Complex Numbers - Practice Exam 2

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Question 1. Suppose $z = \sqrt{2} - i\sqrt{6}$. Write z^{-3} in polar form.

Question 2. Let $z_1 = 1 + i\sqrt{3}$ and $z_2 = 2 - 3i$. Determine the imaginary part of

$$\frac{1+z_1}{1-z_2} \cdot \frac{1-|z_1|}{1+|z_2|}.$$

Question 3. Let $p(z) = z^3 - 3z^2 + 4z - 12$.

a. Verify that $z = 3$ is a root of p.
b. Determine the degree 2 polynomial $q(z)$ such that $p(z) = (z-3)q(z)$.
c. Determine the other roots of $p(z)$ by determining the roots of $q(z)$.

Question 4. (Dr. Lloyd Gunatilake). Let z be a complex number with modulus $r > 0$ and argument $\vartheta \in (-\pi, \pi]$ such that $z^2 = 3 - i$. Use DeMoivre's theorem to show that

$$\sin 2\vartheta = -\frac{1}{\sqrt{10}} \text{ and } \cos 2\vartheta = \frac{3}{\sqrt{10}}.$$