Calculus Practice Exam 5

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In this practice exam, we attempt to explore an application of calculus. That is, finding maxima, minima and inflection points of functions $f : \mathbb{R} \to \mathbb{R}$.

Exercise 1. Determine the stationary points of the following functions.

a. $f(x) = x^2 - 5x + 6$.
b. $f(x) = x^2 + 2x + 1$.
c. $f(x) = x^3 + 2x + 1$.
d. $f(x) = x^4 + 1$.
e. $f(x) = x^3 - 2x^2 + x + 1$.
f. $f(x) = 4x^3 - 12x$.
g. $f(x) = 3x^3 - x^2 + 4x - 5$.
h. $f(x) = 2x^3 + 5x + \sqrt{3}$.

Exercise 2. Determine the stationary points of the following functions and use the second derivative test to determine the nature of stationary points.

a. $f(x) = x^3 - 10x^2 + 31x - 30$.
b. $f(x) = x^3 + 5x^2 - 8x - 12$.
c. $f(x) = x^3 - 4x^2 - 39x - 54$.
d. $f(x) = x^3 + 10x^2 - x - 10$.

Exercise 3. Determine the stationary points of the following functions.

a. $f(x) = \sin x$.
b. $f(x) = \cos x$.
c. $f(x) = 4\sin(3x - \pi)$.
d. $f(x) = \frac{2}{5}\cos\left(\frac{1}{2} - x\right)$.

Exercise 4. Determine the stationary points of the following functions.

a. $f(x) = x \sin x$.
b. $f(x) = x^2 \cos(-x)$.
c. $f(x) = e^x \sin x$.
d. $f(x) = e^{-x}\cos(x^2)$.

Exercise 5.

a. Let $f(x) := \tan x$. Determine where f is differentiable.
b. Evaluate $f'(x)$ at all points that f was determined to be differentiable in part (a).
c. Hence, or otherwise, determine the stationary points of $g(x) := \tan^2 x$.
d. Using the second derivative test, or otherwise, determine the nature of the stationary points that were determined in part (c).

Exercise 6. Investigate the consequence of assuming[1] that $f''(x) = 0$ implies that f has a points of inflection at x by considering the functions

$$f(x) = x^4, \quad \text{and} \quad f(x) = x^{\frac{1}{3}}.$$

[For more information on this, see Chapter 5.3 of *Introduction to Analysis*, Kyle Broder.]

[1] incorrectly

Exercise 7. Let $\psi : [0, 4\pi] \to \mathbb{R}$ be the function defined by $\psi : x \mapsto x - \sin x$. Determine whether ψ has any points of inflection. Are any of these points stationary?

Exercise 8. Let $\xi : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$\xi : x \longmapsto \frac{1}{x^2 + x + 1}.$$

a. Determine the coordinates of the inflection points of ξ.

b. Find the coordinates of the point of intersection of the tangents at the points of inflection.

Exercise 9. Show that for all $a, b, c \in \mathbb{R}$, the function $f(x) := ax^2 + bx + c$ has no points of inflection.

Exercise 10. Determine the points of inflection of the function

$$f(x) := e^{-x^2} \sin\left(\frac{1}{x}\right).$$

⋆ Can you determine the limit: $\lim_{x \to 0} f(x)$?

Exericse 11. Determine the nature of the stationary points of

$$f(x) := x^2 \log_e(\log_e(x)) + 9 \log\left(\frac{3}{2}\right).$$

Exercise 12. The radius $r > 0$ and height $h > 0$ of a solid circular cylinder $\mathscr{C}$ vary in such a way that the volume of the cylinder is always 250π.

a. Show that the total surface area $\mathscr{A}$ of the cylinder is given by

$$\mathscr{A} = 2\pi r^2 + \frac{500\pi}{r}.$$

b. What is the minimum surface area of $\mathscr{C}$?