 3. Let
$$A = \begin{pmatrix} \frac{3}{2} & \frac{1}{4} & \frac{3}{7} \\ \frac{2}{5} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{3}{7} \end{pmatrix} \text{ and } B = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{7} & \frac{1}{2} \\ 4 & -\frac{2}{5} & 1 \\ 3 & 2 & 0 \end{pmatrix}.$$

a. Compute $A + B$.
b. Compute $A - B$.
c. Compute $B - A$.

Example 2. Let
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 3 \\ 2 & 5 \end{pmatrix}.$$
Compute $A \cdot B$.

Proof. Matrix multiplication is a little more interesting.
$$\begin{aligned} \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} \cdot \begin{pmatrix} 0 & 3 \\ 2 & 5 \end{pmatrix} &= \begin{pmatrix} 1 \cdot 0 + 2 \cdot 2 & 1 \cdot 3 + 2 \cdot 5 \\ 3 \cdot 0 + 6 \cdot 2 & 3 \cdot 3 + 6 \cdot 5 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 13 \\ 12 & 39 \end{pmatrix}. \end{aligned}$$
□

Exercise 4. In Example 2 above, compute $B \cdot A$.

Exercise 5. Let
$$A = \begin{pmatrix} 3 & 0 \\ 5 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 0 \\ 4 & 1 \end{pmatrix}.$$

a. Compute $A \cdot B$.
b. Compute $B \cdot A$.

Exercise 6. Let
$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 5 \\ 12 & 1 \end{pmatrix}.$$

a. Compute $A \cdot B$.
b. Compute $B \cdot A$.

Exercise 7. Let
$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ \frac{1}{2} & 3 & 6 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}.$$

a. Compute $A \cdot B$.
b. Compute $B \cdot A$.
c. Compute A^2.
d. Compute B^2.

Exercise 8. Let
$$A = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{2}{3} \\ 1 & \frac{2}{5} & \frac{2}{7} \\ 5 & 0 & \frac{4}{7} \end{pmatrix} \text{ and } B = \begin{pmatrix} \frac{2}{5} & 1 & -\frac{3}{5} \\ -1 & -5 & -4 \\ -2 & -7 & 1 \end{pmatrix}.$$

a. Compute $A \cdot B$.
b. Compute $B \cdot A$.
c. Compute A^2.
d. Compute B^2.

Example 3. Calculate the determinant of the matrix
$$A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}.$$

Proof. The determinant of an arbitrary matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is given by $\det(A) = ad - bc$. In this particular case, we see that the determinant is given by $\det(A) = 1 \cdot 1 - 2 \cdot 3 = 1 - 6 = -5$. ☐

Exercise 9. Calculate the determinant of the following matrices

a. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
b. $\begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}$.
c. $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$.
d. $\begin{pmatrix} 4 & 1 \\ 0 & -1 \end{pmatrix}$.