

## Calculus Exam 1

Kyle Broder – ANU – MSI – 2017

The contents of this examination require an understanding of the elementary calculus material that was covered in the calculus practice exams 1 and 2. An understanding of graphing techniques and function transformations are also required.

There are no permitted materials for this test. That is, you are not permitted any cheat notes, calculators or resources other than a pen/pencil, eraser, sharpener, ruler and water bottle.

There is to be no collaboration on this examination and any attempts of communication will result in a nullified score. You are permitted 10 minutes of reading time and 105 minutes of writing time. There is a total of 100 available marks. It is recommended that you use the reading time to ask the invigilator about any issues regarding the format of the test, the problems or other issues. No hints will be given. Best of luck!

Name: \_\_\_\_\_

Grade: \_\_\_\_\_/100

**Question 1.** [10 marks]. Evaluate the following limit

$$\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x - 3}.$$

**Question 2.** [25 marks]. Consider the function  $f : \mathbb{R} \setminus \{3\} \rightarrow \mathbb{R}$  defined by

$$f(x) = \frac{1}{3-x}.$$

Show that  $f$  is differentiable on  $\mathbb{R} \setminus \{3\}$  and compute  $f'(x)$ .

**Question 3.** [25 marks]. Consider the function  $f : [0, \infty) \rightarrow \mathbb{R}$  defined by

$$f(x) = \frac{4}{\sqrt{x + \pi}} + \sqrt{x}.$$

Explain why  $f$  is differentiable on  $(0, \infty)$  and compute  $f'(x)$ .

**Question 4.** [20 marks]. Consider the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by

$$f(x) = \begin{cases} x, & x \geq 0, \\ -x, & x < 0. \end{cases}$$

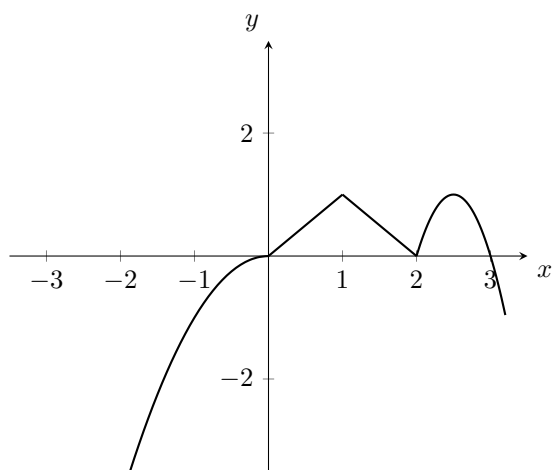
This function forms the prototypical example of a function that is continuous but *not* differentiable.

a. [10 marks]. Explain exactly what is meant by a function not being differentiable.

b. [5 marks]. Determine the point(s) where  $f$  is not differentiable.

c. [5 marks]. Compute the derivative of  $f$  at the points where  $f$  is differentiable.

**Question 5.** [20 marks]. Consider the function  $f$  whose graph is given below.



Determine the domain of  $f'(x)$  and sketch  $f'(x)$  on this domain. [Hint: If you struggle with the sketch, just describe where the derivative is positive, negative, zero, undefined, etc.]