Question 1

a.) $5n^3 + 2n^2 + 3n = O(n^3)$

$5n^3 + 2n^2 + 3n \leq Cn^3$ for $n \geq n_0$

$5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3$

$5n^3 + 2n^2 + 3n \leq 10n^3$

$C = 10$
$n \geq 1$

b.) $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

$c_1 n \leq \sqrt{7n^2+2n-8} \leq c_2 n$ for $n \geq n_0$

$c_1^2 n^2 \leq 7n^2+2n-8 \leq c_2^2 n^2$

$4n^2 \leq 7n^2+2n-8 \leq 7n^2+2n^2$

$\sqrt{4n^2} \leq \sqrt{7n^2+2n-8} \leq \sqrt{9n^2}$

$2n \leq \sqrt{7n^2+2n-8} \leq 3n$

$2(2) \leq \sqrt{7(2)^2+2(2)-8} \leq 3(2)$

$4 \leq \sqrt{24} \leq 6$ ✓

$c_1 = 2$
$c_2 = 3$
$n_0 = 2$

c.) $d(n) \leq c_1 f(n)$ $\quad e(n) \leq c_2 g(n)$

$d(n)e(n) \leq c_1 f(n) e(n)$

$c_1 f(n) e(n) \leq c_1 c_2 f(n) e(n) \quad c_1 c_2 = c_3$

$d(n)e(n) \leq c_3 f(n) e(n)$

$d(n)e(n) = O(f(n)e(n))$

Question 2:

Example 1: $\Theta(n^2)$

Example 2: $\Theta(n)$

Example 3: $\Theta(\log(n))$

Example 4: $\Theta(n)$