Complex Numbers - Practice Exam 7

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Question 1. Determine $(\sqrt{3} - i)^8$ in cartesian form.

Question 2. Let $z = x + iy$, where $x, y \in \mathbb{R}$. Determine the imaginary part of

$$\frac{1}{(z - \overline{z})^2} + \frac{z}{\overline{z}}.$$

Question 3. Find the cartesian equation of the set $\{z \in \mathbb{C} : |z - i| = 2\}$. What geometric significance does this equation have?

Question 4. (Dr. Lloyd Gunatilake).

a. Express $1 + i$ and $1 - i$ in polar form.

b. Let $n \in \mathbb{N}$ be a natural number. Show that

$$(1 + i)^n + (1 - i)^n = 2^{\frac{n+2}{2}} \cos\left(\frac{n\pi}{4}\right).$$

c. Hence, find a natural number $n < 10$ such that

$$(1 + i)^n + (1 - i)^n = 0.$$