GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
GRADE 12

2024

MATHEMATICS

(PAPER 2)

MATHEMATICS P2

C2612E

TIME: 3 hours

MARKS: 150

X05

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs etc. which you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

1.1 The data below shows the marks (out of 100), obtained by 16 learners in a Mathematics test (Test 1).

67	77	26	92	48	38	56	58
75	83	32	94	60	44	64	68

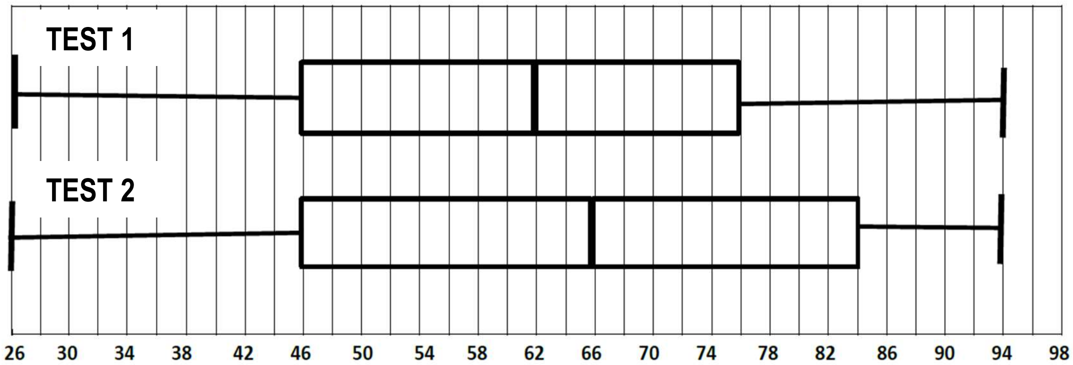
Calculate the:

1.1.1 Mean mark for the test (2)

1.1.2 Standard deviation of the data (1)

1.1.3 The number of learners whose marks are outside one standard deviation of the mean (2)

1.2 The 16 learners, referred to in QUESTION 1.1 above, wrote a second test (Test 2). The box and whisker diagram below shows the distribution of marks obtained by the 16 learners in the two tests.



1.2.1 Describe the skewness of the marks for Test 1. (1)

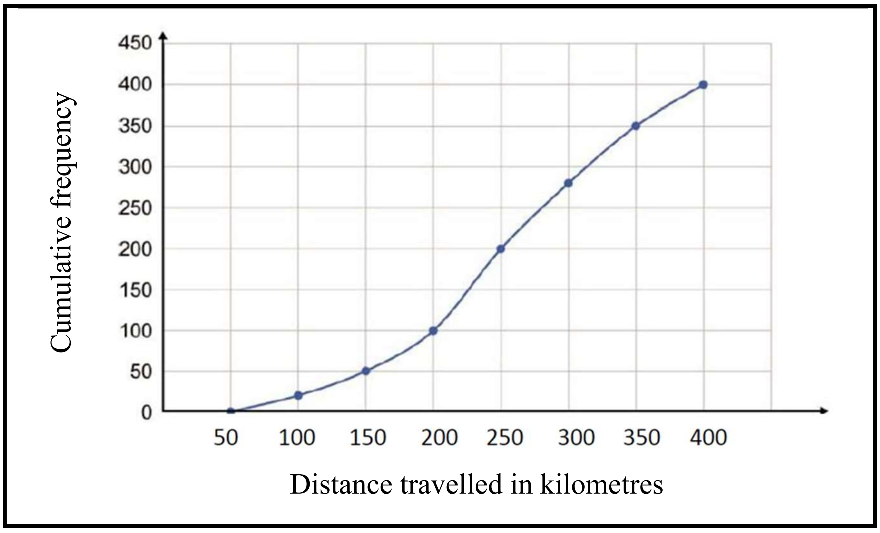
1.2.2 Which test was easier for the learners? Give a reason for your answer. (2)

1.2.3 How many learners scored more than 84% in the second test? (2)
[10]

QUESTION 2

2.1 In a survey, a group of people were asked about the total distance they had travelled from their homes in the last week of December 2023. The data collected is represented in the frequency table and ogive (cumulative frequency curve) below.

Distance travelled (x kilometres)	Frequency
$50 \leq x < 100$	20
$100 \leq x < 150$	30
$150 \leq x < 200$	**A**
$200 \leq x < 250$	**B**
$250 \leq x < 300$	80
$300 \leq x < 350$	70
$350 \leq x < 400$	50



2.1.1 How many people participated in this survey? (1)

2.1.2 Determine the values of **A** and **B** in the table above. (2)

2.1.3 Use the ogive/table to estimate the number of people who travelled between 100 km and 300 km. (2)

2.1.4 If all people who travelled more than 350 km were removed from the survey, how would this affect the median of the data? (1)

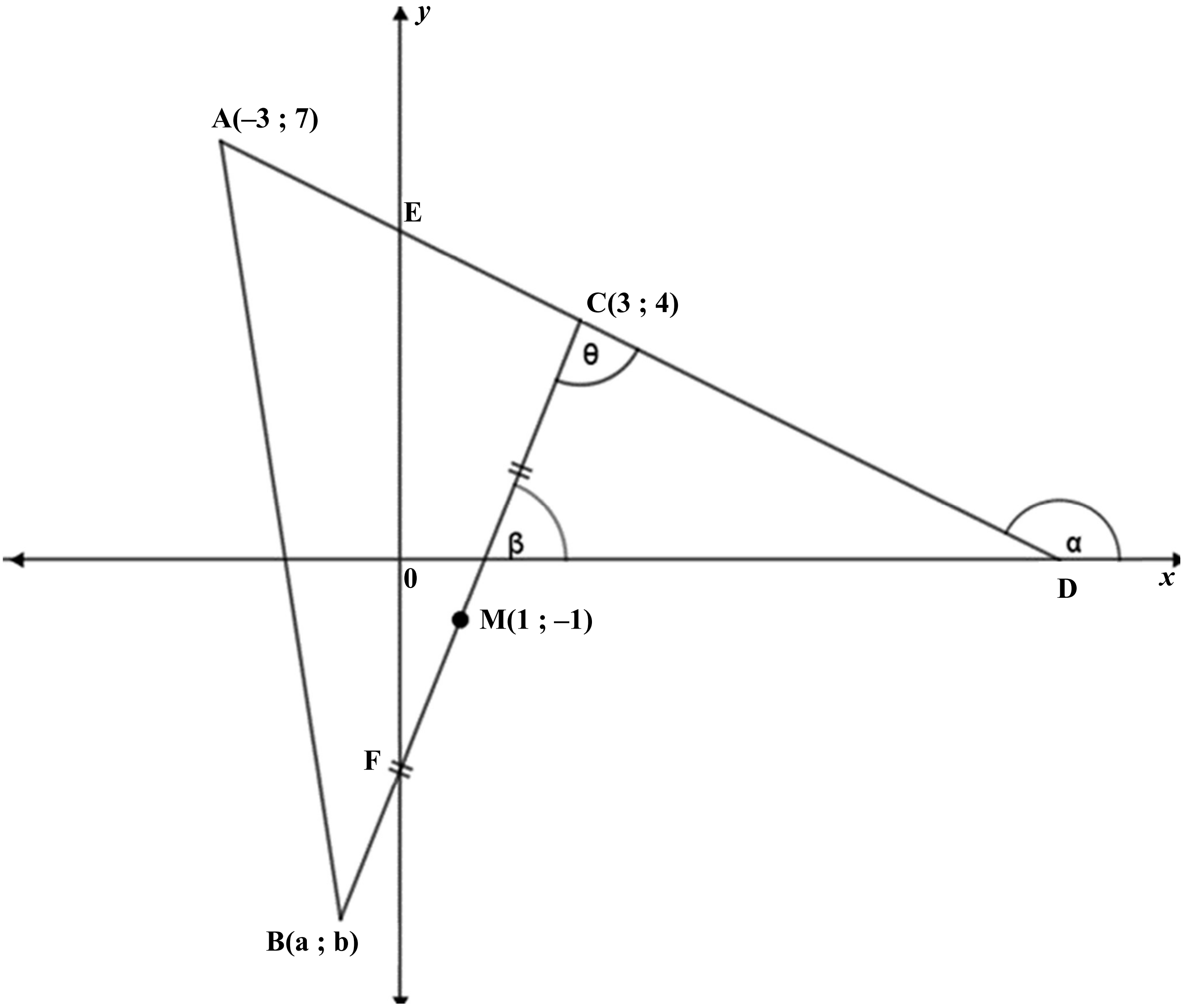
2.2 If the estimated mean of the data below is 16,4, then what is the value of t ?

Class Interval	Frequency
$0 < x \leq 10$	13
$10 < x \leq 20$	$\boldsymbol{t}$
$20 < x \leq 30$	12
$30 \leq x < 40$	4

(4)
[10]

QUESTION 3

In the diagram below, the equation of line AD is $2y + x = 11$. M(1; -1) is the midpoint of the straight line joining B(a; b) and C(3;4). The angles of inclination of AD and BC are α and β respectively.
E and F are the y-intercepts of AD and BC respectively.



3.1 Determine the coordinates of B(a ; b). (3)

3.2 Determine the gradient of BC. (2)

3.3 Determine the size of θ. (Correct to ONE decimal place) (4)

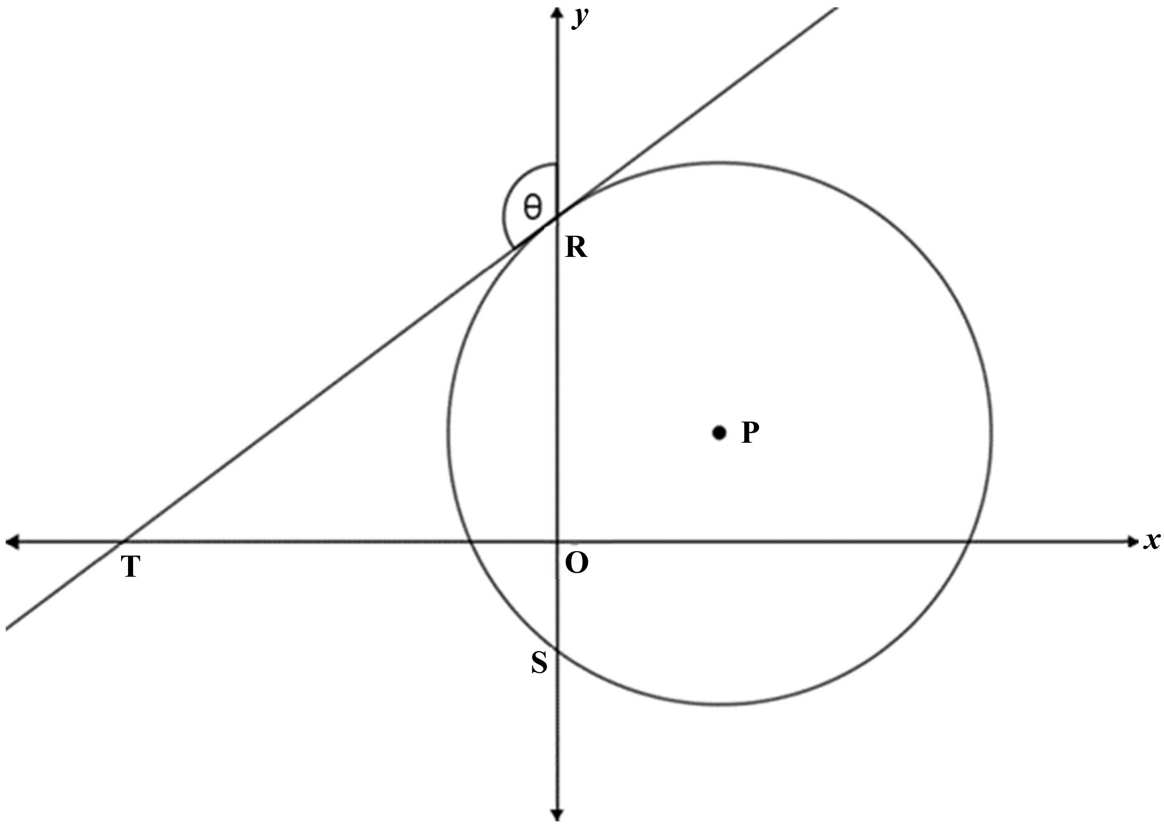
3.4 Determine the equation of line BC. (2)

3.5 Determine the area of ΔCEF. (4)

[15]

QUESTION 4

The diagram below shows a circle with centre P and equation $x^2 + y^2 - 6x - 4y = 12$.
The circle cuts the y-axis at points R and S. RT is a tangent to the circle at R and cuts the x-axis at T. Angle θ is indicated.



4.1 Determine the coordinates of P as well as the radius of the circle. (4)

4.2 Show that the coordinates of R is R$(0\,;6)$. (2)

4.3 Determine the equation of tangent RT in the form $y = mx + c$. (3)

4.4 Determine the size of θ. (Correct to ONE decimal place) (3)

4.5 A vertical line is drawn as a tangent to circle P at Q(a ; b) where a > 0. Write down the coordinates of Q. (2)

4.6 For which values of k will $y = +\frac{3}{4}x + k$, be a secant to the circle? (4)

4.7 Another circle M with equation $(x+3)^2 + (y+2)^2 = 36$ is drawn. Will circle M touch or cut circle P or will it neither touch nor cut circle P? Show all calculations to determine your answer. (4)

[22]

QUESTION 5

5.1 Simplify the following expression, **without using a calculator**:

$$tan\,(-x) \cdot sin\,(90° + x) + \frac{sin2x}{2cos(360° + x)}$$ (6)

5.2 Given that $sin27° = p$, determine, **without the use of a calculator**, each of the following in terms of p.

5.2.1 $cos27°$ (2)

5.2.2 $sin^2 63°$ (2)

5.2.3 $cos13{,}5°$ (3)

5.3 If $cosx + sinx = k$, express the following in terms k:

$cos\,(x - 45\,°)$ (4)

5.4 Prove the identity:

$$\frac{cos2\theta + 1}{sin2\theta} + tan\theta = \frac{1}{sin\theta cos\theta}$$ (5)

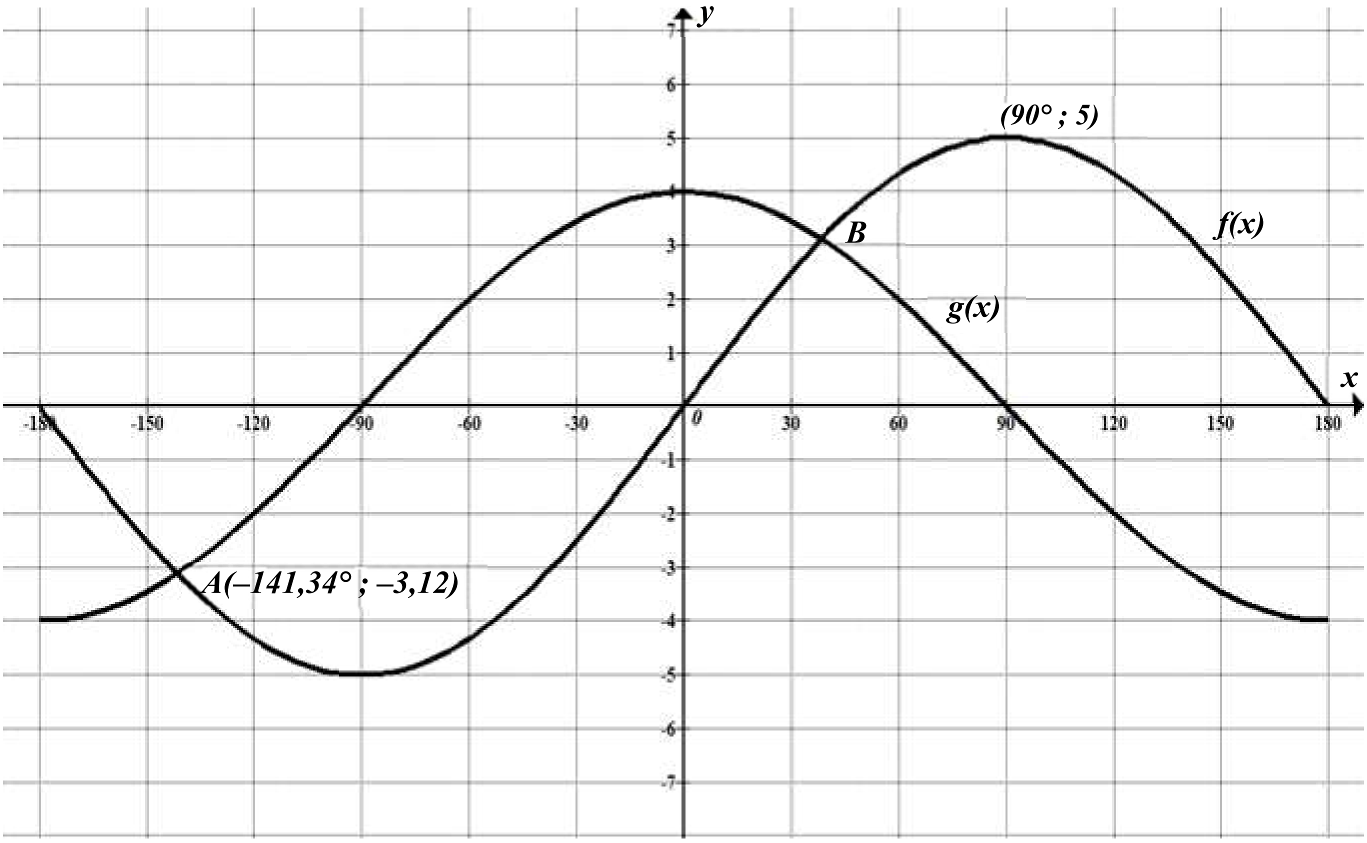
5.5 Determine the general solution of the equation:

$4sin^2\theta = cos\,(90° - 2\theta)$ (6)

[28]

QUESTION 6

In the diagram below, $f(x) = a\sin bx$ and $g(x) = 4\cos x$ are drawn for the interval $x \in [-180°; 180°]$. The graph of $f(x)$ passes through $(90°; 5)$ and the graph of $g(x)$ passes through $(90°; 0)$. A and B are points of intersection of the graphs of f and g. A$(-141,34°; -3,12)$.



6.1 Determine the values of a and b. (2)

6.2 The graph of $g(x)$ is shifted 30° to the right and 2 units vertically down to form the graph of $h(x)$. Determine the equation of $h(x)$. (2)

6.3 Calculate the minimum value of $\frac{8}{g(x)}$ in the interval $x \in [-60°; 60°]$. (1)

6.4 If A$(-141,34°; -3,12)$, determine the coordinates of B. (2)

6.5 For what values of k will $f(x) = k$ have no real solutions? (2)

6.6 Use the graph(s) to determine the values of x in the interval $x \in [-180°; 180°]$ for which:

6.6.1 $f(x) < g(x)$ (2)

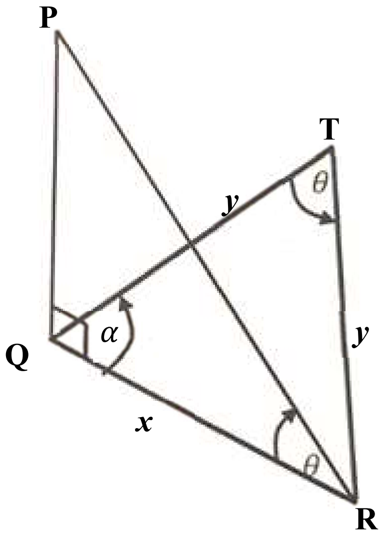
6.6.2 $f'(x) \cdot g(x) \geq 0$ (2)

[13]

QUESTION 7

In the diagram below, TQR represents three points in a horizontal plane on a sportsfield. PQ represents a vertical flagpole.

The angle of elevation of the top of the pole from R is equal to θ. $\hat{T} = \theta$. TQ = TR = y.
QR = x units. $T\hat{Q}R = \alpha$.



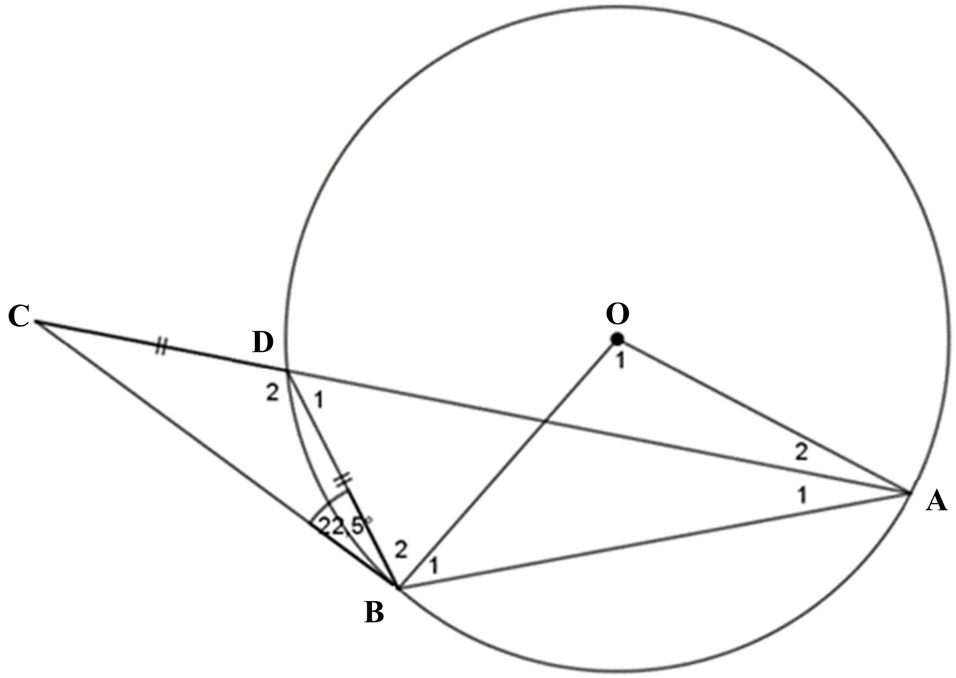
7.1 Express θ in terms of α and show that $\sin\theta = \sin 2\alpha$ (2)

7.2 Hence, prove that in ΔPQR: $\text{PR} = \frac{2y\cos\alpha}{\cos\theta}$ (4)

7.3 If $\alpha = 49°$; $x = 20$ m and $y = 15$ m, calculate the area of ΔTQR. (3)
[9]

QUESTION 8

In the diagram below, O is the centre of the circle. A, B and D are points on the circle. BC is a tangent to the circle at B and $D\hat{B}C = 22{,}5°$. Chord AD is produced to C such that CD = BD.



8.1 Determine, giving reasons, the size of:

8.1.1 $\hat{A}_1$ (2)

8.1.2 $\hat{D}_1$ (2)

8.1.3 $\hat{B}_2$ (2)

8.1.4 $\hat{O}_1$ (2)

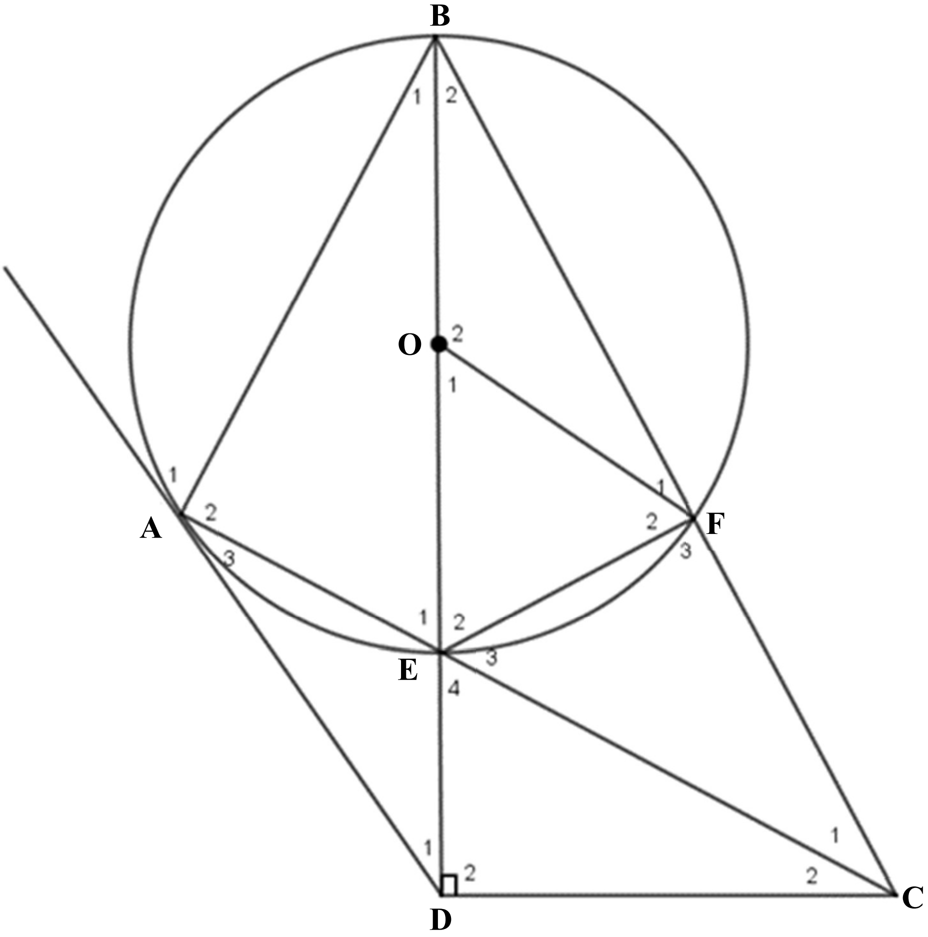
8.2 Prove, giving reasons, that OA ∥ CB. (2)

8.3 If it is further given that the radius of the circle is 12 units, calculate the length of BC. (4)

[14]

QUESTION 9

In the diagram below, the diameter BE of circle O is produced to D. DA is a tangent to the circle and $CD \perp BD$. AC and BC cut the circle at E and F respectively. OF and EF are drawn.



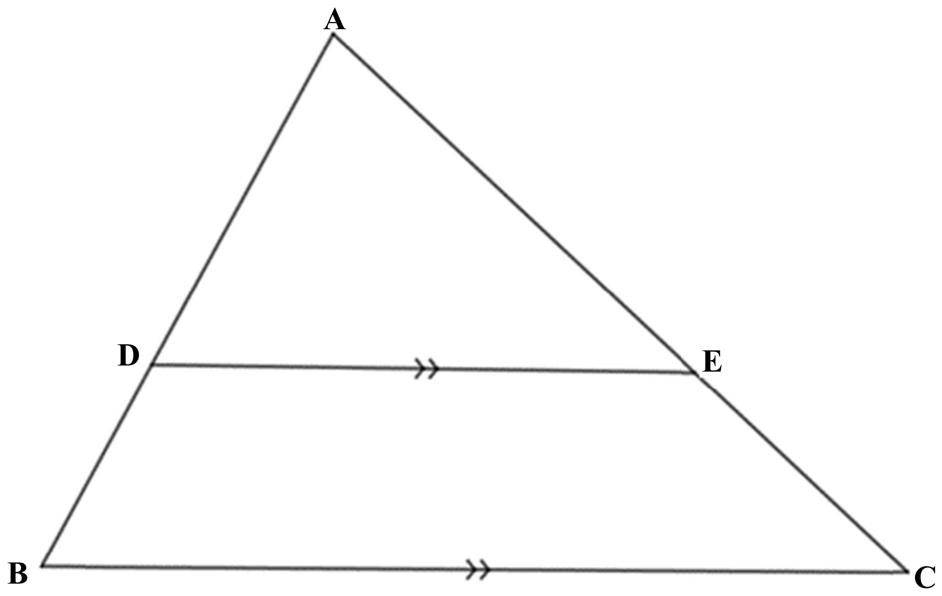
9.1 Prove, with reasons, that ABCD is a cyclic quad. (3)

9.2 Prove, with reasons, that BD bisects $A\hat{B}C$. (3)

9.3 Prove, with reasons, that EC is a tangent to circle OEF. (4)
[10]

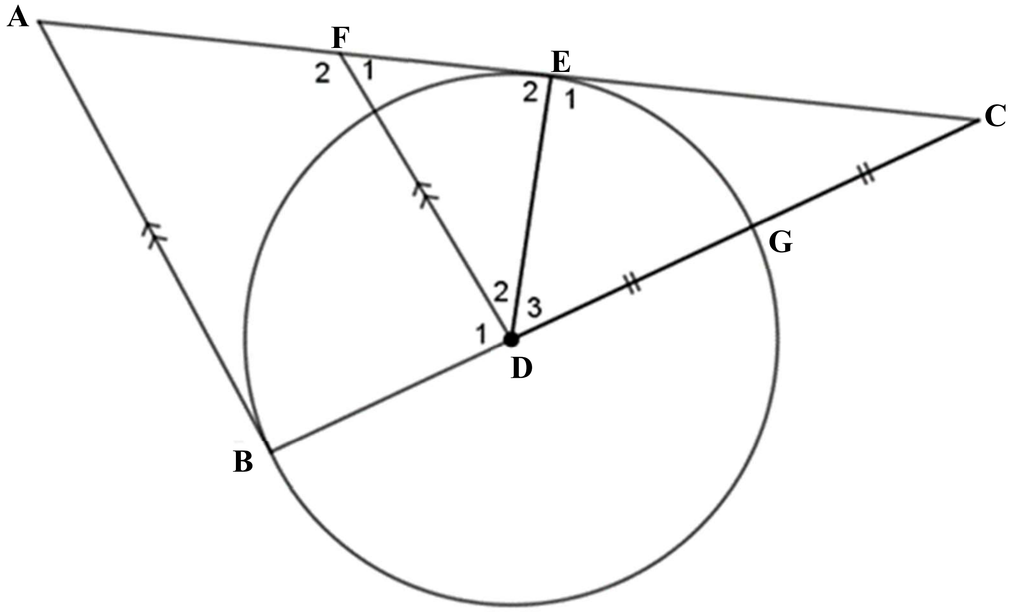
QUESTION 10

10.1 In the diagram below, ΔABC is drawn with DE ∥ BC.



Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$. (5)

10.2 In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that DF ∥ AB.



10.2.1 Find, with reasons, the ratio of $\frac{AC}{FC}$. (3)

10.2.2 Prove, giving reasons, that $\Delta ABC\ ///\ \Delta DEC$. (3)

10.2.3 Prove that $DE^2 = \frac{AE.EC}{3}$ (5)

10.2.4 Find the ratio of: $\frac{\text{Area}\Delta FDC}{\text{Area}\Delta ABC}$ (3)

[19]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\hat{y} = a + bx$$