ADDITIONAL TRANSFORMATION EXERCISES

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All questions and solutions are written by Kyle Broder in 2018.
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NOTES

Let $y = f(x)$ be an arbitrary function defined on $\mathbb{R}$. Consider the expression

$$y = af(b(x - c)) + d, \qquad a, b, c, d \in \mathbb{R}.$$

For simplicity, we will assume that a, b, c and d are all positive. Then we see that

  † The number a corresponds to a *dilation by factor a from the x-axis.*
  † The number b corresponds to a *dilation by factor $1/b$ from the y-axis.*
  † The number c corresponds to a *translation by c units to the right.*
  † The number d corresponds to a *translation by d units up.*

If a is negative, then multiplication by a will not only induce a dilation from the x-axis, but also a reflection about the x-axis. Similarly, if b is negative, then we will have a reflection about the y-axis and dilation by factor $1/b$.

One must also perform the transformations in the order of dilations, reflections and transformations. (DRT).

Example 1. Describe the transformations required to map the function $f(x) = x^2$ to the function

$$\widetilde{f}(x) = -\frac{4}{3}\left(8x + \frac{7}{3}\right)^2 - \frac{8}{9}.$$

Proof. We simply observe that

  † Dilate by factor $\frac{4}{3}$ from the x-axis.
  † Dilate by factor $1/8$ from the y-axis.
  † Reflected about the x-axis.
  † Translate by $\frac{7}{24}$ to the left. (this comes from factorising the inside of the parentheses).
  † Translate by $\frac{8}{9}$ down.

$\square$

Question 1. Let $f(x) = x^2$. Describe the transformations necessary to map $f(x)$ to

$$g(x) := (x - 3)^2 + 1.$$

Question 2. Let $f(x) = x^2$. Describe the transformations necessary to map $f(x)$ to

$$g(x) := -2(x + 1)^2 - 5.$$

Question 3. Let $f(x) = x^2$. Describe the transformations necessary to map $f(x)$ to

$$g(x) := 1 - 4(5x + 6)^2.$$

Question 4. Let $f(x) = x^3$. Describe the transformations necessary to map $f(x)$ to

$$g(x) := 2 + (x + 6)^3.$$

Question 5. Let $f(x) = x^5$. Describe the transformations necessary to map $f(x)$ to

$$h(x) := 1 - \frac{1}{4}(5x + 6)^5 - \frac{3}{17}.$$

Question 6. Let $f(x) = \frac{1}{x}$. Describe the transformations necessary to map $f(x)$ to

$$\varphi(x) := \frac{3}{7 + 4x} - 5.$$

Question 7. Let $f(x) = \frac{1}{x}$. Describe the transformations necessary to map $f(x)$ to

$$\psi(x) = 1 - \frac{3}{5 - 12x}.$$

Question 8. Let $f(x) = |x|$. Describe the transformations necessary to map $f(x)$ to
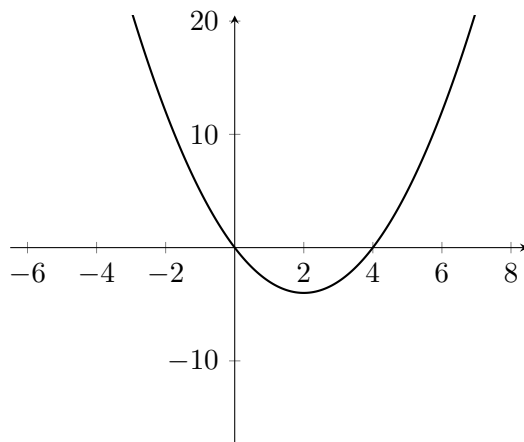
$$g(x) := 1 - |4x - 9|.$$

Question 9. Let $f(x) = 2x^2 + 4$. Describe the transformations necessary to map $f(x)$ to

$$g(x) := (x - 3)^2 + 1.$$

Question 10. Let $f(x) = 1 - 9x^3$. Describe the transformations necessary to map $f(x)$ to

$$g(x) = (3x + 5)^3 - \frac{1}{7}.$$

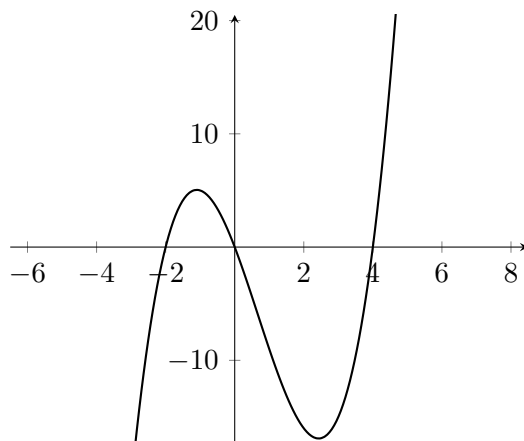**Question 11.** Consider the function f whose graph is given by

On the above pair of axes, sketch the graph of:

   a. $f(-x)$.
   b. $-f(x)$.

Can you describe what these transformations are?

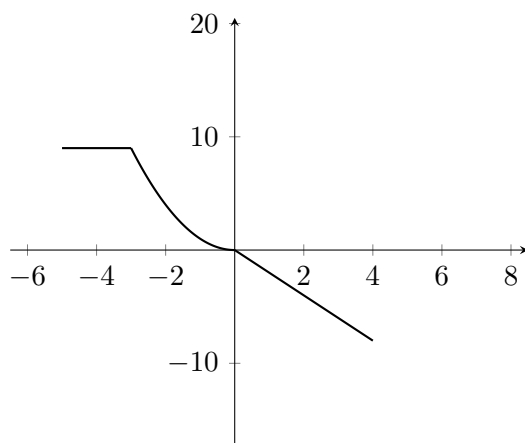**Question 12.** Consider the function f whose graph is given by

On the above pair of axes, sketch the graph of:

   a. $\frac{1}{2}f(-x)$.
   b. $1 - f(x)$.

Can you describe what these transformations are?

Question 13. Consider the function f whose graph is given by

On the above pair of axes, sketch the graph of:

    a. $|f(x)|$.

    b. $f(|x|)$.

Can you describe what these transformations are?