ADDITIONAL LINEAR EQUATIONS EXERCISES

KYLE BRODER - ANU MSI 2018

All questions and solutions are written by Kyle Broder in 2018.
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Q1. Toby wishes to hire a car for 2 days. He has a total of $400 to spend. The Golden Car Rental company charges a flat fee of $45 per day plus $0.35 per kilometre.

a. Write a linear equation to represent this situation, where $\mathcal{C}$ is the total cost of hiring a car for 2 days and d is the number of kilometres travelled.
b. How far, to the near kilometre, can Toby travel on his budget?

Q2. The cost of making CDs is quoted at $3.15 each plus $3500 to set up the pressing plant for each new title.

a. Write a linear equation to present the cost $\mathcal{C}$, of producing N CDs for one new title.
b. The Alberto Company has a budgetof $45000 for its 4 recording artists. If it produces an equal amount of CDs for each artist, how many CDs can be produced?

Q3. Georgio wishes to purchase a second-hand iPhone, which costs $485. He has $60. He can save a further $30 per month, but the price of the iPhone rises by an average of $0.90 per month.

a. White an equation to represent the cost $\mathcal{C}_p$ of the iPhone after m months.
b. Write an equaton to represent the amount that Georgio has saved, $\mathcal{S}$, after m months.
c. How many months will it take him to save enough for the iPhone?

Q4. A bushwalker travels a certain distance on day 1 of a trip. On day 2, he walks one quarter of the distance of day 1. On day 3, he walks one sixth of the distance on day 1. On day 4, he walks one fifth of the distance of day 1. Finally, on day 5, he walks the same distance as he did on day 4. If the total distance travelled was 120 km, how far did he walk on the first day?

Q5. For each of the following sentences, (i) write a linear equation using x to represent the unknown and (ii) solve for x.

a. When 8 is added to the product of 4 and a certain number, the result is 86.
b. When 7 is divided into the sum of a certain number and 2, the result is 8.
c. When the product of 4 and a certain number is divided by 5, the result is 16.

d. When the sum of 10 and a certain number is divided by 12, the result is 6.

Q6. When 5 is added to a certain number and then quadrupled, the result is 224.

a. Write an equation to represent this situation, where a is the unknown number.
b. What is the number?

Q7. When Simon was born, his father was 36 years old. If s represents Simon's age and f represents his father's age.

a. Write an equation in terms of s and f to represent this situation.
b. Five years ago, his father was twice as old as Simon will be in 7 years time.
    i. Write an equation in terms of s and f that represents this situation.
    ii. Determine how old Simon is.

Q8. At the beginning of the chapter, we considered the case of a school dance, from which the organisers hope to raise at least \$2000 for charity. The tickets to the dance cost \$15 each with costs of \$350 for hire of the venue and \$400 for the DJ.

a. Write an inequation to represent this situation. Explain what your chosen pronumeral represents.
b. Solve the inequation to find the number of people that must attend the dance to raise at least \$2000.
c. Find the number of people attending the dance if \$2500 profit is made.
d. If the organisers hire a band for \$900 instead of a DJ, find, to the nearest dollar, the minimum cost of a ticket to maintain a \$2500 profit if the same number of people attend.

Q9. The cost of hiring a floodlit tennis court consists of a booking fee and an hourly rate. The booking fee is \$25 and the hourly rate is \$5 per hour.

a. Write an equation for the total hire cost in terms of the hourly rate.
b. Sketch a graph of the relationship.
c. What would be the charge for 3 hours?

Q10. A singing telegram service charges a \$60 appearance fee, and \$8 per minute sung.

a. Write an equation for the total cost of a singing telegram in terms of the number of minutes sung.
b. Sketch a graph of the relationship.
c. What would be the charge for a 5 minute singing telegram?

Q11. Colleen delivers junk mail and is paid $32 to traverse a particular route, and a further 10 cents per leaflet delivered.

a. Write an equation for the total payment she receives.
b. Sketch a graph of the relationship expressed in part a.
c. What would be Colleen's pay if she delivers 1650 leaflets along the route?

Q12. A pay-TV salesperson receives $300 per week plus $20 for every household he signs up to have pay-TV connected. How much does the salesperson receive for a week in which he signs up 33 households?

Q13. A computer firm, SuperComputers Inc., offers a back-up plan covering the ongoing service and troubleshooting of its systems after sale. The cost of signing up for the service plan is $215, and there is an hourly rate of $65 for the serviceperson's time. Purchasers not signing up for the plan are charged a flat rate of $150 per hour for service. Would it be advisable to sign up for the service plan if you expected to need 3 hours of service assistance during the life of a computer purchased from SuperComputers Inc?

Q14. A telephone company, Opus, offers calls to Biddelonia for a connection fee of $14, and thereafter $1 per minute. Its rival, Elstra, offers calls for $2 per minute (no connection fee) to the same country.

a. Compare the cost of a 10 minute call to Biddelonia using each company.
b. At what point would it be cheaper to use Opus?

Q15. It costs you $6 to get into a taxi (the flagfall), and $1.50 per kilometre if you use PinkCabs, while NoTop taxis charge $8 flagfall, and $1.20 per kilometre.

a. How much would it cost with each company to travel 15 km in one of its cabs?
b. When would it cost the same to use both companies?

Q16. Medirank, a health insurance company, charges $860 per year (for a single person), and requires customers to pay the first $100 of any hospital visit. HAB, on the other hand, charges an annual fee of $560 and requires its members to pay the first $150 of any hospital visit. Determine the number of hospital visits in a year for which the cost of health services is the same whichever company insures you.

Q17. Nifty is a car hire firm that charges insurance of $135, and $50 per day car hire. A competitor, Savus, simply charges $65 per day and offers free insurance. You are planning a holiday, and would prefer to use Savus. Under what conditions (days hired) could you justify this choice?

Q18. Jake and Voula Catering Enterprises charge \$250 for the hire of the function centre plus \$8 per head for the spit roast.

a. How much will it cost for a party of 50 people?
b. Determine the cost, C, equation for a party of n people.
c. Graph the function for $0 \leq n \leq 250$.
d. If the cost for a function was \$1050, how many people were expected to attend?

Q19. A local football club decided to hire a bus to the Grand Final. The bus company charges \$200 for a 45-seater bus and \$5 per person.

a. How much will it cost if every seat is occupied?
b. Construct an equation for the total cost, C, if n people catch the bus.
c. Graph the equation.
d. What does the y-intercept represent?
e. What does the gradient represent?
f. If the final cost was \$380, how many people went by bus?

Q20. A builder's fee, C dollars, can be determined from the rule $C = 60 + 55n$, where n represents the number of hours worked. According to this rule, the builder's fee will be:

A. \$60 for 1 hour of work.
B. \$110 for 2 hours of work.
C. \$500 for 8 hours of work.
D. \$550 for 10 hours of work.
E. \$1150 for 10 hours of work.

Q21. An electrician charges \$70 for a job which takes 2 hours to complete and \$92.50 for 3.5 hours of work.

a. Find a linear model to express the charge, \$$C$, for t hours of work.
b. Find the maximum daily earnings of the electrician if she is not prepared to work more than 8 hours on any given day.
c. Find the time taken to complete the job if the charge was \$73.75.

Q22. A long-distance truck driver is travelling towards home. After 1.5 hours of driving, he is 260 km from home and 2 hours later he is 140 km from his destination.

a. Find a linear model to express the distance from home, D, at any time, t, from the beginning of the journey.
b. Sketch the graph of the relation.
c. How far away from home was the driver when he started the journey?
d. What is the speed of the truck?
e. How long did it take to complete the journey? (Give the answer in hours and minutes.)

Q23. Ausat Telecommunications charges \$60 per quarter plus 35 cents for every call. Their rivals, Tetcom Telecommunications charge \$45 per quarter plus 45 cents for every call.

a. For Ausat, find the equation relating the total quarterly telephone bill, T, to the number of telephone calls, n.
b. For Tetcom, find the equation relating the total quarterly telephone bill, T, to the number of telephone calls, n.
c. Graph both equations on the same set of axes.
d. Use your graph to find when the quarterly cost is the same for both companies.
e. If you made, on average, only 120 calls per quarter, which company would be the better choice and by how much?

Q24. The Rave T-Shirt Company produces T-shirts at a cost of \$6 each after an initial setting up cost of \$300.

a. How much does it cost Rave to produce 100 T-shirts?
b. Find an equation that describes the cost, C, to produce n T-shirts.
c. Graph the equation.
d. The selling price, S, for each T-shirt is \$10.80. Find the equation expressing S in terms of n T-shirts.
e. Sketch the graph on the same set of axes you used in part c above.
f. From the graph, find the number of T-shirts needed to break even.
g. If an order were completed for 130 T-shirts, how much profit would Rave make?

Q25. The cost of manufacturing a number of frying pans consists of a fixed cost of \$400 plus a cost of \$50 per frying pan. The manufacturer could break even by selling:

A. 10 frying pans at \$90 each.
B. 10 frying pans at \$45 each.
C. 15 frying pans at \$60 each.
D. 15 frying pans at \$30 each.
E. 20 frying pans at \$50 each.

Q26. Metros Hire Car Company offers two daily rate packages. Package 1: An up-front charge of \$55 plus 26 cents per kilometre. Package 2: An up-front charge of \$100 per day, unlimited kilometres.

a. Find an equation for the cost, C, of each package per day.
b. Graph each equation on the same set of axes.
c. How many kilometres would you need to travel per day for the cost of both packages to be the same?
d. For what distances would Package 1 be cheaper?

Q27. Two friends, Michael and Julia, work in a large department store. Michael sells men's apparel and earns \$520 per week. Julia is in the white goods department, selling refrigerators, dishwashers, washing machines and so on. She earns a \$200 per week retainer, plus 8% of the total amount of her weekly sales.

a. Find the equation to represent the amount, $\$A$, earned by each friend per week. (Use s to denote the total value of sales.)
b. Graph each equation on the same set of axes.
c. What should be the total value of goods sold by Julia so that her weekly earnings will be the same as Michael's?
d. During one week, Julia sells white goods to the total value of \$12500. Whose earning would be greater for that week, and by how much?

Q28. A local karate club, Quick Kick charges a \$120 joining fee and \$6 for every lesson attended. Their competitors across the road, the Super Kids club, charge \$10 per lesson, but there is no joining fee.

a. Find the equation to represent the total cost $\$C$ of attending n lessons for each club.
b. Graph each equation on the same set of axes.
c. How many lessons would need to be attended, for the total tuition costs to be the same for both clubs?
d. If Alex intends to go to karate lessons once a week for an entire year, which club should he choose (provided that his decision is based on the total cost of tuition only)?

Q29. Last Easter, Nathan and Rachel travelled to Lorne. Nathan drove for the first one and a half hours at an average speed of 60 km/h. They then stopped at a service station for 30 minutes to fill the petrol tank and rest. Rachel then drove for 1 hour and 15 minutes, and averaged 80 km/h.

a. Draw a graph to represent the journey. Place time on the horizontal axis and distance travelled on the vertical axis.
b. Find the total distance that Nathan and Rachel drove.
c. Find the total time it took for Nathan and Rachel to travel to Lorne.
d. What was the average speed of the entire trip?

Q30. The Cybercheap company offers discounts on all of its products purchased over the Internet. The conditions are as follows:

1. a 10% discount is given on all purchases to the total value up to \$500, and
2. a 7% discount is given on any amount in excess of \$500.

a. Draw a line segment graph to show the discount, D, (in dollars) for purchase, P, of any size up to \$1000. (Place discount on the vertical axis and purchase price on the horizontal axis.)
b. Find the equation for each segment of the graph.
c. Find the discount for a purchase to the total value of:
   i. \$375.
   ii. \$820.

Q31. Every Saturday morning, Gary goes delivering newspapers in his neighbourhood. He is paid \$0.03 for each newspaper delivered up to (and including) 300, and \$0.05 for every newspaper delivered in excess of 300.

a. Draw a line segment graph to show Gary's total earnings, E, (in dollars) for delivering any number of newspapers, n, up to 500. (Place earnings on the vertical axis and number of newspapers delivered on the horizontal axis.)
b. Find the equation for each segment of the graph.
c. Calculate the total amount that Gary will earn for delivering:
   i. 221 newspapers.
   ii. 475 newspapers.

Q32. Find the equation of the straight line which passes through the point $(2, 5)$ and is:

a. parallel to the line with equation $y = 32x$.
b. perpendicular to the line with equation $y = 3x7$. Write equations in the form

$$ax + by + c = 0.$$

Q33. Find the equation of the straight line which passes through the point $(3, 1)$ and is:

a. parallel to the line with equation $4x2y = 13$.
b. perpendicular to the line with equation $4x2y = 13$.

Q34. If the straight lines $3xy = 2$ and $ax + 2y = 3$ are parallel, then a is equal to:

A. 6.
B. 2.
C. -2.
D. -3.
E. -6.

Q35. If the straight lines $5x + y3 = 0$ and $bxy2 = 0$ are perpendicular, then b is equal to:

A. 5.
B. $\frac{1}{5}$.
C. -5.

D. $-\frac{1}{5}$.
E. 3.

Q36. Find the equation, in the form $ax + by + c = 0$, of each straight line described below.

a. The line passing through $(3, 4)$ and $(1, 10)$.
b. The line passing through $(7, 5)$ and $(2, 0)$.

Q37. Consider the points $A(2, 5)$ and $B(1, b)$.

a. Find b if:
   i. the gradient of the straight line AB is 2.
   ii. the equation of the straight line AB is $yx = 7$.
b. Find the general equation of the straight line which passes through $(4, 5)$ and is parallel to the line with equation $y3x + 4 = 0$.
c. Find the equation in the form $ax + by + c = 0$ that passes through $(2, 4)$ and is perpendicular to the line with equation $2yx + 1 = 0$.

Q38. Find the value(s) of k such that

$$\begin{cases} kx + 9y = 0, \\ 3x + (k - 6)y = 0 \end{cases}$$

has a unique solution.

Q39. Determine the values of m such that

$$\begin{cases} mx - 5y = 10 \\ 3x - (m - 2)y = 6 \end{cases}$$

has either infinitely many solutions, or no solutions.

Q40. Solve the system of equations given by

$$\begin{cases} x + y + z = 9 \\ 2y - x + 3z = -15 \\ x + 5y + 3z = 29. \end{cases}$$

Q41. Solve the system of equations given by

$$\begin{cases} 6x + 2y - z = 1 \\ x + y + z = 2 \\ 4x + y - z = 1. \end{cases}$$

Q42. When five is added to three more than a certain number, the result is 19. What is the number?

Q43. If five is subtracted from three times a ce3rtain number, the result is 10. What is the number?

Q44. When 18 is subtracted from six times a certain number, the result is 42. What is the number?

Q45. A certain number added twice to itself equals 96. What is the number?

Q46. The second angle of a triangle is the same size as the first angle. The third angle is 12 degrees larger than the first angle. How large are the angles?

Q47. Two angles of a triangle are the same size. The third angle is 12 degrees smaller than the first angle. Find the measure the angles.

Q48. The perimeter of a rectangle is 304 cm. The length is 40 cm longer than the width. Find the length and width.

Q49. The perimeter of a rectangle is 152 meters. The width is 22 meters less than the length. Find the length and width.

Q50. You are driving along at 55 km per hour when you are passed by a car doing 85 km per hour. How long will it take for the car that passed you to be one km ahead of you?

Q51. If the perimeter of a rectangle is 18 inches, and one side is one inch longer than the other, how long are the sides?

Q52. John is 3 years older than Jim. Jim is 4 years less than twice Davids age. How old are the three boys if their ages add up to 35?

Q53. The instructions on a can of powdered drink mix say to mix $\frac{1}{4}$ cup of the mix with 2 quarts of water. How much of the mix should be used with $1\frac{1}{2}$ gallons of water?

Q54. Find a number such that 5 more than one-half the number is three times the number.

Q55. What is the velocity of a car (in km per hour) if it goes 40 km in 35 minutes?

Q56. Jim is 3 times as old as his cousin and the difference in their ages is 18. How old is Jim?

Q57. Maria has scores of 96, 86, and 78 on three tests. What must her average score on the next two tests be in order for her to have an average of at least 90?

Q58. What are the dimensions of a rectangle whose length is 4 more than twice the width and whose perimeter is 3 less than 7 times the width?

Q59. A carpenter cuts a board into three pieces of equal length and then cuts off $\frac{1}{4}$ of one of the pieces. If the smallest board he has is 1 foot in length, what was the length of the original board?

Q60. A restaurant uses 2 pints of milk with 3 pints of cream to make coffee creamier. To make a large quantity of the mixture, how many pints of cream should be used with 18 pints of milk?

Q61. Chuck travels 80 km in the same time that Mary travels 180 km. Mary travels 50 km/h faster than Chuck. Find the speed of each person.

Q62. A bacteria population doubles every minute. Explain why this population growth cannot be modeled using a linear equation.

Q63. Find the value of two numbers if their sum is 12 and their difference is 4.

Q64. The difference of two numbers is 3. Their sum is 13. Find the numbers.

Q65. Flying to Kampala with a tailwind a plane averaged 158 km/h. On the return trip the plane only averaged 112 km/h while flying back into the same wind. Find the speed of the wind and the speed of the plane in still air.

Q66. The school that Stefan goes to is selling tickets to a choral performance. On the first day of ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of \$38. The school took in \$52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.

Q67. A boat traveled 210 miles downstream and back. The trip downstream took 10 hours. The trip back took 70 hours. What is the speed of the boat in still water? What is the speed of the current?

Q68. The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

Q69. The senior classes at High School A and High School B planned separate trips to New York City. The senior class at High School A rented and filled 1 van and 6 buses with 372 students. High School B rented and filled 4 vans and 12 buses with 780 students. Each van and each bus carried the same number of students. How many students can a van carry? How many students can a bus carry?

Q70. Find the perimeter of a triangle if one side is 11 centimeters, another side is one-fifth the perimeter, and the third side is one-fourth the perimeter.

Q71. In 1984, the Soviets led the world in drilling the deepest hole in the Earths crustmore than 12 kilometers deep. They found that below 3 kilometers the temperature T increased 2.5° C for each additional 100 meters of depth.

a. If the temperature at 3 kilometers is 30° C and x is the depth of the hole in kilometers, write an equation using x that will give the temperature T in the hole at any depth beyond 3 kilometers.
b. What would the temperature be at 15 kilometers? (The temperature limit for their drilling equipment was about 300° C.)
c. At what depth (in kilometers) would they reach a temperature of 280° C?

Q72. An earthquake emits a primary wave and a secondary wave. Near the surface of the Earth the primary wave travels at about 5 miles per second, and the secondary wave travels at about 3 miles per second. From the time lag between the two waves arriving at a given seismic station, it is possible to estimate the distance to the quake. Suppose a station measures a time difference of 12 seconds between the arrival of the two waves. How far is the earthquake from the station? (The epicenter can be located by obtaining distance bearings at three or more stations.)

Q73. . A ship using sound-sensing devices above and below water recorded a surface explosion 39 seconds sooner on its underwater device than on its above-water device. If sound travels in air at about 1,100 feet per second and in water at about 5,000 feet per second, how far away was the explosion?

Q74. A naturalist for a fish and game department estimated the total number of trout in a certain lake using the popular capturemarkrecapture technique. She netted, marked, and released 200 trout. A week later, allowing for thorough mixing, she again netted 200 trout and found 8 marked ones among them. Assuming that the ratio of marked trout to the total number in the second sample is the same as the ratio of all marked fish in the first sample to the total trout population in the lake, estimate the total number of fish in the lake.

Q75. How many gallons of distilled water must be mixed with 50 gallons of 30% alcohol solution to obtain a 25% solution?

Q76. . An old computer can do the weekly payroll in 5 hours. A newer computer can do the same payroll in 3 hours. The old computer starts on the payroll, and after 1 hour the newer computer is brought on-line to work with the older computer until the job is finished. How long will it take both computers working together to finish the job? (Assume the computers operate independently.)

Q77. One pump can fill a gasoline storage tank in 8 hours. With a second pump working simultaneously, the tank can be filled in 3 hours. How long would it take the second pump to fill the tank operating alone?

Q78. A major chord in music is composed of notes whose frequencies are in the ratio 4:5:6. If the first note of a chord has a frequency of 264 hertz (middle C on the piano), find the frequencies of the other two notes. [Hint: Set up two proportions using 4:5 and 4:6.].

Q79. A minor chord is composed of notes whose frequencies are in the ratio 10:12:15. If the first note of a minor chord is A, with a frequency of 220 hertz, what are the frequencies of the other two notes?

Q80. In an experiment on motivation, Professor Brown trained a group of rats to run down a narrow passage in a cage to receive food in a goal box. He then put a harness on each rat and connected it to an overhead wire attached to a scale. In this way he could place the rat different distances from the food and measure the pull (in grams) of the rat toward the food. He found that the relationship between motivation (pull) and position was given approximately by the equation

$$p = -\frac{1}{5}d + 70, \qquad 30 \leq d \leq 170,$$

where p is measured in grams and distance d in centimeters. When the pull registered was 40 grams, how far was the rat from the goal box?

Q81. An oil-drilling rig in the Gulf of Mexico stands so that onefifth of it is in sand, 20 feet of it is in water, and two-thirds of it is in the air. What is the total height of the rig?

Q82. During a camping trip in the North Woods in Canada, a couple went one-third of the way by boat, 10 miles by foot, and one-sixth of the way by horse. How long was the trip?

Q83. After exactly 12 oclock noon, what time will the hands of a clock be together again?