ADDITIONAL COMPOSITE FUNCTION EXERCISES

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Q1. If the function f is defined by $f(x) = \sqrt{x^2 - 9}$ and the function g is defined by $g(x) = x + 5$.

a. Determine the integers $c, d \in \mathbb{Z}$ such that $f(g(x)) = \sqrt{(x + c)(x + d)}$.
b. State the maximal domain for which $f(g(x))$ is defined.

Q2. Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{2}{\sqrt{x+1}}$.

a. State the domain and range of $f(x)$ and $g(x)$.
b. State the maximal domain of $g(x)$ such that $f(g(x))$ is well-defined.
c. State the rule for $f(g(x))$.
d. Determine the domain and range of $f(g(x))$.

Q3. Let $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x}$.

a. State the domain and range of $f(x)$ and $g(x)$.
b. State the maximal domain of $g(x)$ such that $f(g(x))$ is well-defined.
c. State the rule for $f(g(x))$.
d. Determine the domain and range of $f(g(x))$.

Q4. Let $f(x) = \frac{1}{\sqrt{x^2+1}}$ and $g(x) = 3x$.

a. State the domain and range of $f(x)$ and $g(x)$.
b. State the maximal domain of $g(x)$ such that $f(g(x))$ is well-defined.
c. State the rule for $f(g(x))$.
d. Determine the domain and range of $f(g(x))$.

Q5. Let $f(x) = \frac{2x+1}{x-3}$ and $g(x) = x + \sqrt{x - 3}$.

a. State the domain and range of $f(x)$ and $g(x)$.
b. State the maximal domain of $g(x)$ such that $f(g(x))$ is well-defined.
c. State the rule for $f(g(x))$.
d. Determine the domain and range of $f(g(x))$.

Q6. Let $f(x) = \frac{1}{1+\sqrt{x}}$ and $g(x) = 2x + 1$.

a. State the domain and range of $f(x)$ and $g(x)$.
b. State the maximal domain of $g(x)$ such that $f(g(x))$ is well-defined.
c. State the rule for $f(g(x))$.
d. Determine the domain and range of $f(g(x))$.

Q7. Let $f(x) = x^2 - 5$ and $g(x) = \frac{1}{(x-3)^2}$.

a. State the domain and range of $f(x)$ and $g(x)$.
b. State the maximal domain of $g(x)$ such that $f(g(x))$ is well-defined.
c. State the rule for $f(g(x))$.
d. Determine the domain and range of $f(g(x))$.

Q8. Let $f(x) = 2x - \frac{1}{x}$ and $g(x) = \frac{1}{x}$.

a. State the domain and range of $f(x)$ and $g(x)$.
b. State the maximal domain of $g(x)$ such that $f(g(x))$ is well-defined.
c. State the rule for $f(g(x))$.
d. Determine the domain and range of $f(g(x))$.

Q9. Let $f(x) = 4x - 2\sqrt{x^2+3}$ and $g(x) = \frac{4x+1}{3x-2}$.

a. State the domain and range of $f(x)$ and $g(x)$.
b. State the maximal domain of $g(x)$ such that $f(g(x))$ is well-defined.
c. State the rule for $f(g(x))$.
d. Determine the domain and range of $f(g(x))$.

Q10. Let $f(x) = 2x + |x+3|$ and $g(x) = 2x + \sqrt{x}$.

a. State the domain and range of $f(x)$ and $g(x)$.
b. State the maximal domain of $g(x)$ such that $f(g(x))$ is well-defined.
c. State the rule for $f(g(x))$.
d. Determine the domain and range of $f(g(x))$.

Q11. Let $f(x) = \frac{1}{|x|}$ and $g(x) = 2x + \frac{1}{5+x}$.

a. State the domain and range of $f(x)$ and $g(x)$.
b. State the maximal domain of $g(x)$ such that $f(g(x))$ is well-defined.
c. State the rule for $f(g(x))$.
d. Determine the domain and range of $f(g(x))$.

Q12. Determine the domain of the function

$$f(x) := \frac{1}{\sqrt{x^2 - 5x + 6}}.$$

Q13. Determine the domain of the function

$$f(x) := \frac{1}{x^2 + 2x + 1}.$$

Q14. Determine the domain of the function

$$f(x) := \frac{1}{\sqrt{x^2 - 5x - 6}}.$$

Q15. Determine the domain of the function

$$f(x) = \frac{2x + 1}{\sqrt{x^2 + 6x + 9}}.$$