GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
GRADE 12

2024

MATHEMATICS

(PAPER 1)

MATHEMATICS P1

C2611E

TIME: 3 hours

MARKS: 150

10 pages + a formula sheet

X05

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $2x(3x + 4) = 0$ (2)

1.1.2 $2x^2 - 4x = -1$ (Correct to TWO decimal places) (4)

1.1.3 $(x - 2)^2 \geq 1$ (4)

1.2 Given: The equation $\sqrt{x-2} = 4 - x$

1.2.1 Without solving for x, show that the solution lies in the interval $2 \leq x \leq 4$. (2)

1.2.2 Solve the equation. (4)

1.3 Solve for x and y simultaneously:

$3x + y = 2$ and $y^2 = 2x^2 - 1$ (6)

1.4 If $r + 2s = a$ and $r - 2s = b$, prove that $rs = \dfrac{a^2 - b^2}{8}$ (4)

[26]

QUESTION 2

2.1 Given the arithmetic sequence $85 ; 82 ; 79 ; 76 ; \ldots$

2.1.1 Determine a simplified expression for T_n. (3)

2.1.2 Which term would be the first negative number in the sequence? (3)

2.2 A quadratic sequence, with general term T_n, has the following properties:

- $T_{11} = 190$
- $T_n - T_{n-1} = 4n - 3 \; ; \; n \geq 2$

Determine the first term of the quadratic sequence. (5)

2.3 The sum of the first 50 terms of an arithmetic sequence is 1 275.

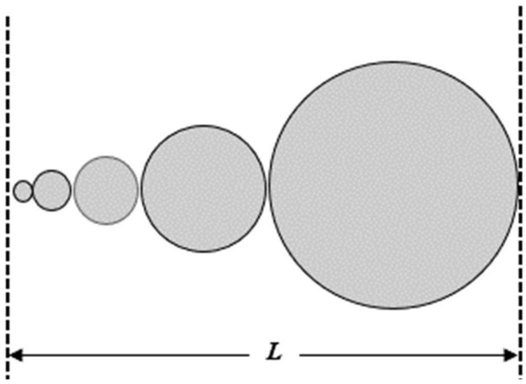
Calculate the sum of T_{25} and T_{26}. (3)

[14]

QUESTION 3

3.1 For which values of x will $\sum_{k=1}^{\infty} (4x-1)^k$ exist? (4)

3.2 The figure below shows a pattern of 5 circles, touching externally, whose centres lie on a straight line of length L units. The radii of these circles form a geometric pattern, where the radius of the smallest circle is 3 units and that of the fifth (largest) circle is 48 units.



3.2.1 Determine the common ratio of the geometric pattern formed by the radii of the circles. (3)

3.2.2 Determine the value of L. (3)

3.2.3 The pattern is extended by 5 more circles to 10 circles. Calculate, in terms of π, the total area of the 10 circles of the new pattern. (4)

[14]

QUESTION 4

The lines $y = x + 1$ and $y = -x + 3$ are the axes of symmetry of the function $g(x) = \dfrac{-4}{x+p} + q$.

4.1 Show that $p = -1$ and $q = 2$. (3)

4.2 Calculate the x-intercept of g. (2)

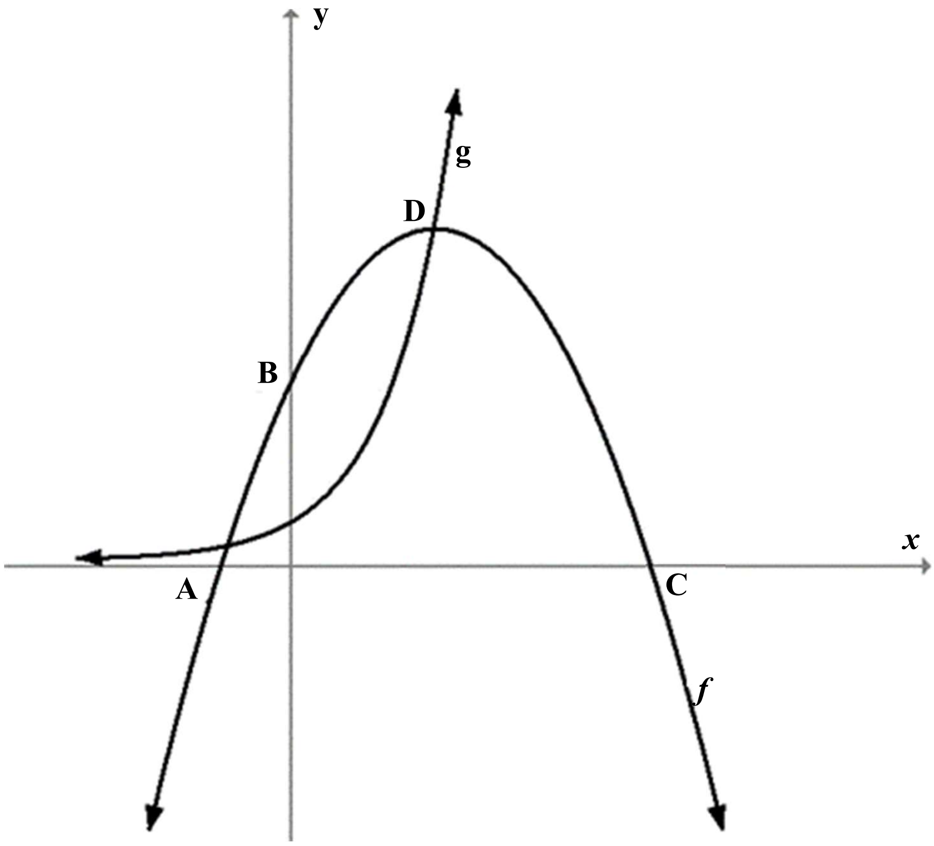
4.3 Sketch the graph of g in your ANSWER BOOK clearly showing the asymptotes and the intercepts with the axes. (4)

4.4 For which values of x is $g(x) > 0$? (2)

[11]

QUESTION 5

5.1 The sketch below, shows the graphs of $f(x) = -(x-2)^2 + 9$ and $g(x) = b^x$ where b is a constant. D is the turning point of f and a point of intersection of f and g. B is the y-intercept and A and C, the x-intercepts of f.



5.1.1 Determine the length of AC. (4)

5.1.2 Determine the value of b. (2)

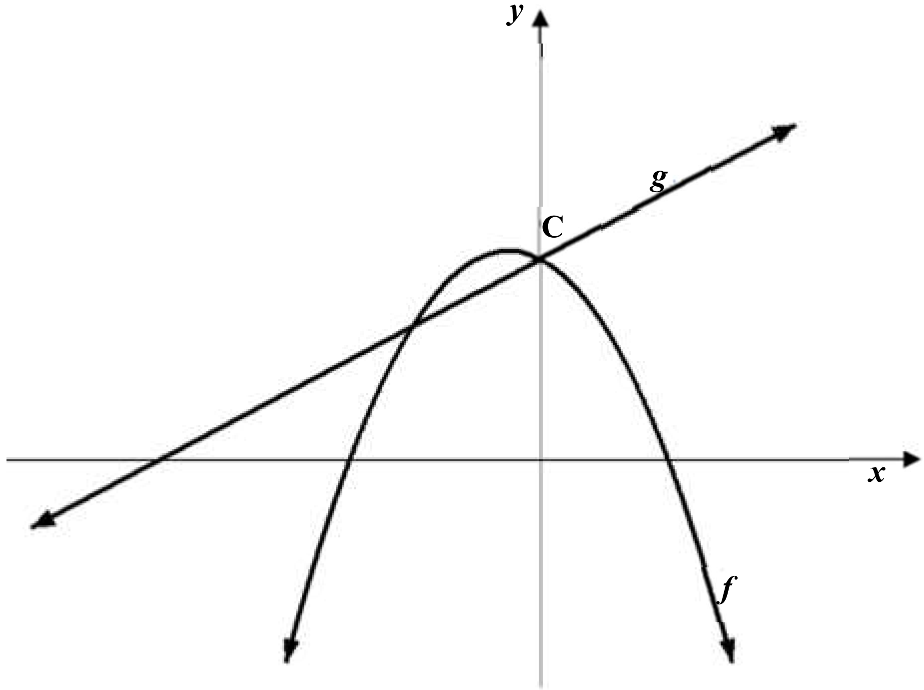
5.1.3 Determine the values of x for which $g(x) \geq 9$. (1)

5.1.4 Write down the equation of h if $h(x) = f(x+2) - 9$. (2)

5.1.5 How can the domain of h be restricted so that h^{-1} will be a function? (1)

5.1.6 Show, algebraically, that $g\left(x + \frac{1}{2}\right) = \sqrt{3}g(x)$. (2)

5.2 Given: $f(x) = ax^2 + bx + c$ and $g(x) = mx + c$



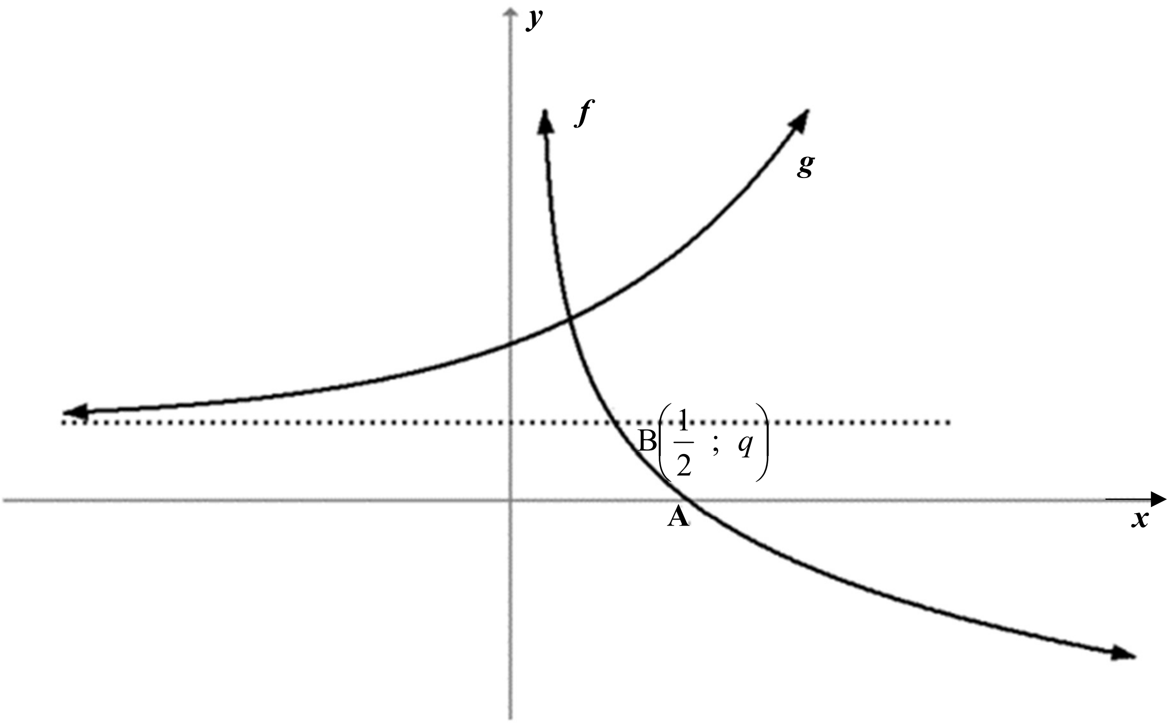
If given $f(x).\ g(x) < 0$ for all values of x when $-6 < x < -3$ or $x > 2$, determine the value of 'a' in terms of m (show all workings). (5)

[17]

QUESTION 6

The sketch below shows the graphs of $g(x) = 2^x + q$ and $f(x) = log_{\frac{1}{2}} x$.

Graph f and the asymptote of g intersect at B$\left(\frac{1}{2}; q\right)$.



6.1	Write down the coordinates of A, the x-intercept of f.	(1)
6.2	Determine the domain of f.	(1)
6.3	Determine the equation of f^{-1} in the form of $y = \cdots$	(2)
6.4	Sketch the graph of f^{-1}. Indicate on your graph the intercept(s) with the axes and the coordinates of one other point on the graph.	(3)
6.5	Determine the equation of the asymptote of g.	(1)
6.6	Describe in words the transformation of g to f^{-1}.	(2)
		[10]

QUESTION 7

7.1 An investment earns interest at a rate of 7,5 % p.a. compounded quarterly. Calculate the effective annual interest rate of this investment. (3)

7.2 Rhino poaching is a serious problem and has resulted in the black rhino becoming an endangered species. Statistics show that there were 60 000 black rhinos in 1970 and only 4 200 remained in 2011. Determine the annual rate of population decrease as a percentage over these 42 years, if the rate is compound decay. (4)

7.3 Adam inherits R27 000, which he invests in a savings account. The interest is calculated at 5,4% compounded monthly for the full period. Three years after he invested the initial amount, he withdraws Rx to make a down payment on a car. Calculate the value of x if he accumulated R17 614,76 in the savings account after 10 years. (5)

[12]

QUESTION 8

8.1 Determine $f'(x)$ from FIRST PRINCIPLES if $f(x) = 2 - 3x^2$. (5)

8.2 Determine:

8.2.1 $f'(x)$ if $f(x) = 2x^4 - 3x + a^2$ (3)

8.2.2 $D_x\left[\dfrac{2x^3 - \sqrt{x}}{x}\right]$ (3)

8.3 The following information is given for a function f:

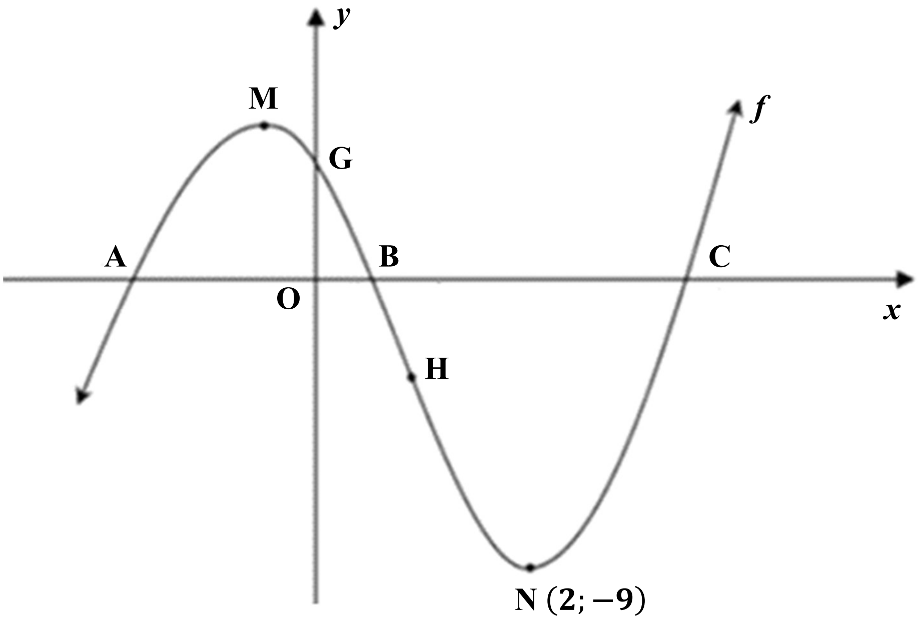
- $f(4) = 5$
- $f'(4) = 7$

Determine the equation of the tangent of f at $x = 4$ in the form $y = mx + c$. (3)

[14]

QUESTION 9

The function $f(x) = 2x^3 + px^2 + qx + 3$ is drawn below. N$(2; -9)$ and M are turning points of f.
G is the y-intercept of f and A, B and C are the x-intercepts of f.



9.1 Show that $p = -5$ and $q = -4$. (5)

9.2 Determine the coordinates of G, the y- intercept of f. (2)

9.3 If it is given that C$(3; 0)$, calculate the distance between A and B, the x-intercepts. (4)

9.4 Calculate the x-coordinate of M, a turning point of f. (3)

9.5 Determine the x-coordinate of H, the point of inflection of f. (3)

9.6 For which value(s) of x will f be concave up? (1)

9.7 For which values of x is $f(x).f'(x) < 0$? (3)

[21]

QUESTION 10

10.1 The events S and T are independent:

- $\text{P(S and T)} = \frac{1}{6}$
- $\text{P(not S)} = \frac{3}{4}$

10.1.1 Calculate P(T). (3)

10.1.2 Hence, calculate P(S or T). (2)

10.2 A group of 200 tourists visited the same restaurant on two consecutive evenings. On both evenings the tourists could choose beef (B) or chicken (C) for their meals. The manager observed that 35% of the tourists chose beef on the first evening and 70% of the tourists chose chicken on the second evening.

10.2.1 Draw a tree diagram to represent the different choices of main meals on the two evenings. Show on your diagram the probabilities associated with each branch, as well as the possible outcomes of the choices. (3)

10.2.2 Calculate the number of tourists who chose the same meal on both evenings. (3)

[11]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$