

## ADDITIONAL MATH1013 TUTORIAL PROBLEMS - WEEK 5

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**Question 1.** Let

$$f(x) = \frac{1}{3}(x+1) - x^{\frac{1}{3}}, \quad -8 \leq x \leq 8.$$

- (a) Find all local and global maximum and minimum points, and corresponding values, of  $f$ . Use the convention that local maxima and minima may occur at the endpoints. Justify your answers by applying an appropriate test at each such point.
- (b) Where is  $f$  increasing and where is it decreasing?
- (c) Where is  $f$  concave up and where is it concave down? Find any inflection points of  $f$ .
- (d) Hence, or otherwise, sketch the graph of  $f$ .

**Question 2.**

- (a) Find  $\lim_{x \rightarrow \infty} \left( \frac{x^2}{x+1} - \frac{x^2}{x-1} \right)$ .
- (b) Find  $\lim_{x \rightarrow 0} \frac{1}{x} \sin(\sin x)$ .

**Question 3.**

- (a) Suppose that  $g$  and  $h$  are differentiable on the whole real line. Find the derivative of

$$f(x) = g(x + x^2 h(x^3)).$$

- (b) Prove that the derivative of  $f(x) = x^{1/3}$  is  $f'(x) = \frac{1}{3}x^{-2/3}$  for  $x \neq 0$ .

Hint: Consider the function  $g(x) = (x^{1/3})^3$  and use the chain rule.

**Question 4.** Show that the function  $f(x) = \cos^2 x + x^3 + 2x$  takes the value 0 exactly once.

**Question 5.** Let

$$f(x) = x^{\frac{1}{3}}(x-4).$$

- (a) Find all local and global maximum and minimum points, and corresponding values, of  $f$ . Justify your answers by applying an appropriate test at each such point.
- (b) Where is  $f$  increasing and where is it decreasing?
- (c) Where is  $f$  concave up and where is it concave down? Find any inflection points of  $f$ .

- (d) Sketch the graph of  $f$ .