Calculus Practice Exam 4

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Recall that in the notes we proved the following result: if f and g are both differentiable, then the derivative of the product $f \cdot g$ is given by

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) \quad = \quad f(x) \cdot g'(x) + f'(x) \cdot g(x).$$

This is known as the *product rule*, or more formally, *Leibniz rule*.

This practice exam aims at illustrating the use of this rule.

Question 1. Using the product rule, differentiate the following functions.

a. $f(x) = xe^x$.
b. $f(x) = (x-1)e^{3x}$.
c. $f(x) = x^2 e^{4-6x}$.
d. $f(x) = \sqrt{x}e^{\sqrt{x}}$.

Question 2. Using the product rule, differentiate the following functions.

a. $f(x) = x \sin x$.
b. $f(x) = x \cos x$.
c. $f(x) = 3x \cos(2x)$.
d. $f(x) = (x^2 - 5x + 6)\sin(x)$.

Question 3. Using the product rule, differentiate the following functions.

a. $f(x) = e^x \log_e(x)$.
b. $f(x) = e^x \sin x$.
c. $f(x) = \sqrt{x}\log_e(x)$.
d. $f(x) = 3x^{\frac{1}{3}} \log_e(x-5) + 1$.
e. $f(x) = \sin(x)\cos(3x)$.
f. $f(x) = \sin(x+\pi)\log_e(x+\pi)$.

Question 4. Using the product rule, prove that

$$\frac{d}{dx}\tan(x) = \sec^2(x).$$

Question 5. Consider the function

$$f(x) = \cot(x) := \frac{1}{\tan(x)}.$$

Using a similar method to that considered in Question 4, determine the derivative of $f(x)$.

Question 6. Using the product rule, differentiate the following functions.

a.
$$f(x) = \frac{x-3}{x+2}.$$

b.
$$f(x) = \frac{3x+1}{4x+7}.$$

c.
$$f(x) = \frac{4x+6}{3-x}.$$

d.
$$f(x) = \frac{3-6x}{5-x}.$$

e.
$$f(x) = \frac{2}{x^2-5x+6}.$$

f.
$$f(x) = \frac{2x-6}{x^2+12x+1}.$$

g.
$$f(x) = \frac{x^2+2x+1}{x^2+4x+1}.$$

h.
$$f(x) = \frac{x+3}{x^2-9}.$$

i.
$$f(x) = \frac{4x^3+5x+1}{x^3+2x+1}.$$

Question 7. Using the product rule, differentiate the following functions.

a.
$$f(x) = \frac{\sqrt{x}+\sqrt{-x}}{2x+3}.$$

b.
$$f(x) = \frac{\sin x + \cos x}{\tan x}.$$

c.
$$f(x) = \frac{3x+\cos(x^2)}{2x-4}.$$

d.
$$f(x) = \frac{2e^{x-3}\cos x + \sin(\cos(x))}{e^x - e^{-x}}.$$