**Calculus Practice Exam 3**

**Kyle Broder – ANU – MSI – 2017**

This practice exam aims to emphasise the differentiation techniques that allow us to differentiate exponential, logarithmic and trigonometric functions. The main technique at our disposal will be the chain rule. Recall that in the notes, we established that

$$\begin{aligned} f(x) = e^x \quad &\implies \quad f'(x) = e^x, \\ f(x) = \log_e x \quad &\implies \quad f'(x) = \frac{1}{x}, \quad x > 0, \\ f(x) = \sin x \quad &\implies \quad f'(x) = \cos x, \\ f(x) = \cos x \quad &\implies \quad f'(x) = -\sin x. \end{aligned}$$

**Question 1.** Using the chain rule, differentiate the following functions.

a. $f(x) = e^{3-x} + 1$.
b. $f(x) = e^{2x-1} + 4x + 12$.
c. $f(x) = e^{7x+2} + 3$.
d. $f(x) = \frac{1}{3}e^{4-x} + \frac{3}{7}$.
e. $f(x) = \sqrt{3}e^{4x+13} + 2$.

**Question 2.** Using the chain rule, differentiate the following functions.

a. $f(x) = e^{\sqrt{x+1}}$.
b. $f(x) = e^{x^2-5x+6}$.
c. $f(x) = 3e^{4x^2+\sqrt{x-3}+2} + 3x + 5$.
d. $f(x) = \frac{4}{3}e^{2(x-5)^3+1} + 4$.
e. $f(x) = \frac{\sqrt{2}}{5}e^{-x} + 4x^3$.

**Question 3.** Consider the function

$$f(x) = \frac{1}{\sqrt{3}\exp\left(\sqrt{5x + x^2 - 1}\right)}.$$

Evaluate $f'(x)$.

**Question 4.** Using the chain rule, differentiate the following functions.

a. $f(x) = \log_e(x-3) + 1$.
b. $f(x) = \frac{1}{3}\log_e(4-2x) + 4$.
c. $f(x) = \frac{3}{5}\log_e(x+7) + 2x + 3$.
d. $f(x) = \sqrt{3}\log_e(4x+5) + x^2$.
e. $f(x) = \log_e(x) + 1$.

**Question 5.** Consider the function

$$f(x) = \frac{2}{3\log_e(x)}.$$

Evaluate $f'(x)$.

**Question 6.** Let $|x|$ denote the absolute value function and consider the function

$$f(x) := \log_e |x|.$$

Evaluate $f'(x)$ and state the exact domain on which $f$ is differentiable.

**Question 7.** Using the chain rule, differentiate the following functions.

a. $f(x) = \sin(x + \pi) - 3.$
b. $f(x) = 2\cos(x - \pi/2) + 1.$
c. $f(x) = 3\sin(2(x + \pi)) + x.$
d. $f(x) = \frac{1}{4}\cos(-x) + 3x + 5 + \frac{1}{x^2}.$
e. $f(x) = \sin^2(x) + \cos^2(x).$
f. $f(x) = \cos^3(x) + \sin^2(x - \pi).$
g. $f(x) = 2\sin^3(x) + 5x - 3.$

**Question 8.** Consider the function

$$f(x) := \sec(x) := \frac{1}{\cos x}.$$

a. Determine the maximal domain on which $f(x)$ is defined.
b. Evaluate $f'(x)$.
c. Let $g(x)$ be the function given by applying the following transformations to $f(x)$:
   1. Dilate by factor 3 from the $x$-axis.
   2. Reflect about the $y$-axis.
   3. Translate by 2 units in the positive $x$-direction.

   Write the equation for $g(x)$.
d. Evaluate $g'(x)$.

**Question 9.** Consider the function

$$f(x) := \csc(x) := \frac{1}{\sin x}.$$

a. Determine the maximal domain of $f(x)$.
b. Evaluate $f'(x)$.

**Question 10.** Consider the functions

$$\sinh(x) := \frac{e^x - e^{-x}}{2} \text{ and } \cosh(x) := \frac{e^x + e^{-x}}{2}.$$

a. Show that the derivative of $\sinh(x)$ is $\cosh(x)$.
b. Determine the derivative of $\cosh(x)$.

**Question 11.** Using the chain rule, differentiate the following functions.

a. $f(x) = \sinh(2x + 4) + 4x + 6.$
b. $f(x) = 3\cosh(4x - 3) + 4x^2.$

**Question 12.** Consider the function

$$f(x) = \sinh(\sqrt{x}) = \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2}.$$

Evaluate $f'(x)$.

**Question 13.** Consider the function

$$f(x) = \sin(x)\cos(x).$$

Evaluate $f'(x)$.