Calculus Exam 2

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The contents of this examination require an understanding of the elementary calculus material that was covered in the calculus practice exams 1-5. An understanding of graphing techniques and function transformations may also be required.

There are no permitted materials for this test. That is, you are not permitted any cheat notes, calculators or resources other than a pen/pencil, eraser, sharpener, ruler and water bottle.

There is to be no collaboration on this examination and any attempts of communication will result in a nullified score. You are permitted 10 minutes of reading time and 105 minutes of writing time. There is a total of 100 available marks. It is recommended that you use the reading time to ask the invigilator about any issues regarding the format of the test, the problems or other issues. No hints will be given. Best of luck!

Name: ____________________

Grade: __________/100

Question 1. [10 marks]. Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ be defined by $f(x) = \tan x$. Show that $f'(x) = \sec^2 x$, where $\sec x := \frac{1}{\cos x}$.

Question 2. [20 marks]. Consider the function $f : (0, \infty) \to \mathbb{R}$ defined by

$$f(x) = \log_e(x) + \frac{1}{\sqrt{x}} \sin x.$$

Explain why f is differentiable on $(0, \infty)$ and evaluate $f'(x)$.

Question 3. [20 marks]. Consider the function $f : (-\pi/2, \pi/2) \to \mathbb{R}$ defined by

$$f(x) = \frac{\tan x}{x^2 + 1}.$$

Evaluate $f'(0)$. [Hint: Question 1 may be useful.]

Question 4. [25 marks]. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = e^{-x}(x^2 + 4x + 6).$$

a. Determine the stationary point(s) of f.

b. Determine the nature of the stationary point(s) which were found in part (a).

Question 5. [25 marks]. Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable on $\Omega_1 \subseteq \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ is differentiable on $\Omega_2 \subseteq \mathbb{R}$.

a. What is the maximal domain on which $f \cdot g$ is differentiable?

b. Let $f(x) := |x|$ and $g(x) := \sqrt{x}$. State the maximal domain on which f and g are differentiable.

c. Hence, or otherwise, state the maximal domain on which $h(x) := |x| \cdot \sqrt{x}$ is differentiable.

d. Evaluate $h'(x)$ at all points x in the domain determined in part c.

$\star$ **Question 6.** [20 marks]. Determine the value of the limit

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right).$$

[Hint: $-1 \leq \sin x \leq 1$.]