LINEAR ALGEBRA – LECTURE 3

KYLE BRODER – ANU MSI & PKU BICMR – DECEMBER 2019

Abstract. The purpose of this lecture is to discuss matrices, their properties, and particular examples.

Consider the following system of linear equations:

$$\begin{aligned} 2x + 3y + z &= 10 \\ x + 4y + 3z &= 4 \\ 10y + z &= 3. \end{aligned}$$

We would like to understand the following questions:

(i) (Existence). Does this system of equations admit a solution?
(ii) (Uniqueness). Does there exist only one solution?
(iii) What is the geometric picture underlying the solution set?
(iv) Is there a convenient way to express the solutions of this system?

Matrices provide a convenient tool to study such systems of linear equations and answer the above questions. To convert the above system of equations into an equation of matrices, let

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 4 & 3 \\ 0 & 10 & 1 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 10 \\ 4 \\ 3 \end{pmatrix}.$$

Then we see that the system of equations is expressed by the single matrix equation

$$A\mathbf{v} = \mathbf{b}.$$

Before attempting to answer questions (i)–(iv) above, we begin some elementary computations to familiarise ourselves with matrices.

0.1. **Matrices: Elementary Properties.** To begin with, we will start with $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 \\ 1 & 4 \end{pmatrix}$.