

Calculus Practice Exam 1

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Question 1. Recall that a function f is said to be differentiable at $x \in \mathbb{R}$ if the limit

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

exists and is finite. Determine whether the following functions are differentiable and calculate their derivatives.

- a. $f(x) = 4x - 5$.
- b. $f(x) = 5 - x^2$.
- c. $f(x) = 1 + 2x + 3x^3$.
- d. $f(x) = x^4$.
- e. $f(x) = 2|x| + 1$.
- f. $f(x) = x^4 + x^2 + 1$.

Question 2. Determine where the following functions are not differentiable. [Hint: Recall that for a function to be differentiable, the left derivative must equal the right derivative.]

- a. $f(x) = |x - 3| + 1$.
- b. $f(x) = 2|x + 1| + 3$.
- c. $f(x) = \frac{1}{3}|2 - 4x| + \frac{2}{5}$.
- d. $f(x) = |x - 1|^2 + 1$.

Question 3. In this exercise, we assume that all polynomials are differentiable on the entire real line \mathbb{R} . One may differentiate the following functions by using the formula given in Question 1, or we may simply use the rule

$$f(x) = x^n \implies f'(x) = nx^{n-1}, \quad n \in \mathbb{N},$$

which was proved in the notes. Differentiate the following functions

- a. $f(x) = x^2 - 5x + 6$.
- b. $f(x) = 2x + 1$.
- c. $f(x) = 4x - 3x^2 + 1$.
- d. $f(x) = 2x^3 + 5x^5 + 6x^7$.
- e. $f(x) = \frac{1}{3}x^3 + 4x$.

Question 4. The formula

$$f(x) = x^n \implies f'(x) = nx^{n-1}$$

that was given in the previous exercise may be extended to fractional powers. That is,

$$f(x) = x^{\frac{1}{2}} \implies f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}.$$

Use this extension of the formula to differentiate the following functions.

- a. $f(x) = \sqrt[3]{x}$.
- b. $f(x) = \frac{1}{2}\sqrt{x} + 1$.

- c. $f(x) = 3x - 5\sqrt{x} + 4x^3$.
- d. $f(x) = \frac{2}{x}$.
- e. $f(x) = \frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x}$.
- f. $f(x) = \frac{1}{3\sqrt{x}}$.

Question 5. Consider the function

$$f(x) := \sum_{k=1}^n 4x^{\frac{4}{5}k - \frac{1}{3}}.$$

- a. Determine whether $f(x)$ is differentiable.
- b. Evaluate $f'(x)$ where $f(x)$ is differentiable.

Question 6. Consider the function

$$f(x) := \frac{1}{x^2 - 5x + 6}.$$

- a. Determine where $f(x)$ is continuous.
- b. Determine the values of A and B such that

$$f(x) = \frac{A}{x-3} + \frac{B}{x-2}.$$

- c. Determine where $f(x)$ is differentiable.
- d. Evaluate $f'(x)$.