

ADDITIONAL MATH1013 TUTORIAL PROBLEMS

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LINEAR ALGEBRA

Question 1. Consider the system of linear equations given by

$$\begin{aligned}x_1 - 2x_2 - x_3 + 3x_4 &= 0 \\ -2x_1 + 4x_2 + 5x_3 - 5x_4 &= 3 \\ 3x_1 - 6x_2 - 6x_3 + 8x_4 &= -3\end{aligned}$$

- Form the associated augmented matrix and by using row operations find its reduced row echelon form. Explicitly show the sequence of row operations used to reduce the matrix.
- Determine the number of solutions of the given system.
- Determine which columns are pivot columns.
- Determine the number of free-variables.
- Determine the associated picture, as a subspace of \mathbb{R}^4 , of the solution set. That is, determine whether the solution set is a point, a line, a plane or a hyperplane.

Question 2. Determine which of the following are true or false. Provide justification for each answer.

- A system of linear equations can have exactly two solutions.
- If v_1, v_2 are vectors in \mathbb{R}^2 , then $v_1 + v_2 \in \mathbb{R}^4$.
- The zero vector $\mathbf{0}$ is contained in all vector spaces.
- \mathbb{R}^2 is a subset of \mathbb{R}^3 .
- A system of equations with one free-variable is a line.

Question 3.

- Let A be a 4×7 matrix. Determine the maximum number of leading 1's in its reduced row echelon form. Justify your answer.
- Let B be a 7×4 matrix. Determine the minimum number of rows of zeroes in any row echelon form of B . Justify your answer.

CALCULUS

Question 1.

- a. Provide an example of a function which is continuous but not differentiable.
- b. Is every differentiable function continuous?

Question 2. Determine whether $f(x) := \frac{1}{x}$ is a continuous function.

Question 3. Evaluate the limit

$$\lim_{x \rightarrow \infty} \frac{\cos^2 x}{x^2 + \sin^2 x}.$$

Question 4. Determine the natural domain of the function

$$f(x) := \frac{1}{\sqrt{e^x + \sin(x) + 4}}.$$

Question 5. Determine the natural domain of the function

$$f(x) = \frac{x}{x}.$$

Question 6. Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by

$$g(x) := e^{|x|}.$$

Determine the points where $g(x)$ is:

- a. continuous.
- b. differentiable.

Question 7. Determine whether

$$\cot x = \frac{1}{\tan x},$$

where $\cot x := \frac{\cos x}{\sin x}$.