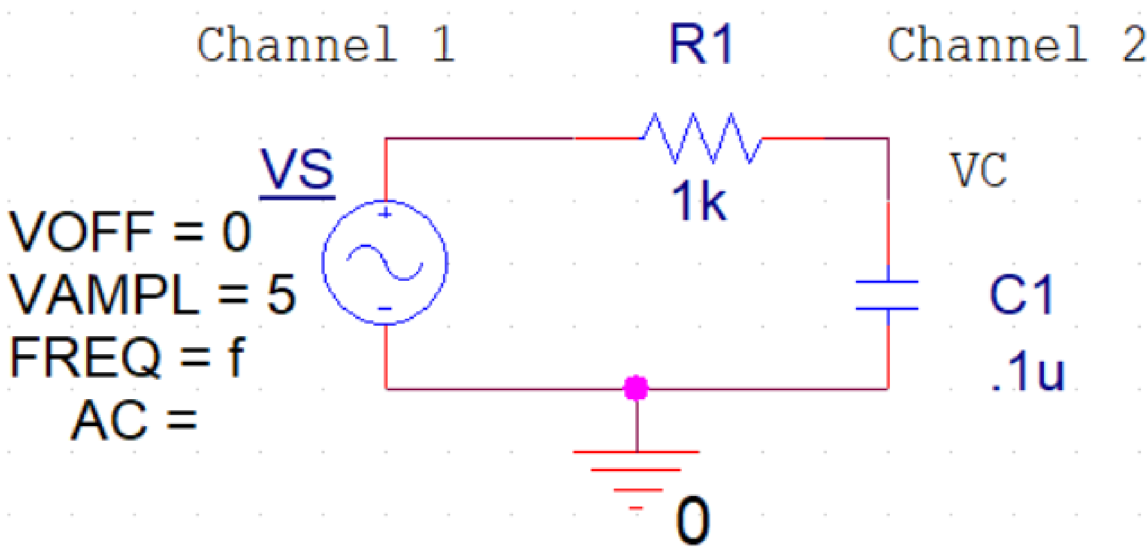
ECE 2101L Lab #1

The Sinusoidal Steady State Responses Of First Order RC Circuits



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September 12th, 2020

Objective

The objective of the lab be able to theoretically find the steady state response of RC circuits when powered by a sinusoidal power supply and then compare that to data found through a simulation or provided by the instructor. Another objective is to be able to configure the necessary lab equipment to collect the necessary data for the lab.

Materials

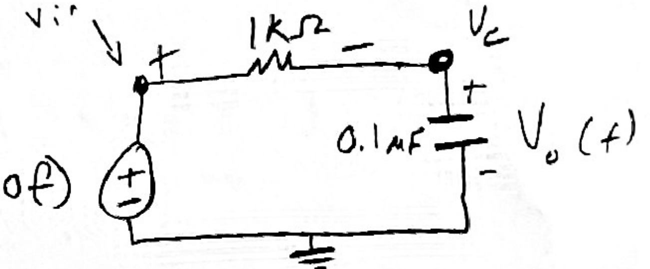
The necessary equipment needed for the lab are as follows

1. Clip leads
2. Breadboard
3. Resistors of value 1kΩ and 2kΩ
4. Capacitor of value 0.1μF
5. Oscilloscope with two channel input and 4 BNC to clip connectors
6. Function generator
7. Digital multimeter and LCR meter

Pre-Lab

1.

$V_{in}(t) = 5\cos(2\pi 1500 t)$



$$\frac{V_c - V_{in}}{1K} + V_c = 0, \quad \frac{V_c - V_{in}}{1K} + .1\mu\frac{dV_c}{dt} = 0$$

$$V_c - V_{in} + 10^{-4}\frac{dV_c}{dt} = 0, \quad \frac{dV_c}{dt} + 10^4 V_c = 10^4 V_{in}$$

$$\boxed{\frac{dV_c}{dt} + 10^4 V_c = 5\times10^4\cos(2\pi\, 1500 t)}$$

Assuming the solution to be of the form $A\cos(2\pi f(t) + \theta)$

$$\frac{d}{dt}\left(A\cos(2\pi f(t)+\theta)\right) + 10^4 A\cos(2\pi f t+\theta) = 5\times10^4\cos(2\pi 1500 t)$$

$$-2\pi f A\sin(2\pi f(t)+\theta) + 10^4 A\cos(2\pi f t+\theta) = 5\times10^4\cos(2\pi 1500 t)$$

using the trig identity $C\sin(x) + B\cos(x) = \sqrt{B^2+C^2}\cdot\cos(x + \tan^{-1}(\frac{-C}{B}))$

$$\sqrt{(A\cdot10^4)^2 + (A\cdot -2\pi f)^2}\,\cos\left(2\pi f t + \theta + \tan^{-1}\left(\frac{2\pi f A}{10^4 A}\right)\right) = 5\times10^4\cos(2\pi 1500 t)$$

$A \neq 0$, so we can cancel the variable from the $\tan^{-1}$ argument since if $A = 0$, then the argument can't be computed.

$$A\sqrt{10^8 + (2\pi f)^2} = 5\times10^4$$

$$A = \frac{5\times10^4}{\sqrt{10^8 + (2\pi(1500))^2}}$$

$$A = 3.6386$$

$$2\pi(1500 t) + \theta + \tan\left(\frac{2\pi f}{10^4}\right) = 2\pi 1500 t$$

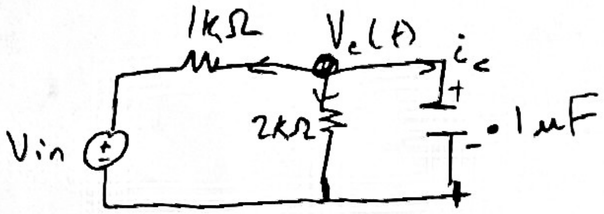
$$\theta = -\tan\left(\frac{2\pi(1500)}{10^4}\right)$$

$$\theta = -43.304^\circ$$

$$\boxed{V_c(t) = 3.639\cos(2\pi(1500)t - 43.30^\circ)\ [V]}$$

2.

$V_{in} = 5\cos(2\pi(2000)t)$



$2K\left(\frac{V_c - V_{in}}{1K} + \frac{V_c}{2K} + cV_c'\right) = 0$, $2V_c - 2V_{in} + V_c + (2k)c\frac{dV_c}{dt} = 0$

$3V_c + 2\times10^{-4}\frac{dV_c}{dt} = 2V_{in}$, $\boxed{\frac{dV_c}{dt} + 1.5\times10^4 V_c = 5\times10^4\cos(2\pi(2000)t)}$

Assuming a solution of the form $A\cos(2\pi f t + \theta)$

$\frac{d}{dt}(A\cos(2\pi f t + \theta)) + \frac{3}{2}\times10^4(A\cos(2\pi f t + \theta)) = 5\times10^4\cos(2\pi(2000)t)$

$(-2\pi f)(A)\sin(2\pi f t + \theta) + \frac{3}{2}\times10^4(A\cos(2\pi f t + \theta)) = 5\times10^4\cos(2\pi(2000)t)$

using the trig identity $C\sin x + B\cos x = \sqrt{B^2 + C^2}\cos\left(x + \tan^{-1}\left(-\frac{C}{B}\right)\right)$

$\sqrt{(-2\pi f A)^2 + (\frac{3}{2}\times10^4 A)^2}\cos\left(2\pi(2000)t + \theta + \tan^{-1}\left(\frac{2\pi f A}{\frac{3}{2}\times10^4 A}\right)\right) = 5\times10^4\cos(2\pi(2000)t)$

We can cancel A since $A \neq 0$ since if $A = 0$ then $\tan^{-1}$ argument can't be computed, and if $A=0$, then $0\cos(x)$ can never equal $5\times10^4\cos(x)$

$A\sqrt{(2\pi f)^2 + (2.25\times10^8)} = 5\times10^4$ $\theta + 2\pi(2000)t + \tan^{-1}\left(\frac{2\pi f}{\frac{3}{2}\times10^4}\right) = 2\pi(2000)t$

$A = \frac{5\times10^4}{\sqrt{(2\pi\cdot2000)^2 + (2.25\times10^8)}}$ $\theta = -\tan^{-1}\left(\frac{2\pi f}{\frac{3}{2}\times10^4}\right)$

$A = 2.5552$ $\theta = -39.955°$

$\boxed{V_c(t) = 2.555\cos(2\pi(2000)t - 39.96°)\,[V]}$

Procedure

To begin the lab, we first made a list of the materials that we would need to complete the lab. We then constructed the circuits shown in figures 1 and 2, see below. We then simulated the circuits in PSPICE and placed voltage probes at the different nodes, marked with channel 1 and channel two. This was done to find the information necessary to discern the voltage response of the components of the circuit. We used the measurement tools to find information to build the response such as the amplitude of the input signals, the periods of the waves, and two times where the signal equaled zero which were in the same period and either both decreasing or increasing. We found the phase shift by using the equation $(t_S - t_c) * 360/T$, where T is the period of the signal, and t_S is the time where the voltage source equaled zero and t_c is when the component equaled zero. These calculations were performed and displayed in PSPICE for convenience. We then demonstrated this process to the professor.

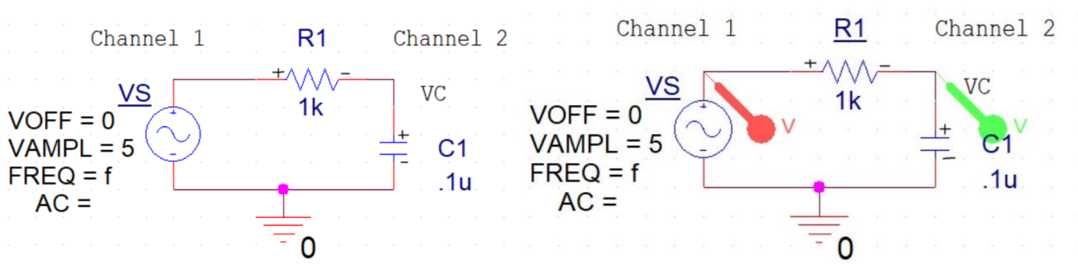


Figure 1: Circuit 1 with an AC power source and 1k Ohm resistor and .1 micro farad capacitor and version with probes

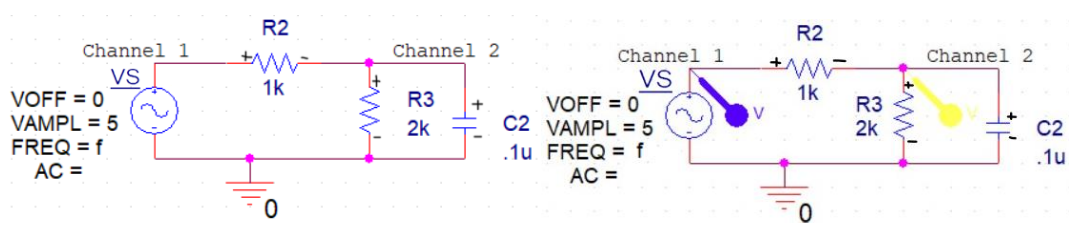


Figure 2: Circuit 1 with an AC power source and 1k & 2k Ohm resistor and .1 micro farad capacitor

Results

The oscilloscope pictures have been altered to display the channel numbers that correspond with the circuit, other than that, no other alterations have been made. To represent waiting for steady state, we took measurements from time periods once steady state had been reached.

Circuit 1: f= 500 HZ

Oscilloscope:

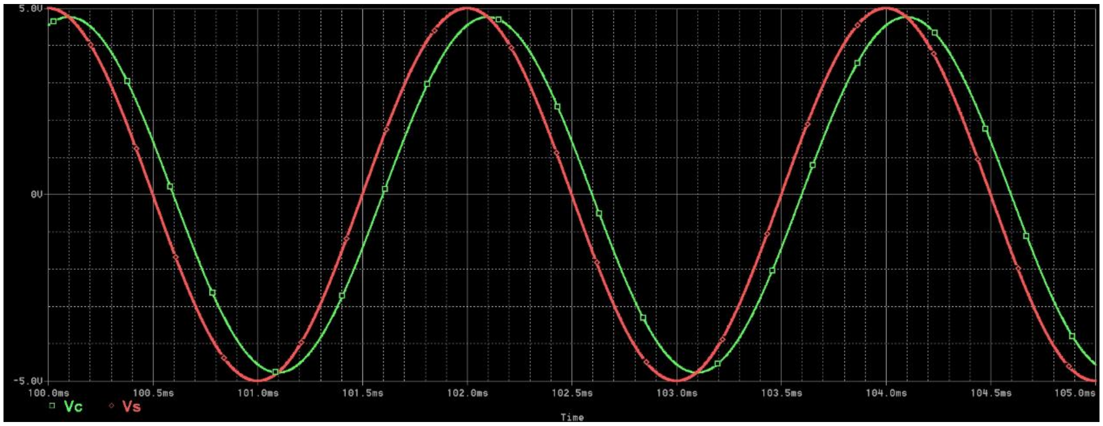


Figure 3: Vs and Vc when f= 500 Hz

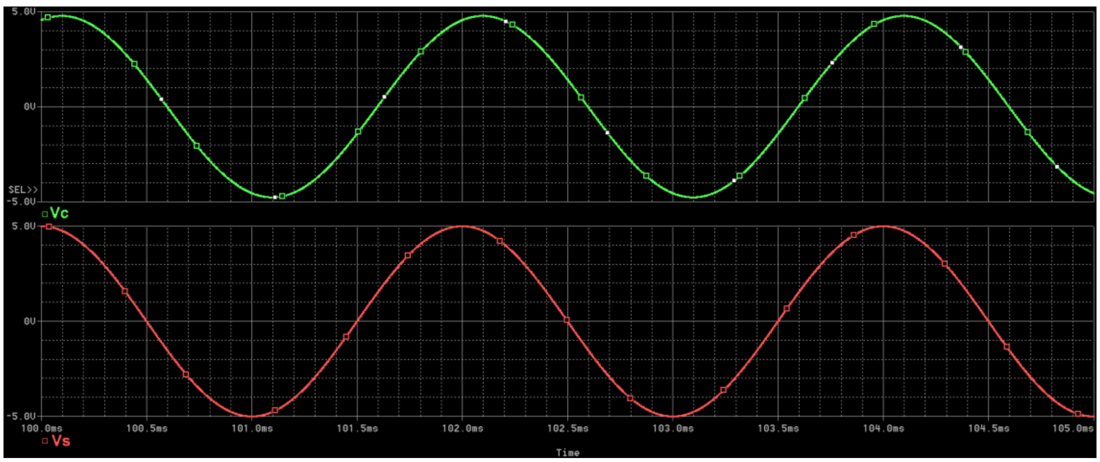


Figure 4: Vs and Vc separately when f= 500 Hz

Measurements:

Evaluate	Measurement	Value
☑	XatNthY(V(C1:2),0,1)	100.59689m
☑	Max_XRange(V(VS:+),100ms,1s)	5
☑	Max_XRange(V(C1:2),100ms,1s)	4.77014
☑	Period(V(R1:2))	2.00000m
☑	Period(V(VS:+))	2.00000m
☑	XatNthY(V(VS:+),0,1)	100.50000m
☑	1/Period(V(R1:2))	500.00000
☑	1/Period(V(VS:+))	500.00000
☑	(XatNthY(V(VS:+),0,1) - XatNthY(V(...	-17.44061

Figure 5: Measurement Results when f = 500 Hz

Characteristic	Measured Value
V_S Amplitude	5V
V_C Amplitude	4.77014V
V_C Period	2 ms
V_C Frequency	500 Hz
Time Where V_S=0 From Max (Same Period)	100.50000ms

Time Where V_C=0 From Max (Same Period)	100.59689ms
Phase Angle V_C to V_S	-17.44061^0

Table 2: Measurements of circuit 1 when f = 500 Hz

$V_S = 5\cos(2\pi*500*t)$ [V] $V_C = 4.770\cos(2\pi*500t - 17.44^0)$ [V]

Circuit 1: f= 1500 HZ

Oscilloscope:

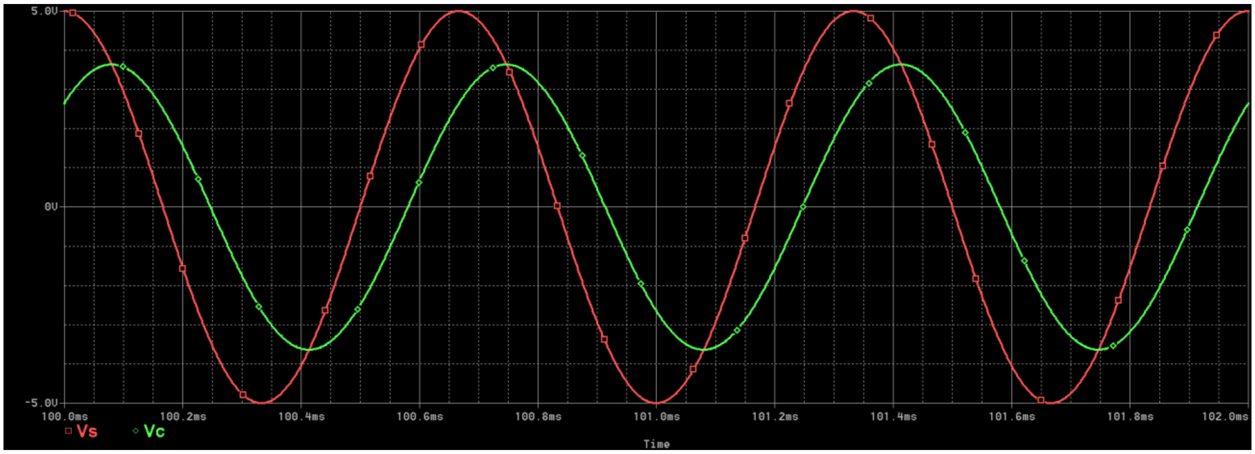


Figure 6: Vs and Vc when f= 1500 Hz

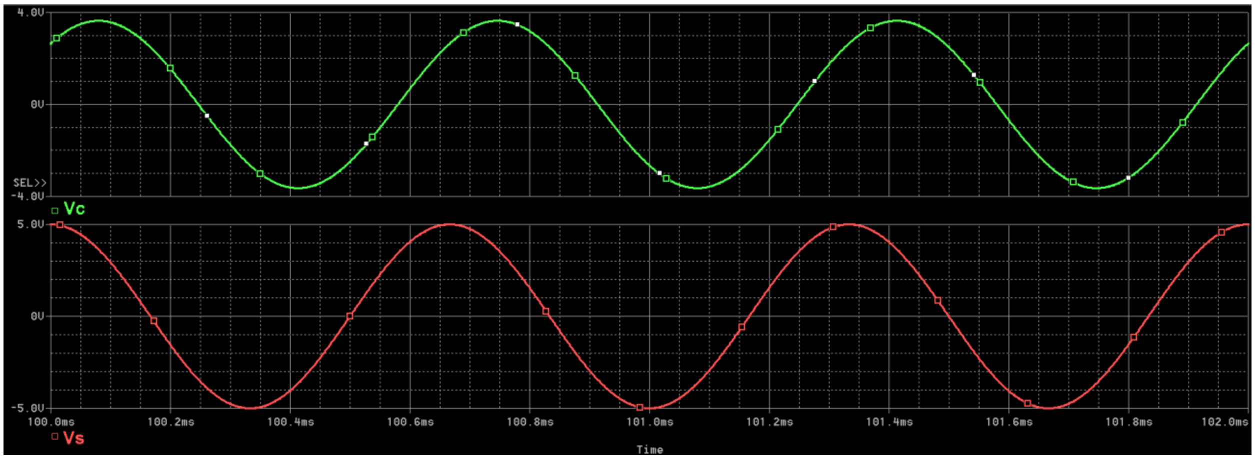


Figure 7: Vs and Vc separately when f= 1500 Hz

Measurements:

Evaluate	Measurement	Value
☑	XatNthY(V(C1:2),0,1)	100.24686m
☑	Max_XRange(V(VS:+),100ms,1s)	5
☑	Max_XRange(V(C1:2),100ms,1s)	3.63862
☑	Period(V(R1:2))	666.66667u
☑	Period(V(VS:+))	666.66667u
☑	XatNthY(V(VS:+),0,1)	100.16667m
☑	1/Period(V(R1:2))	1.50000k
☑	1/Period(V(VS:+))	1.50000k
☑	(XatNthY(V(VS:+),0,1) - XatNthY(V(...	-43.30401

Figure 8: Measurement Results when f = 1500 Hz

Characteristic	Measured Value
V_S Amplitude	5V
V_C Amplitude	3.63862V
V_C Period	666.66667μs
V_C Frequency	1500 Hz
Time Where V_S=0 From Max (Same Period)	100.16667 ms
Time Where V_C=0 From Max (Same Period)	100.24686 ms
Phase Angle V_C to V_S	-43.30401 ^{0}

Table 3: Measurements of circuit 1 when f = 1500 Hz

$V_S = 5\cos(2\pi*1500*t)$ [V]

$V_C = 3.639\cos(2\pi*1500t - 43.30\,^0)$ [V]

Circuit 1: f= 4000 HZ

Oscilloscope:

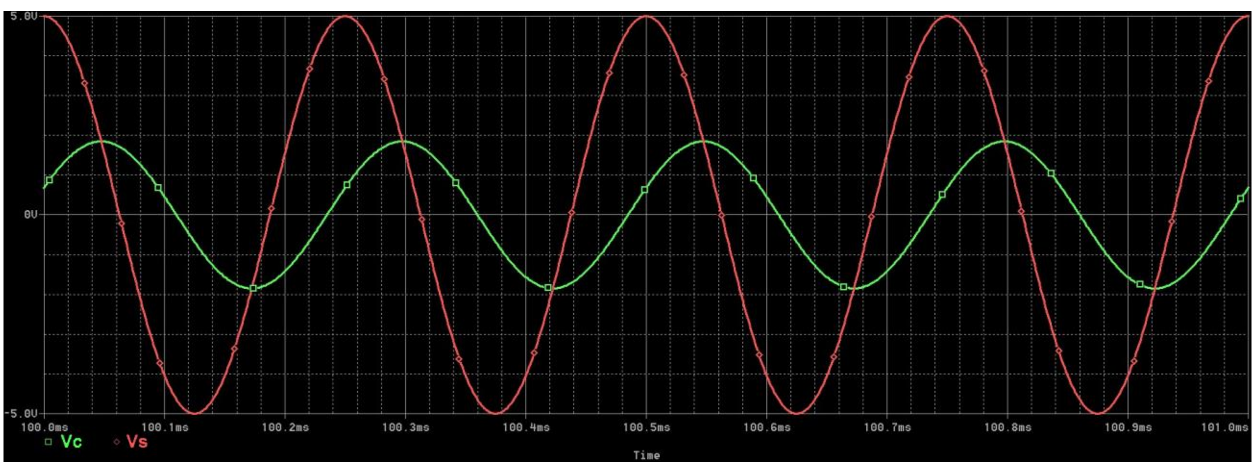


Figure 9: Vs and Vc when f= 4000 Hz

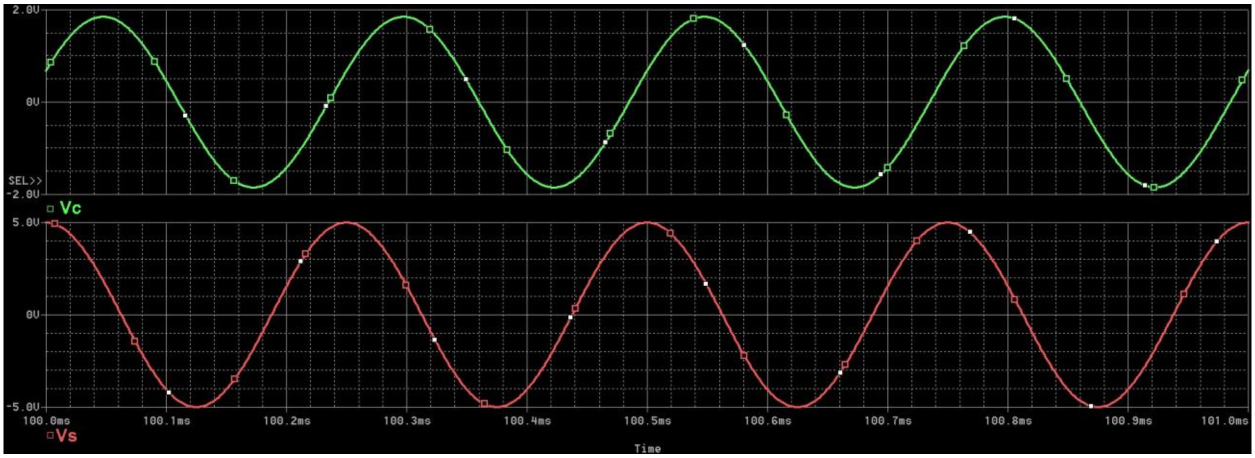


Figure 10: Vs and Vc separately when f= 4000 Hz

Measurements:

	Evaluate	Measurement	Value
	☑	XatNthY(V(C1:2),0,1)	100.10993m
	☑	Max_XRange(V(VS:+),100ms,1s)	5
	☑	Max_XRange(V(C1:2),100ms,1s)	1.84841
	☑	Period(V(R1:2))	249.99999u
	☑	Period(V(VS:+))	250.00000u
	☑	XatNthY(V(VS:+),0,1)	100.06250m
	☑	1/Period(V(R1:2))	4.00000k
	☑	1/Period(V(VS:+))	4.00000k
▶	☑	(XatNthY(V(VS:+),0,1) - XatNthY(V(...	-68.30397

Figure 11: Measurement Results when f = 4000 Hz

Characteristic	**Measured Value**
V_S Amplitude	5V
V_C Amplitude	1.84841V
V_C Period	249.99999µs
V_C Frequency	4000 Hz
Time Where V_S=0 From Max (Same Period)	100.06250ms
Time Where V_C=0 From Max (Same Period)	100.10993ms
Phase Angle V_C to V_S	-68.30397^0

Table 3: Measurements of circuit 1 when f = 4000 Hz

$V_S = 5\cos(2\pi*4000*t)$ [V]

$V_C = 1.848\cos(2\pi*4000t–68.30^0)$ [V]

Circuit 2: f= 1000 HZ

Oscilloscope:

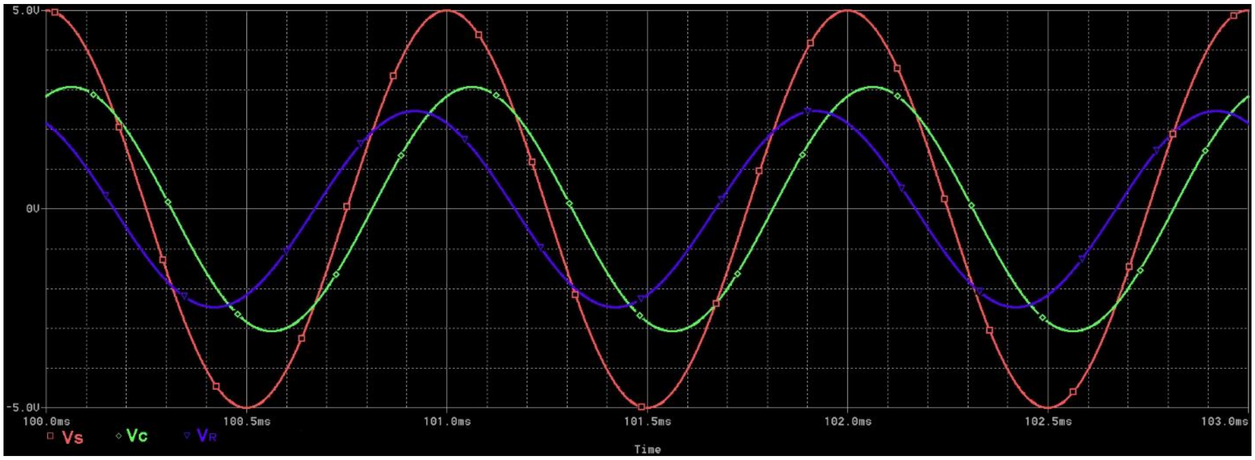


Figure 12: Vs, Vc, and V_R when f= 1000 Hz

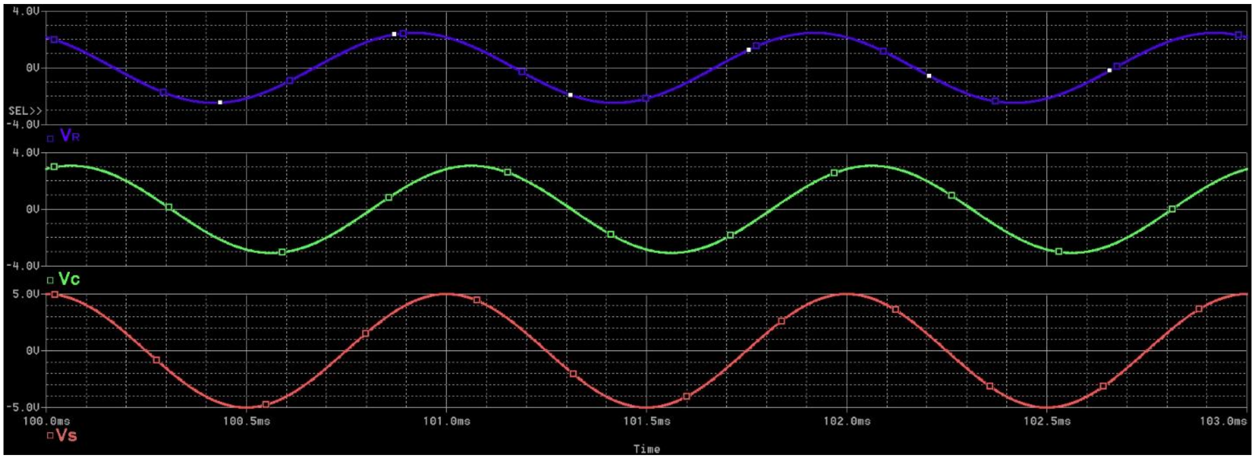


Figure 13: Vs, Vc, and V_R separately when f= 1000 Hz

Measurements:

Measurement	Value
XatNthY(V(C2:2),0,1)	100.31313m
Max_XRange(V(V2:+),100ms,1s)	5
Max_XRange(V(C2:2),100ms,1s)	3.07450
Period(V(C2:2))	1.00000m
Period(V(V2:+))	1.00000m
XatNthY(V(V2:+),0,1)	100.25000m
1/Period(V(C2:2))	1000.00000
1/Period(V(V2:+))	1000.00000
(XatNthY(V(V2:+),0,1) - XatNthY(V(C2:2),0,1))*360 /(Period_XRange(V(V2:+),100ms,1s))	-22.72785
Max_XRange((V(V2:+)-V(C2:2)),100ms,1s)	2.46878
XatNthY((V(V2:+)-V(C2:2)),0,1)	100.17011m
Period((V(V2:+)-V(C2:2)))	1.00000m
1/Period((V(V2:+)-V(C2:2)))	1000.00000
(XatNthY(V(V2:+),0,1) - XatNthY((V(V2:+)-V(C2:2)),0,1))*360 /(Period_XRange(V(V2:+),100ms,1s))	28.76035

Figure 14: Measurement Results when f = 1000 Hz

Characteristic	Measured Value
V_S Amplitude	5V
V_C Amplitude	3.07450V

V_R Amplitude	2.46878V
V_C Period	1.00000ms
V_R Period	1.00000ms
V_C Frequency	1000 Hz
V_R Frequency	1000 Hz
Time Where V_S=0 From Max (Same Period)	100.25000ms
Time Where V_C=0 From Max (Same Period)	100.31313ms
Time Where V_R=0 From Max (Same Period)	100.17011ms
Phase Angle V_C to V_S	-22.72785^0
Phase Angle V_R to V_S	28.76035^0

Table 4: Measurements of circuit 2 when f = 1000 Hz

$V_S = 5\cos(2\pi*1000*t)$ [V] $V_C = 3.075\cos(2\pi*1000t - 22.73^0)$ [V]

$V_R = 2.47\cos(2\pi*1000t + 28.76^0)$ [V]

Circuit 2: f= 2000 HZ

Oscilloscope:

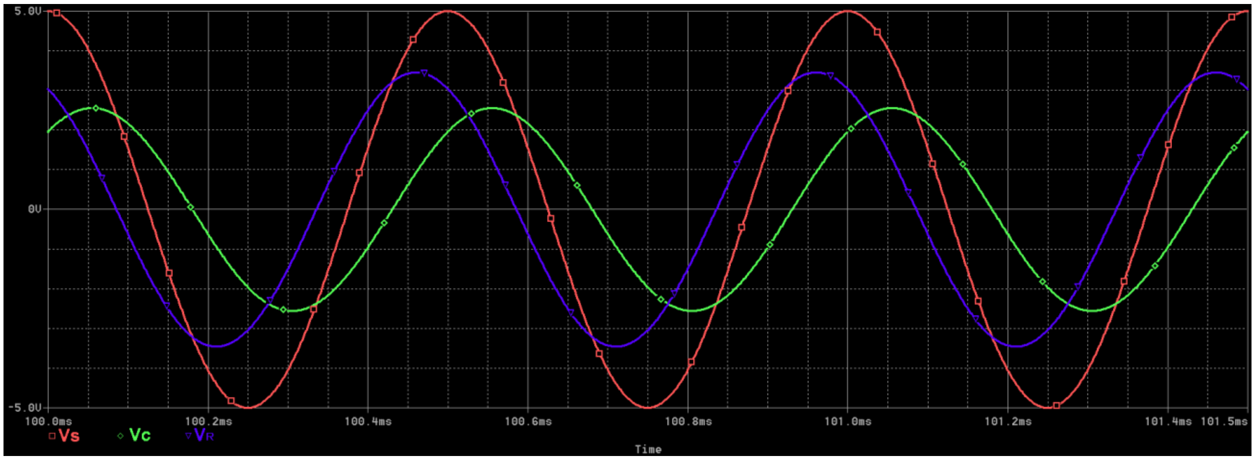


Figure 15: Vs, Vc, and V_R when f= 2000 Hz

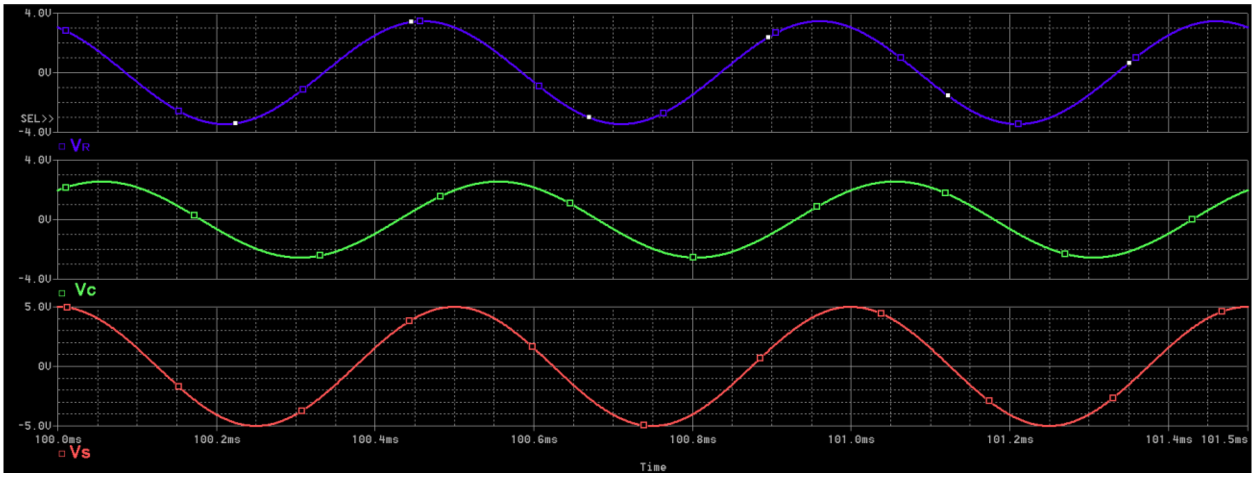


Figure 16: Vs, Vc, and V_R separately when f= 2000 Hz

Measurements:

Evaluate	Measurement	Value
☑	XatNthY(V(C2:2),0,1)	100.18049m
☑	Max_XRange(V(V2:+),100ms,1s)	5
☑	Max_XRange(V(C2:2),100ms,1s)	2.55515
☑	Period(V(C2:2))	500.00000u
☑	Period(V(V2:+))	500.00000u
☑	XatNthY(V(V2:+),0,1)	100.12500m
☑	1/Period(V(C2:2))	2.00000k
☑	1/Period(V(V2:+))	2.00000k
☑	(XatNthY(V(V2:+),0,1) - XatNthY(V(C2:2),0,1))*360 /(Period_XRange(V(V2:+),100ms,1s))	-39.95522
☑	Max_XRange((V(V2:+)-V(C2:2)),100ms,1s)	3.45577
☑	XatNthY((V(V2:+)-V(C2:2)),0,1)	100.08563m
☑	Period((V(V2:+)-V(C2:2)))	500.00000u
☑	1/Period((V(V2:+)-V(C2:2)))	2.00000k
☑	(XatNthY(V(V2:+),0,1) - XatNthY((V(V2:+)-V(C2:2)),0,1))*360 /(Period_XRange(V(V2:+),100ms,1s))	28.34801

Figure 17: Measurement Results when f = 2000 Hz

Characteristic	**Measured Value**
V_S Amplitude	5V
V_C Amplitude	2.55515V
V_R Amplitude	3.45577V
V_C Period	500µs
V_R Period	500µs
V_C Frequency	2000 Hz
V_R Frequency	2000 Hz
Time Where V_S=0 From Max (Same Period)	100.12500ms
Time Where V_C=0 From Max (Same Period)	100.18049ms
Time Where V_R=0 From Max (Same Period)	100.08563ms
Phase Angle V_C to V_S	-39.95522^0
Phase Angle V_R to V_S	28.34801^0

Table 5: Measurements of circuit 2 when f = 2000 Hz

$V_S = 5\cos(2\pi*2000*t)$ [V]

$V_C = 2.555\cos(2\pi*2000t - 39.96^0)$ [V]

$V_R = 3.46\cos(2\pi*2000t + 28.35^0)$ [V]

<u>Circuit 2: f= 4000 HZ</u>

Oscilloscope:

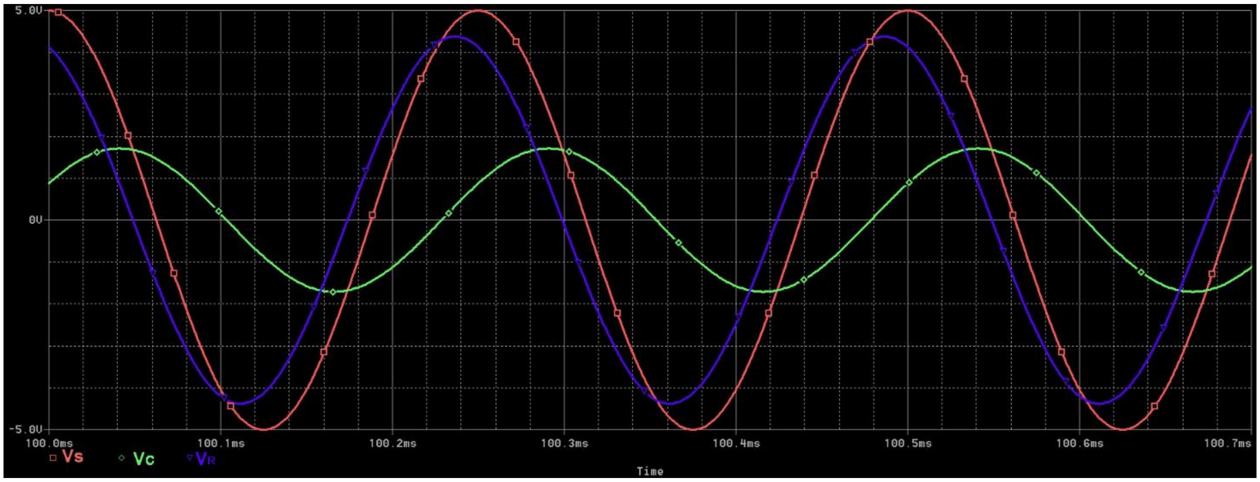


Figure 18: Vs, Vc, and V_R when f= 4000 Hz

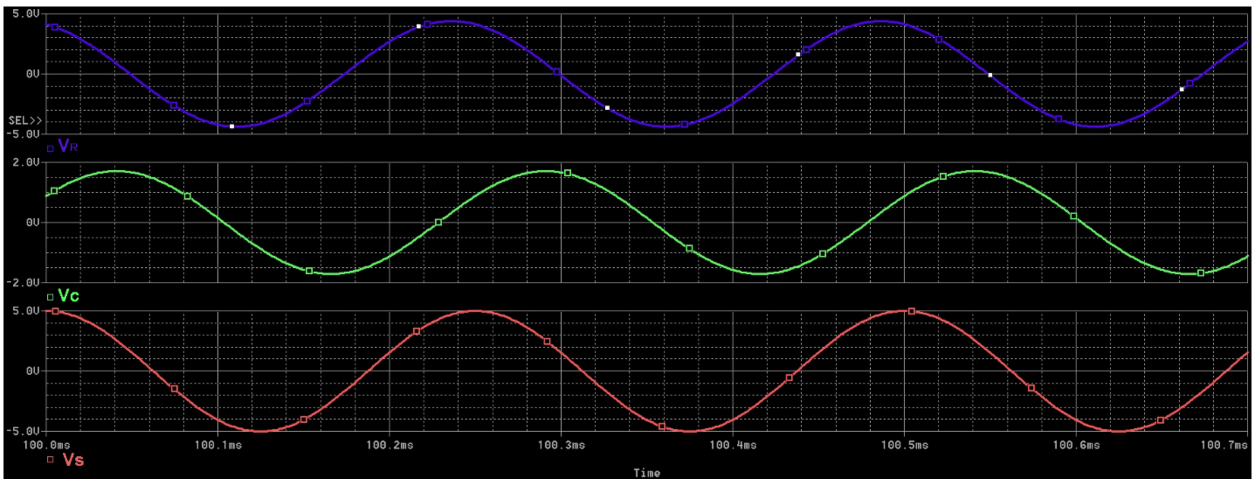


Figure 19: Vs, Vc, and V_R separately when f= 4000 Hz

Measurements:

Measurement	Value
XatNthY(V(C2:2),0,1)	100.10359m
Max_XRange(V(V2:+),100ms,1s)	5
Max_XRange(V(C2:2),100ms,1s)	1.70825
Period(V(C2:2))	249.99999u
Period(V(V2:+))	250.00000u
XatNthY(V(V2:+),0,1)	100.06250m
1/Period(V(C2:2))	4.00000k
1/Period(V(V2:+))	4.00000k
(XatNthY(V(V2:+),0,1) - XatNthY(V(C2:2),0,1))*360 /(Period_XRange(V(V2:+),100ms,1s))	-59.17119
Max_XRange((V(V2:+)-V(C2:2)),100ms,1s)	4.37765
XatNthY((V(V2:+)-V(C2:2)),0,1)	100.04890m
Period((V(V2:+)-V(C2:2)))	250.00001u
1/Period((V(V2:+)-V(C2:2)))	4.00000k
(XatNthY(V(V2:+),0,1) - XatNthY((V(V2:+)-V(C2:2)),0,1))*360 /(Period_XRange(V(V2:+),100ms,1s))	19.57751

Figure 20: Measurement Results when f = 4000 Hz

Characteristic	**Measured Value**
V_S Amplitude	5V

V_C Amplitude	1.70825V
V_R Amplitude	4.37765V
V_C Period	249.99999µs
V_R Period	250.00001µs
V_C Frequency	4000 Hz
V_R Frequency	4000 Hz
Time Where V_S=0 From Max (Same Period)	100.06250ms
Time Where V_C=0 From Max (Same Period)	100.10359ms
Time Where V_R=0 From Max (Same Period)	100.04890ms
Phase Angle V_C to V_S	-59.17119^0
Phase Angle V_R to V_S	19.57751^0

Table 6: Measurements of circuit 2 when f = 4000 Hz

$V_S = 5 \cos(2\pi*4000*t)$ [V]

$V_C = 1.708\cos(2\pi*4000t - 59.17^0)$ [V]

$V_R = 4.378\cos(2\pi*4000t + 19.58^0)$ [V]

Problems

1.

Percent difference was calculated using the following equation

$$\frac{|V_{C\ Theortical} - V_{C\ Simulated}|}{V_{C\ Theortical}} * 100$$

For phase angle, we substituted the phase angle values for the voltage.

	Theoretical	Simulated	Percent Difference
Vc Amplitude	3.6386 V	3.63862V	5.5×10^{-4} %
Vc Phase Angle	-43.304^0	-43.30401^0	2.3×10^{-5} %

Table 7: Comparisons of circuit 1 when f = 1500 Hz

	Theoretical	Simulated	Percent Difference
Vc Amplitude	2.5552V	2.55515V	1.96×10^{-3} %
Vc Phase Angle	-39.955^0	-39.95522^0	5.51×10^{-4} %

Table 8: Comparisons of circuit 2 when f = 2000 Hz

The percent difference between the theoretical and simulated values for the voltage across the capacitor and the phase shift of the voltage compared to the voltage source is small. This is because the simulated values are calculated similarly to the theoretical calculations and thus would not include factors such as noise. If we had access to the materials in a lab environment, the percent different would have been much greater than what we found.

2.

As the frequency increased, the magnitude of V_C decreased and if the frequency had increased towards infinity, the capacitor would have acted as a wire with no resistance. As frequency increases, the phase shift of V_C decreases and becomes more negative.

Question #3

$V_{in} = 5\cos(2\pi(3000)t)$

$V_R = Ri(t)$

$A_s = 5\text{ V}$ $\quad f = 3000\text{ Hz}$

$V_c + V_R = A_s\cos(2\pi f t)$

$\left(\frac{1}{c}\int_0^t i\,dt + V_{c_0} + V_R\right)dt = \left(A_s\cos(2\pi f t)\right)dt$

$\frac{1}{c}i(t) + \frac{dV_R}{dt} = -(A_s)(2\pi f)\sin(2\pi f t) = A_s(2\pi f)\cos(2\pi f t + 90°)$

Assuming a solution of the form $A\cos(2\pi f t + \phi)$

$\frac{A}{RC}\cos(2\pi f t + \phi) - A(2\pi f)\sin(2\pi f t + \phi) = A_s(2\pi f)\cos(2\pi f t + 90°)$

Using the trig identity $C\sin(x) + B\cos(x) = \sqrt{B^2 + C^2}\cos\left(x + \tan^{-1}\left(-\frac{C}{B}\right)\right)$

$\sqrt{\left(\frac{A}{RC}\right)^2 + (A\,2\pi f)^2}\cos\left(2\pi f(t) + \tan^{-1}\left(\frac{2\pi f}{1/RC}\right) + \phi\right) = A_s(2\pi f)\cos(2\pi f t + 90°)$

$A\sqrt{(1/RC)^2 + (2\pi f)^2} = A_s(2\pi f)$, $\quad A = \frac{A_s(2\pi f)}{\sqrt{(1/RC)^2 + (2\pi f)^2}}$

$\boxed{A = 4.4169\text{ V}}$

$\cancel{2\pi f(t)} + \tan^{-1}\left(\frac{2\pi f}{1/RC}\right) + \phi = \cancel{2\pi f(t)} + 90°$

$\phi = 90° - \tan^{-1}\left(\frac{2\pi f}{1/RC}\right) = 27.9467°$

$$\boxed{V_R(t) = 4.417\cos(2\pi(3000)t + 27.95°)}$$