Trigonometry Practice Exam 2

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Question 1. Let $f : [0, 2\pi] \to \mathbb{R}$ be the function defined by

$$f(x) := \cos^2\left(2x - \frac{\pi}{2}\right) - 1.$$

a. Determine the y-intercept of the equation $y = f(x)$.
b. Determine the x-intercepts of the equation $y = f(x)$.
c. State the amplitude and period of $f(x)$.
d. By determining the exact coordinates of the maxima and minima of $f(x)$, sketch the curve $f(x)$ on its maximal domain.
e. State the transformations necessary to map $g(x) = \cos^2(x)$ to $f(x)$.

Question 2. (Dr. Lloyd Gunatilake). The function

$$f(x) = \lambda + \mu \cos\left(\frac{\pi}{3} + \zeta x\right)$$

has a maximum of 7 and a minimum of -1. It is also known that $f(x)$ has a period of 4π. Determine the values of λ, μ and ζ.

Question 3. Consider the following experiment on a simple harmonic oscillator. A spring-block system is put into simple harmonic motion in two experiments. In the first experiment, Thusha pulls the block back a distance of $\frac{1}{10}$th of a metre from its equillibrium position. In the second experiment, he pulls the block back a distance of $\frac{2}{5}$ths of a metre from its equillibrium position.

Thusha knows that if we let $x(t)$ denote the position of the block, then the motion of the block is given by the equation

$$x(t) = \alpha \cos(\omega t + \phi),$$

where α, ω and ϕ are constants. Using some calculus, Thusha was also able to show that the velocity was given by

$$v(t) = -\alpha\omega \sin(\omega t + \phi)$$

and similarly, the acceleration was given by

$$a(t) = -\alpha\omega^2 \cos(\omega t + \phi).$$

a. For those who have done calculus, verify that Thusha's calculations of velocity and acceleration were indeed correct.
b. Determine the physical significance of the quantities α, ω and ϕ.
c. Let α_1 and α_2 denote the values of α in the first experiment and second experiment, respectively. Thusha claims that $\alpha_1 < \alpha_2$. Is Thusha correct?
d. In a similar manner to the previous question, let ω_1 and ω_2 denote the values of ω in the first and second experiments, respectively. In this case, Thusha claims that $\omega_1 = \omega_2$. Is Thusha correct here?

e. In his further studies, Thusha determines that the potential energy of a simple harmonic oscillator is given by

$$U(t) = \frac{1}{2}k\alpha^2 \cos^2(\omega t + \phi),$$

where k, α, ω and ϕ are constants. Thusha wants to determine how $U(t)$ is affected by changing the value of the initial displacement. Unfortunately, he has not studied hard enough and can not figure it out. Help Thusha out by determining whether $U(t)$ depends on the initial displacement away from equillibrium.