

ADDITIONAL MATH1013 TUTORIAL PROBLEMS - WEEK 2

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Based on the results of the tutorial quiz, students primarily struggled with composite functions and transformations of graphs. The problems in the calculus section are aimed at remedying this.

LINEAR ALGEBRA

Question 1. Define what it means for the vectors $\{v_1, \dots, v_n\}$ in \mathbb{R}^n to be linearly dependent.

Question 2. Suppose that $v_1, v_2, v_3, v_4 \in \mathbb{R}^3$, determine whether $\text{span}(v_1, v_2, v_3, v_4)$ is a plane.

Question 3. Find two linearly independent vectors on the plane

$$x + 2y - 3z - w = 0$$

in \mathbb{R}^4 . Then find three linearly independent vectors. Why can we not find four linearly independent vectors?

CALCULUS

Question 1. Let f be the function defined by

$$f(x) = \sqrt{\frac{7+x}{1-x}}.$$

Determine the domain of $f(x)$.

Question 2. Let f be the function defined by

$$f(x) = \frac{1}{1-|x|}.$$

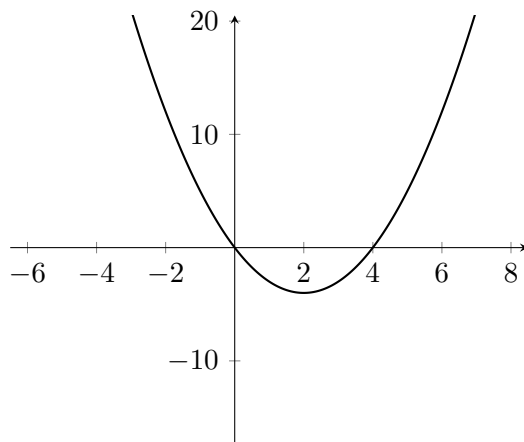
Determine the domain of $f(x)$.

Question 3. Let f be the function defined by

$$f(x) = \sqrt{\frac{1-4x}{x+2}} + \frac{1}{\sqrt{x^2+2x+1}}.$$

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Question 4. Consider the function f whose graph is given by

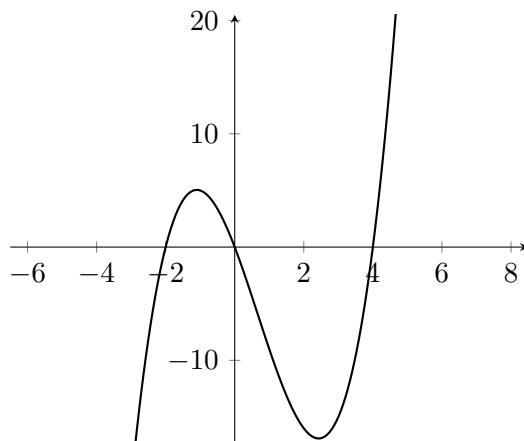


On the above pair of axes, sketch the graph of:

- a. $f(-x)$.
- b. $-f(x)$.

Can you describe what these transformations are?

Question 5. Consider the function f whose graph is given by

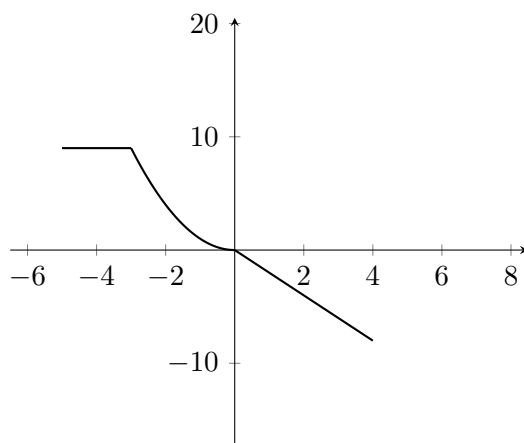


On the above pair of axes, sketch the graph of:

- a. $\frac{1}{2}f(-x)$.
- b. $1 - f(x)$.

Can you describe what these transformations are?

Question 6. Consider the function f whose graph is given by



On the above pair of axes, sketch the graph of:

- $|f(x)|$.
- $f(|x|)$.

Can you describe what these transformations are?

Question 7. Sketch the following curves, stating all relevant features.

- $f(x) = |x + 1| - |x|$.
- $f(x) = |x + 3| + 2|x - 1|$.
- $f(x) = |x + 4| - \frac{1}{2}|x - 5|$.
- $f(x) = |x^2 - 5x| - |x + 1|$.

Question 8. Consider the function $f : \mathbb{R} \setminus \{3\} \rightarrow \mathbb{R}$ defined by

$$f(x) = \frac{1}{3 - x}.$$

Show that f is differentiable on $\mathbb{R} \setminus \{3\}$ and compute $f'(x)$. [Hint: To show that f is differentiable, we need to show that $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ exists and is finite].