Calculus Practice Exam 2

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This practice exam aims to emphasise the chain rule, which was proved in the notes. The chain rule allows us to differentiate composite functions. For your convenience, we begin with an illustrative example.

Example 1. Let $h(x) = \sqrt{x^2 - 5x + 6}$. Notice that h is given by the composition of $f(x)$ and $g(x)$, where $f(x) = x^2 - 5x + 6$ and $g(x) = \sqrt{x}$. The chain rule tells us that

$$h'(x) = f'(x)g'(f(x)).$$

Observe that

$$f(x) = x^2 - 5x + 6 \implies f'(x) = 2x - 5$$

and

$$g(x) = \sqrt{x} \implies g'(x) = \frac{1}{2\sqrt{x}}.$$

Hence we see that

$$h'(x) = (2x - 5) \cdot \frac{1}{2\sqrt{x^2 - 5x + 6}} = \frac{2x - 5}{2\sqrt{x^2 - 5x + 6}}.$$

Question 1. Using the chain rule, evaluate the following derivatives.

a. $f(x) = 2\sqrt{x - 3} + 1$.
b. $f(x) = 3\sqrt{x + 5} + 2x$.
c. $f(x) = \sqrt{2x + 3} + 4x^3$.
d. $f(x) = \frac{1}{2}\sqrt{3x - 5} + 2x + 5$.
e. $f(x) = \sqrt{4x - 6} + \sqrt{x} + 3$.
f. $f(x) = \sqrt{3 - 7x} + \sqrt{4x - 3} + \sqrt{x + 10}$.

Question 2. Using the chain rule, evaluate the following derivatives.

a. $f(x) = \frac{3}{x-5} + 1$.
b. $f(x) = \frac{1}{x+4} + \frac{3}{x-5}$.
c. $f(x) = \frac{5}{x+1} + \frac{2}{x-1}$.
d. $f(x) = \frac{3}{5x+4} + \frac{3}{4-5x}$.
e. $f(x) = \frac{7}{2x+1} - \frac{3}{5x-1}$.

Question 3. Consider the function

$$f(x) = \frac{1}{x^2 - 8x + 15}.$$

a. Determine the values of A and B such that

$$f(x) = \frac{A}{x - 3} + \frac{B}{x - 5}.$$

b. Hence, differentiate $f(x)$.

Question 4. Using the chain rule, evaluate the following derivatives.

a. $f(x) = \sqrt{4x^3 + 2x + 1}$.
b. $f(x) = 2\sqrt{3 - x - x^2 - x^3 - x^4}$.
c. $f(x) = 5\sqrt{2x + 5x^3 + 6x^7}$.
d. $h(x) = \sqrt{f(x) + 1} + g(x)$, where $f(x)$ and $g(x)$ are differentiable on the entire real line.

Question 5. Let

$$f(x) = \sqrt{1 + \sqrt{3x + 1}}.$$

Determine $f'(x)$.

Question 6. Let

$$f(x) = \frac{1}{\sqrt{1 + \sqrt{1 + \sqrt{x}}}}.$$

Determine $f'(x)$.

Question 7. Using the chain rule, evaluate the following derivatives.

a. $f(x) = 2(x - 3)^4$.
b. $f(x) = 4(x^2 - 5x + 6)^{13} + 2x + 5$.
c. $f(x) = (x^2 + 2x + 1)^6 - 3x - 13$.