Calculus Practice Exam 6

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In this practice exam we explore a more flexible method of differentiation that allows us to differentiate functions of a real variable x without writing our function explicitly in terms of such a variable. Such equations of this form are referred to as implcit equations. Examples of such equations include

$$x^2 + y^2 = 4, \quad x^3 + xy^2 = y\sqrt{x} + \frac{x}{y}, \quad \text{and} \quad x^3y + y^3x = \frac{1}{\sqrt{x+y}}.$$

1. Differentiate the following functions using the method of implicit differentiation.

a. $x + y = 2x + y^2$.
b. $2x + 4y = 4x^2 + 5y^3$.
c. $6y - \frac{1}{x} = \sqrt{y} + 2x$.
d. $y + 2x = -1$.
e. $x^2 + y^2 = 9$.
f. $x^2 - 4y^2 = 1$.
g. $x^3 + \frac{3}{y^2} + 3 = 4x + y^{\frac{3}{7}}$.

2. Differentiate the following functions using the method of implicit differentiation.

a. $xy = 1$.
b. $x + y\sqrt{x} = 3y^5$.
c. $5x + y^2 = xy^3$.
d. $4xy + 2\sqrt{y} = xy$.
e. $\frac{1}{x+y} = 2x$.
f. $\frac{1}{\sqrt{y+1}} - 4x = 2y^2$.

3. Find $\frac{dy}{dx}$, where

$$y^2 - x^3 + \frac{5}{y} = 2x - y.$$

4. Find the value(s) of $x \in \mathbb{R}$ such that $\frac{dy}{dx} = -1$, where

$$\frac{1}{4}(x-1)^2 + \frac{1}{16}(y+3)^2 = 1.$$

5. Suppose

$$\frac{x^2 + y}{x - y} = \frac{1}{y}\ln(x).$$

Find the value of $\frac{dy}{dx}$ at $(1, -1)$.