Calculus Exam 2 (Preparation) (Extended)

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The contents of this examination require an understanding of the elementary calculus material that was covered in the calculus practice exams 1-5. An understanding of graphing techniques and function transformations may also be required.

There are no permitted materials for this test. That is, you are not permitted any cheat notes, calculators or resources other than a pen/pencil, eraser, sharpener, ruler and water bottle.

There is to be no collaboration on this examination and any attempts of communication will result in a nullified score. You are permitted 10 minutes of reading time and 105 minutes of writing time. There is a total of 100 available marks. It is recommended that you use the reading time to ask the invigilator about any issues regarding the format of the test, the problems or other issues. No hints will be given. Best of luck!

Name: _____

Grade: _____/100

Question 1. Evaluate the derivatives of the following functions.

a. $f(x) = \sin 2x$.
b. $f(x) = \sec x$.
c. $f(x) = \cos^2 x$.
d. $f(x) = \csc(3x)$.
e. $f(x) = \sec x \tan x + \cos^2(x + \pi)$.
f. $f(x) = \tan^4 x + \sec^3 x + \frac{3}{5} \cot x$.

Question 2. Determine the domain on which the following functions are differentiable and evaluate their derivatives.

a. $f(x) = \log_e(x)$.
b. $f(x) = \exp(x)$.
c. $f(x) = \sin x$.
d. $f(x) = |x|$.
e. $f(x) = \tan^3(x)$.
f. $f(x) = \log_e |x|$.
g. $f(x) = 2\exp(-x^2) + \cos x$.
h. $f(x) = \log_e |x| + \cot(x)$.

Question 3. Evaluate the derivatives of the following functions.

a. $f(x) = \sec^2 x + (x^2 + 1)e^{-x}$.
b. $f(x) = \log_e(x)\sqrt{x^2 + 5}$.
c. $f(x) = \frac{1}{x^3+x}$.
d. $f(x) = e^{-\sqrt{x}} + x\csc(x^3)$.
e. $f(x) = x^2 + x^3 + \cot(x)$.
f. $f(x) = \frac{\sec^2(x)}{x^2-5x+6}$.

Question 4. Determine the stationary point(s), and their nature for the following functions.

a. $f(x) = xe^{-x}$.
b. $f(x) = e^{-x^2}$.
c. $f(x) = x^4 + 2x + 1$.
d. $f(x) = x^{\frac{1}{5}}$.
e. $f(x) = x^{\frac{2}{5}}$.
f. $f(x) = e^x$.
g. $f(x) = x^2 \log_e(x)$.

Question 5. Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable on some set $\Omega_1 \subseteq \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ is differentiable on some set $\Omega_2 \subseteq \mathbb{R}$.

a. Determine the domain on which $f + g$ and $f - g$ are differentiable.
b. Determine the domain on which $f \cdot g$ is differentiable.
c. Determine the domain on which f/g is differentiable.
d. Determine the domain on which $\sqrt{f} \cdot g$ is differentiable.
e. Determine the domain on which $|f|$ is differentiable.
f. Let $f(x) = \exp(x)$ and $g(x) = \sqrt{x}$. On what domain is $h(x) = f(x) \cdot g(x)$ differentiable?
g. Let $f(x) = |x|$ and $g(x) = \log_e(-x)$. On which domain in $f(x) + g(x)$ differentiable?