**Complex Numbers - Practice Exam 9**

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**Question 1.** Let $p(z) = z^4 - 2z^3 + 7z^2 - 4z + 10$ be a polynomial. Suppose that $\lambda i$ is a root of $p(z)$, where $\lambda \in \mathbb{R}$. Determine the value of $\lambda$.

**Question 2.**

a. Determine the cube roots of unity. That is, solve the equation $z^3 - 1 = 0$.

b. Let $\zeta_1, \zeta_2$ and $\zeta_3$ denote the cube roots of unity obtained in part (a). Show that the cube roots of unity satisfy the following relations

i. $\zeta_2 = \overline{\zeta_3} = \zeta_3^2$.

ii. $\zeta_2 + \zeta_3 = -\zeta_1$.

iii. $\zeta_2\zeta_3 = \zeta_1$.

**Question 3.** Let $n \in \mathbb{N}$ be a natural number. Suppose that

$$\frac{(z+i)^n}{z^n} = 1.$$

a. Use DeMoivre's theorem to show that

$$z = \frac{i}{\exp\left(\frac{2k\pi}{n}\right) - 1},$$

where $1 \leq k \leq n-1$.

b. Hence, show that

$$z = \frac{1}{2}\left(\cot\left(\frac{k\pi}{n}\right) - i\right).$$

**Question 4.** Let $z = \sqrt{2} + i\sqrt{2}$ and $w = \sqrt{3} + i$.

a. Express $z$ and $w$ in polar form.

b. Express $z/w$ in cartesian form.

c. Express $z/w$ in polar form.

d. Determine the exact value of

$$\cot\left(\frac{5\pi}{12}\right).$$