Calculus Exam 2 (Preparation)

Kyle Broder – ANU – MSI – 2017

The contents of this examination require an understanding of the calculus material that was covered in the calculus practice exams 3,4 and 5. An understanding of graphing techniques and function transformations may also be required.

There are no permitted materials for this test. That is, you are not permitted any cheat notes, calculators or resources other than a pen/pencil, eraser, sharpener, ruler and water bottle.

There is to be no collaboration on this examination and any attempts of communication will result in a nullified score. You are permitted 10 minutes of reading time and 105 minutes of writing time. There is a total of 100 available marks. It is recommended that you use the reading time to ask the invigilator about any issues regarding the format of the test, the problems or other issues. No hints will be given. Best of luck!

Name: _____

Grade: _____/100

Question 1. [10 marks]. Let $f : (0, \pi) \to \mathbb{R}$ be the function defined by $f(x) := \cot x$. Show that $f'(x) = -\csc^2(x)$, where $\csc x := \frac{1}{\sin x}$.

Question 2. [20 marks]. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \cos(x^3) \log_e(x + 1).$$

Explain why f is differentiable and evaluate $f'(x)$.

Question 3. [20 marks]. Consider the function $f : (-\pi/2, \pi/2) \to \mathbb{R}$ defined by

$$f(x) = \frac{\sec x}{4 + \sqrt{3x+1}}.$$

Evaluate $f'(0)$.

Question 4. [25 marks]. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = x^{\frac{1}{5}}.$$

Determine the points of inflection of $f(x)$.

Question 5. [25 marks]. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable $\forall x \in \mathbb{R}$.

a. Determine the maximal domain on which $\widetilde{f}(x) := 2\sqrt{f(x)+1}$ is differentiable.

b. Evaluate the derivative of $\widetilde{f}(x)$ at all points where $\widetilde{f}$ was determined to be differentiable in part (a).

c. $\star$ Let $g(x) := \log_e |f(x)| + \exp(f'(x))$. Is it necessarily true that g is differentiable? If not, are there any extra conditions you can put on f such that g is differentiable? Are there any conditions you can put on f such that g is differentiable for all $x \in \mathbb{R}$?