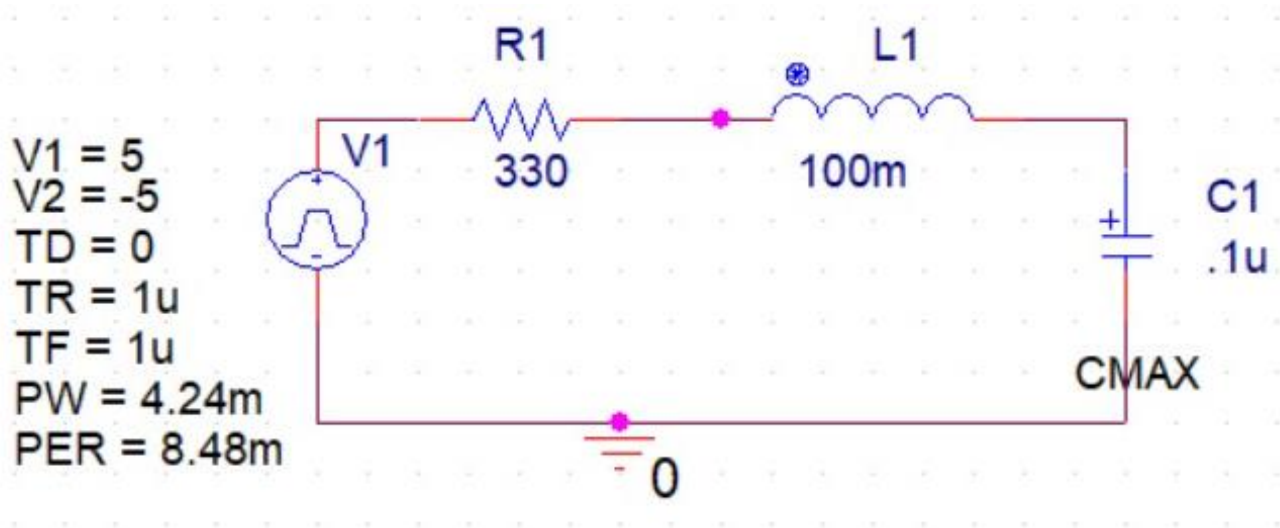
ECE 2101L Lab #3

Transient Response Of 2nd Order Underdamped Circuits



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October 3rd, 2020

Objective

The objective of the lab is to explore the transient response of a 2nd order circuit, more specific an underdamped series RLC circuit and active circuit. We also are to gain experience in using periodic inputs to better study the responses of a circuit, using information derived from the circuit. We are to investigate how the response of an RLC circuit changes as the resistance changes.

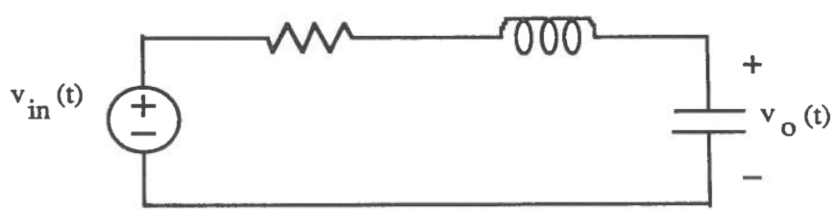
Materials

The necessary equipment needed for the lab are as follows

1. Breadboard
2. 4 BNC Clip Connectors
3. Clip Leads
4. 0.1 & .01 μF Capacitor
5. 330Ω & Two 10k Ω Resistors
6. 100mH inductor
7. Op Amp
8. LCR Meter
9. Digital Multimeter
10. Oscilloscope
11. Function Generator

Pre-Lab

1.

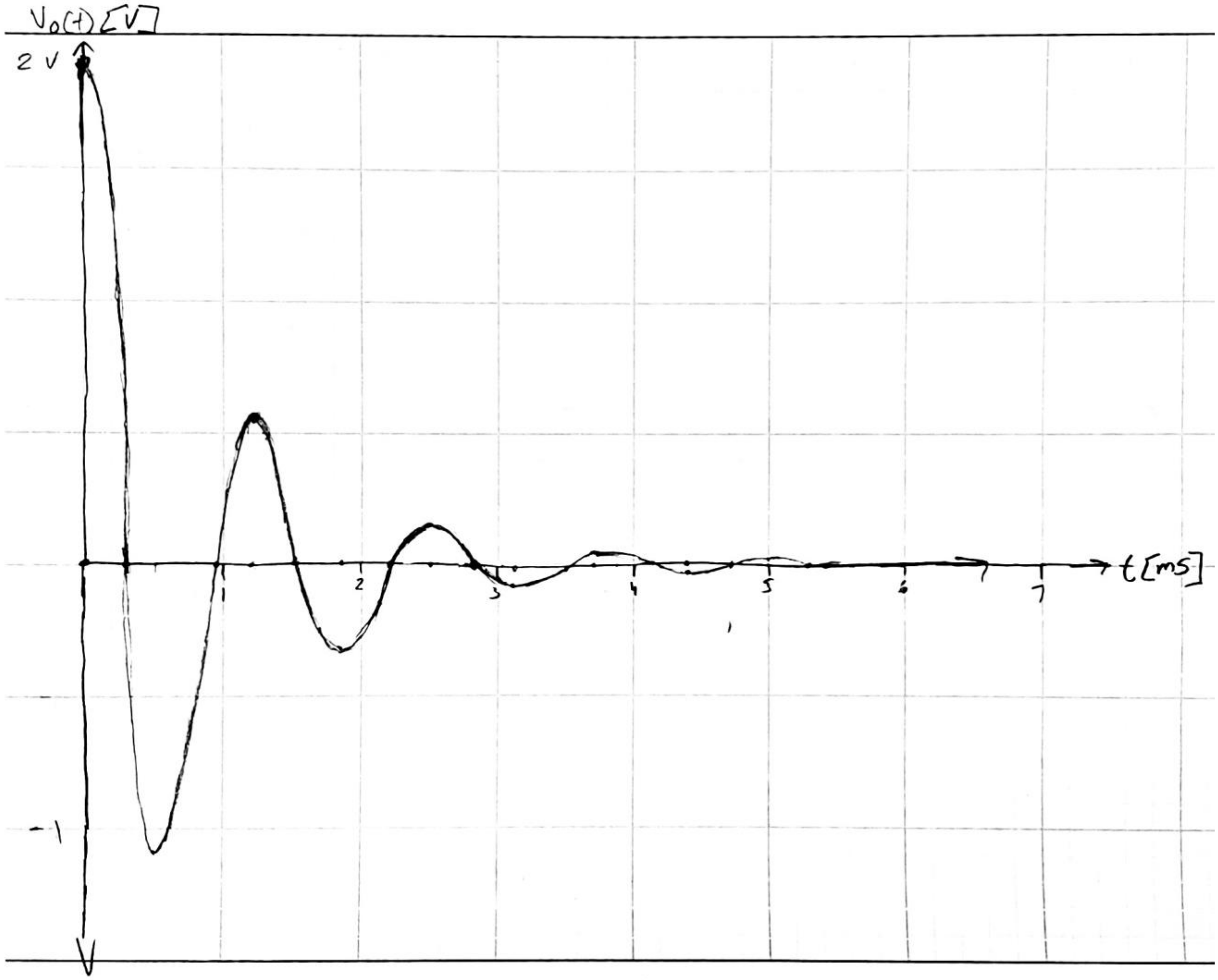


$$v_o(t) = 2e^{-100t}\cos 500t$$

Figure 1: Circuit Diagram

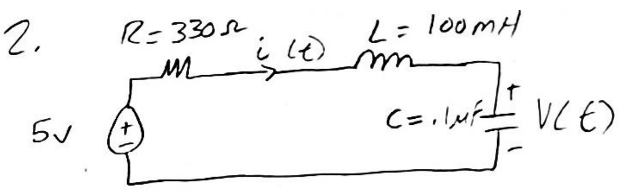
The transient response of v0(t) will take approximately five milliseconds, this was found by using the standard of five time constants to estimate the decay of an exponential, however, seven time constants are also commonly used. Approximately 3.98 cycles will occur before the waveform will decay to small extremely small amplitudes. A good frequency for a square wave would be for it to fully allow the response to decay, thus seven time constants will be used just for good measure. Since the wave will have the negative and positive component we would want the square wave to be at a rate of 7.1428mHz to allow seven time constants for the positive portion of the square wave and seven for the negative portion of the square wave.

Sketch of $v_0(t)$



Graph 1: Sketch of $V_0(t)$ from 0ms to 7ms

2.



$V_c(0^-) = -5\text{ V}$ $\quad v'(0) = 0 \text{ V/s}$

$i_L(0^-) = 0\text{ A}$

a. $i(R) + L\, di/dt + V(t) = 5$

$i = C\frac{dV}{dt}$, $\quad di/dt = C\frac{d^2V}{dt^2}$, $\quad V' = \frac{i}{C}$, $\quad v'(0) = \frac{i(0)}{C}$

$RCV' + LCV'' + V = 5$

$V'' + \frac{R}{L}V' + \frac{1}{LC}V = \frac{5}{LC}$

Which is the same as:

$\ddot{V}_c + \frac{R}{L}\dot{V}_c + \frac{1}{LC}V_c = \frac{5}{LC}$

b. $V'' + 3.3\times10^3 V' + 10^8 V_c = 5\times10^8$

$s^2 + 3.3\times10^3 s + 10^8 = 0$

$s_1, s_2 = \frac{-3.3\times10^3}{2} \pm \frac{\sqrt{(3.3\times10^3)^2 - (4)(10^8)}}{2}$

$s_1, s_2 = -1650 \pm j\,9862.94$, $\alpha = -1650$ and $W_n = 9862.94$

V takes the form of an underdamped response which is

$V = V_n + V_f = Ke^{\alpha t}\cos(W_n t + \theta) + A$

$0 + 3.3\times10^3(0) + 10^8(A) = 5\times10^8$, $A = 5$

$V(t) = Ke^{at}\cos(W_n t + \theta) + 5$

$V(0)$: $K\cos(\theta) + 5 = -5$, $\quad K\cos(\theta) = -10$

$V'(t) = aKe^{at}\cos(W_n t + \theta) + (Ke^{at})(-\sin(W_n t + \theta)W_n)$

$V'(0) = aK\cos(\theta) + -W_n K\sin(\theta) = 0$

$-1650K\cos(\theta) = 9862.94K\sin(\theta)$

$\tan\theta = \frac{-1650}{9862.94}$, $\theta = -9.50°$ $\left\{ K = \frac{-10}{\cos(-9.5°)}, \right.$ $K = -10.139$

$$\boxed{V(t) = -10.139e^{-1650t}\cos(9862.94t - 9.5°) + 5 \text{ [V]}}$$

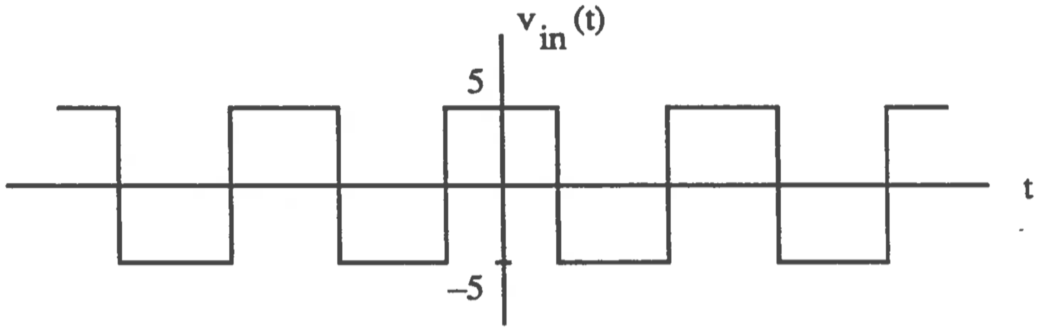
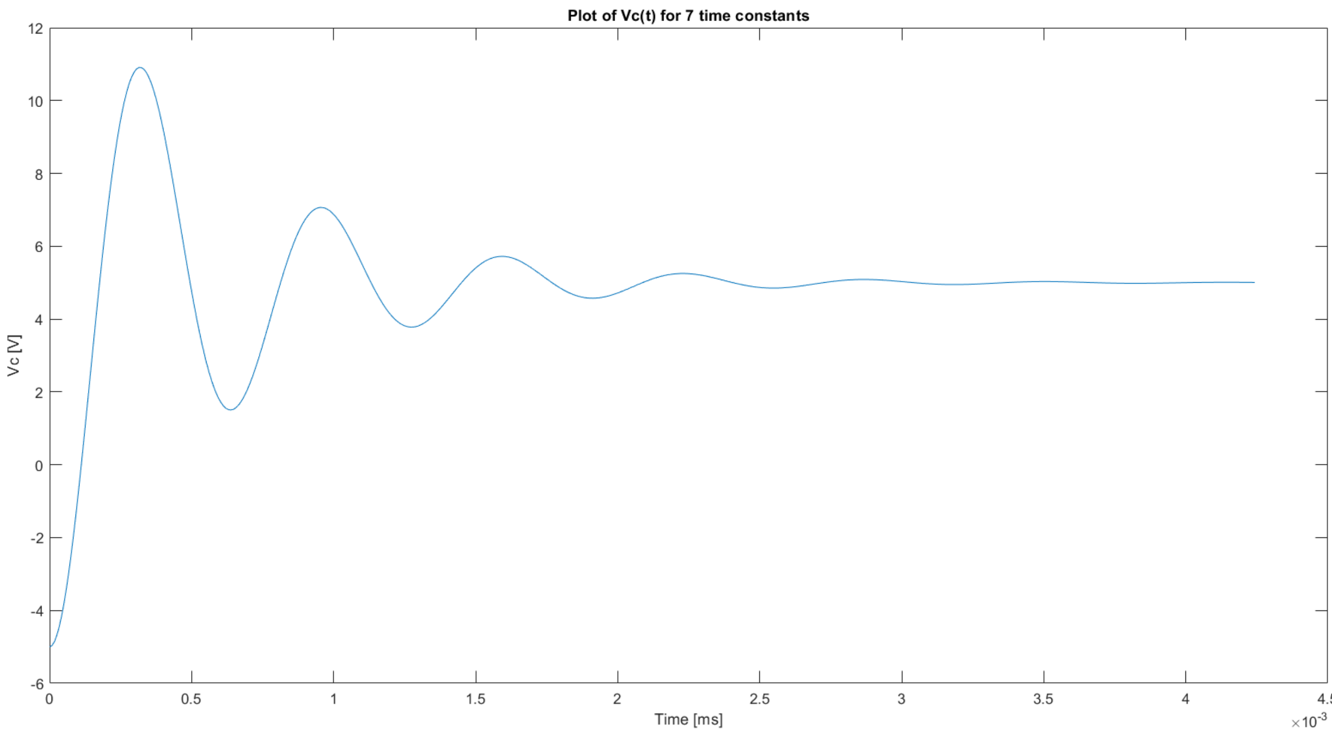


Figure 2: 5V Square Wave Voltage Source Signal

If the voltage source was replaced with a square wave with amplitude 5V, then the frequency should be set to 117.857 Hz to allow for seven time constants at 5V and seven time constants at – 5V.



Graph 2: Vc(t) Plot For 7 τ

Procedure

For the lab, we began by constructing the circuit found in figure three in the program PSPICE and then running the simulation for multiple cycles for the voltage source to have plentiful data to analyze. For the series RLC circuit we move the cursor to the peaks of the wave and measured the time and voltage, we picked a point to be our reference point for placing points on the graph from the prelab. To simulate a potentiometer, we used a parameter to represent the resistance of the resistor and when we ran the simulation we put various resistance values. We then constructed the active circuit in figure four and then used PSPICE to analyze the circuit and took the necessary data to estimate the roots of the characteristic equation. To find the roots of the charcateristic equation, we found the time differnce between two maxima and then using the period to find the frequency and then the natural frequency of the response. We also decrease the frequency of the voltage source so the transient response had more time to decay.

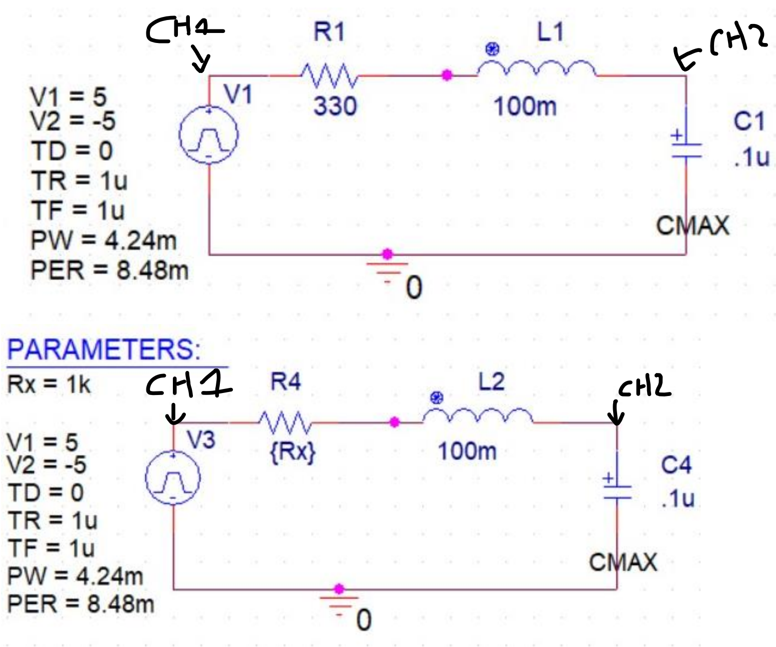


Figure 3: Series RLC Circuit

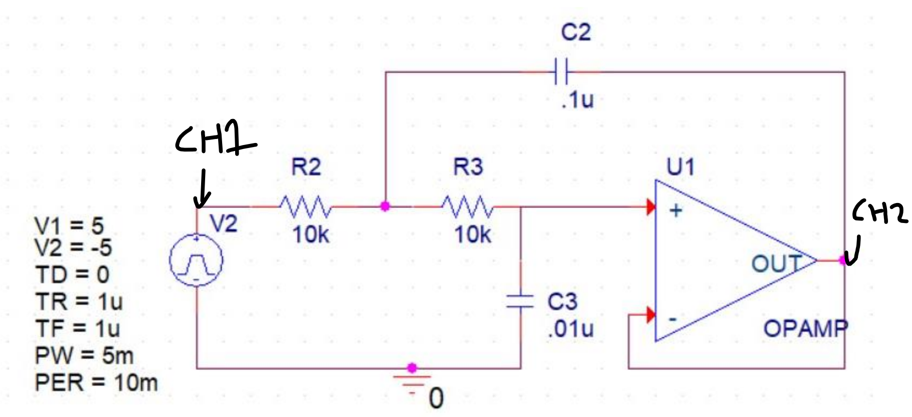


Figure 4: Active Circuit

Results

Series RLC Circuit:

When taking data for the table we used 4.22ms to represent t = 0 on the MATLAB graph, all points on that graph are based on this standard, however in the table we used the exact time from the simulation as well as the adjusted time.

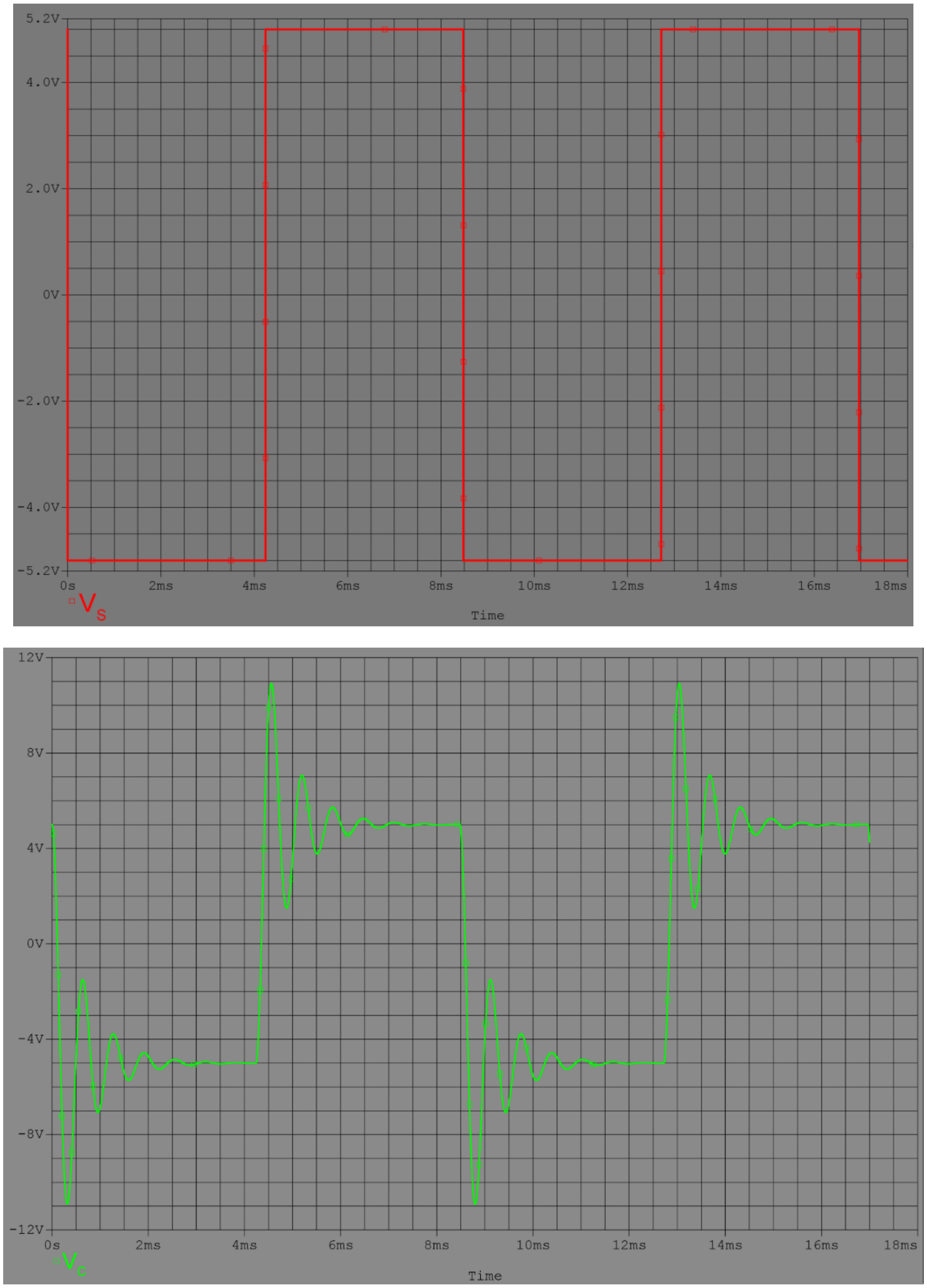


Figure 5: Response of the series RLC circuit's Vs & Vc(t)

$V_c(t)$ when the resistance is changed to various values:

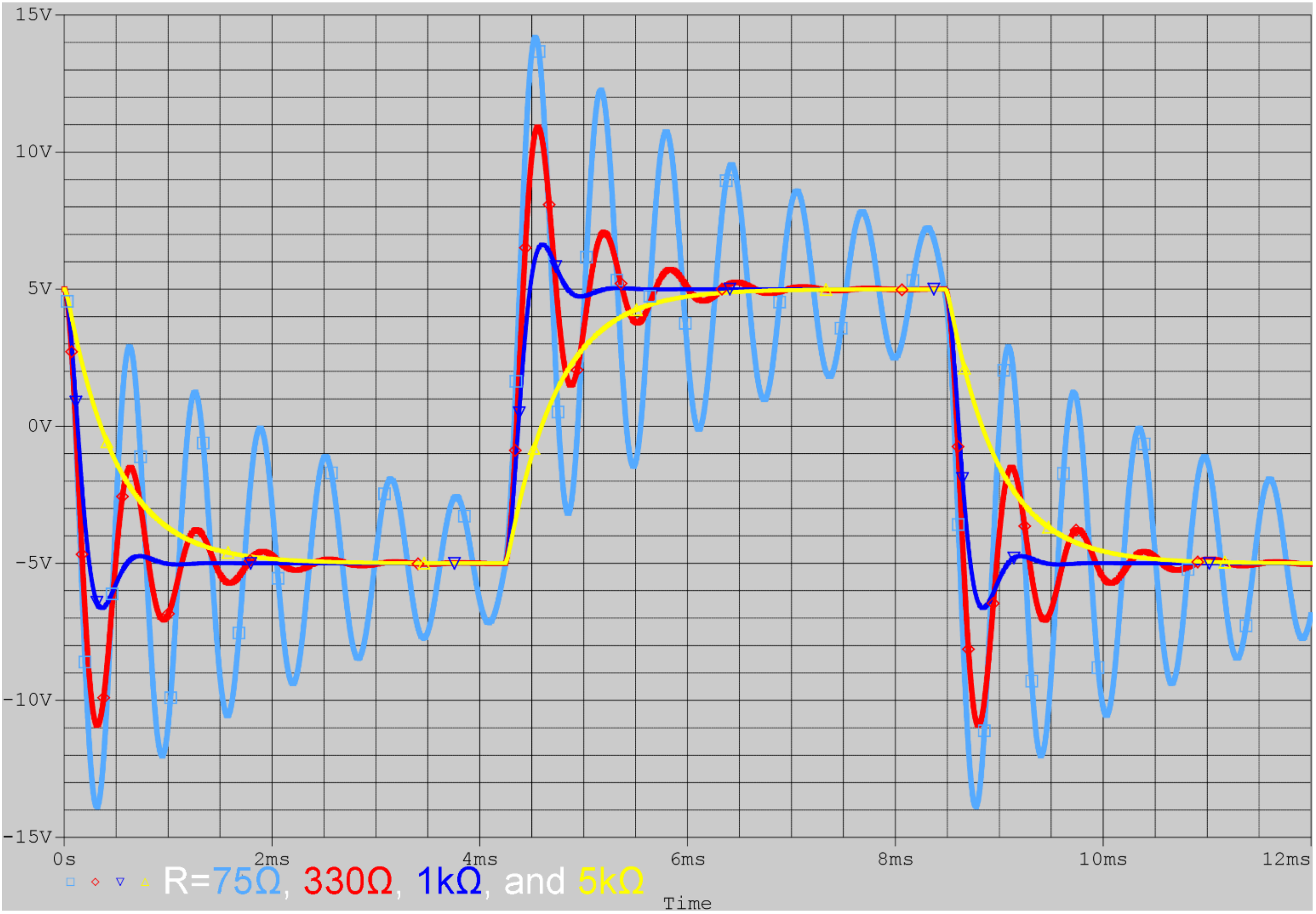


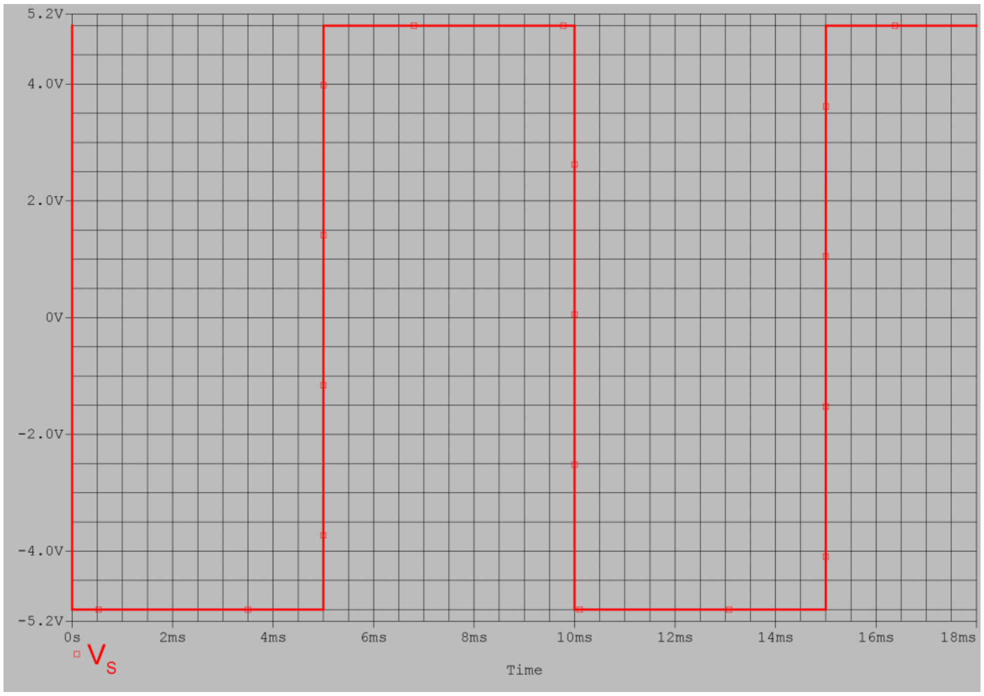
Figure 6: Responses of $V_c(t)$ when R= 75Ω, 330Ω, 1kΩ, and 5kΩ

Time From Simulation [ms]	Time For Graph [ms]	Voltage [V]
4.2449	0	-5.001
4.5599	0.315	10.916
4.879	0.6341	1.5024
5.197	0.9521	7.0679
5.516	1.2711	3.7774
5.834	1.5891	5.7228
6.153	1.9081	4.5727
6.471	2.2261	5.2527
6.79	2.5451	4.8506
7.735	3.4901	5.0307
8.303	4.0581	5.0072

Table 1: Table Compiling All Data Found From Simulation On Series RLC Circuit

Active Circuit:

The time the voltage source is at 5V or -5V was rounded to the nearest whole number, of it's magnitude, to allow for more of the response to be displayed.



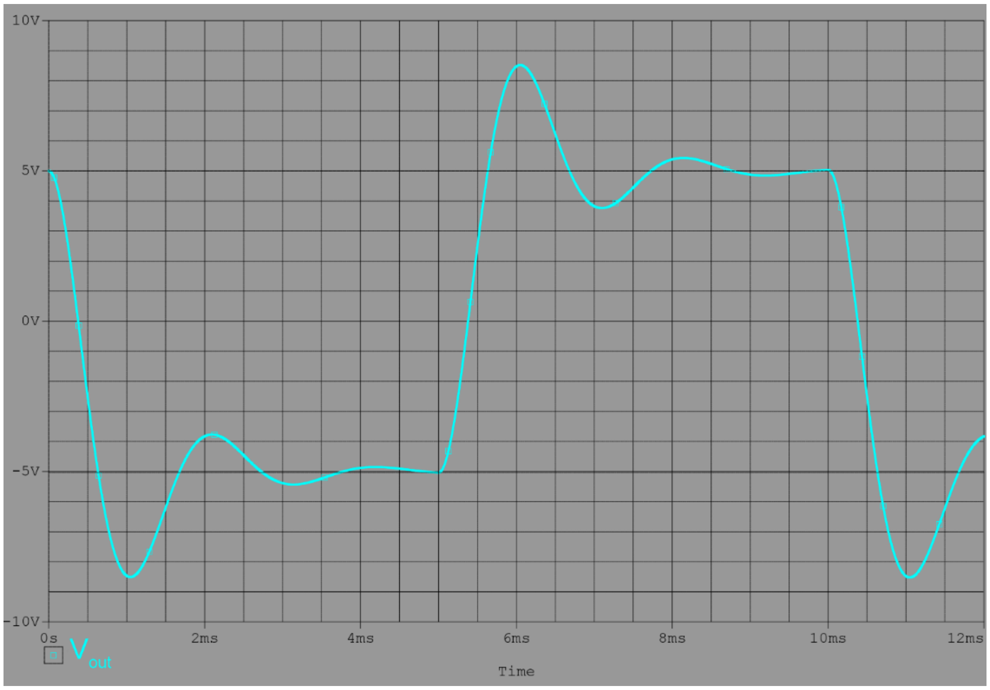


Figure 7: V_{out} for the active circuit graph

T1	T2	5τ	Period	Frequency	Natural Frequency	Dampening Factor
6.0502ms	8.1443ms	5.0010ms	2.0941ms	477.532 Hz	3000.4 rads/s	999.8 Np/s

Table 2: Table Compiling All Data Found From Active Circuit

General Analysis

For the series RLC circuit we noticed that the response is of the form of an underdamped circuit which conforms with the answer we got in the pre-lab. Also using 7τ was an adequate time measurement to show the complete transient response decay to the final value. When we used a variable, we noticed that as we changed the resistance, the response would change. As we increased the resistance the response would change to the overdamped at higher values of resistance, shown when R = 5kΩ. Overall it can be seen that as the resistor is at smaller values, the response of a series RLC circuit will be underdamped and at larger values of R, the response will be overdamped. As the resistance increased, the degree to which the sinusoid is present is diminished, and then becomes overdamped. It was also noticed that as the resistance became smaller, the initial amplitude of the waveform would increase.

The output of the active circuit is underdamped due to the oscillations seen as the responses dies to the final value of 5V or -5V. We found the dampening factor as well by estimating the time when the response was effectively at steady state, which we found to be 5.0001ms since the output was at a minimum at that time and was incredibly close to 5V. We used this base time as 5τ since our voltage was not exactly 5V it would be a better estimate to use as opposed to 7τ. When setting the frequency of the voltage source I was not aware that I had set it so that it would charge up or down for the length of roughly 5τ, which explains why the dampening factor is so close to the value derived in lecture.

Problems

1. Compare the simulated and calculated response of the circuit in figure 3.

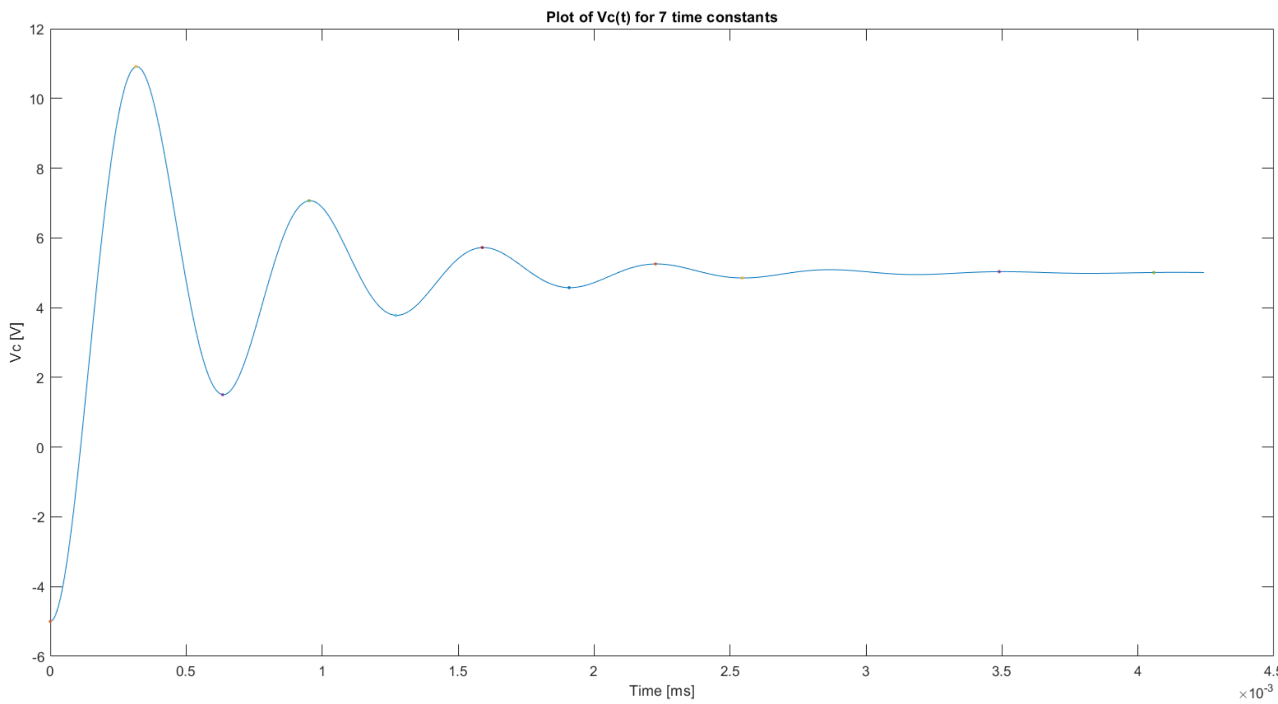


Figure 8: MATLAB Graph of Vc(t) With PSPICE Data

Voltage [V]	Theoretical Voltage [V]	Percent Difference %
-5.001	-4.9999	0.021
10.916	10.9085	0.069
1.5024	1.5061	0.248
7.0679	7.0653	0.037
3.7774	3.7788	0.036
5.7228	5.7219	0.016
4.5727	4.5731	0.009
5.2527	5.2523	0.007
4.8506	4.8508	0.004
5.0307	5.0306	0.003
5.0072	5.0070	0.005

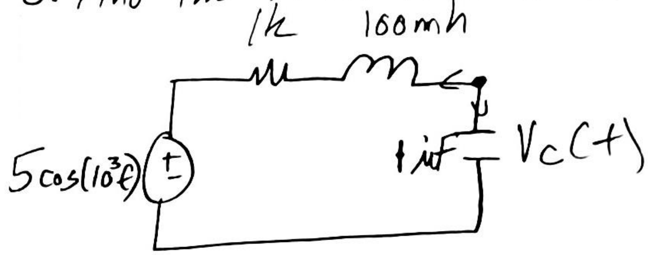
Table 3: Table Showing Percent Difference

To compare our simulated data to our theoretical, the maxima and minima were chosen to plot on our MATLAB graph from the prelab. The percent difference was also found in addition to plotting the simulated data on our previous graph, seen in table 3. We have small percent differences which is expected since we used a simulation and the percent difference that we have is the result of the precision of our data values as opposed to natural phenomena.

2. Estimate the natural frequency of the active circuit.

As seen in table 2, the natural frequency of the active circuit was estimated to 3000.4 rad/s, which is incredibly close to the natural frequency derived in the lecture, 1.33×10^2 percent difference. This was estimated by taking the time measurement of two maxima's that were next to each other and then finding the difference in time which would be our period. We then found the frequency by taking the inverse of the period and then multiplied it by 2π to find the natural frequency.

3. Find the forced response of $V_c(t)$



$$i = C\frac{dv_c}{dt}$$

$$1K\,i + L\frac{di}{dt} + V_C = 5\cos(10^3 t)$$

$$1k\left(C\frac{dv_c}{dt}\right) + L\left(C\frac{d^2V_c}{dt^2}\right) + V_C = 5\cos(10^3 t)$$

$$10^{-7}v'' + 10^{-3}v' + V_C = 5\cos(10^3 t)$$

$$v'' + 10^{4}v' + 10^{7}V = 5\cdot 10^{7}\cos(10^3 t)$$

Assuming a solution takes the form $A\cos(10^3 t + \phi)$

$$(-A)(10^3)^2\cos(10^3 t+\phi) + -(A)(10^7)\sin(10^3 t+\phi) + (10^7)A\cos(10^3 t+\phi) = 5\cdot 10^7\cos(10^3 t)$$

$$A(10^7 - 10^6)\cos(10^3 t+\phi) - A(10^7)\sin(10^3 t+\phi) = 5\cdot 10^7\cos(10^3 t)$$

Using the trig identity $C\sin x + B\cos(x) = \sqrt{B^2+C^2}\cos\left(x + \tan^{-1}\left(-\frac{C}{B}\right)\right)$

$$\sqrt{(A(10^7-10^6))^2 + (-A\,10^7)^2}\cos\left(10^3 t + \tan^{-1}\left(\frac{A\,10^7}{A(10^7-10^6)}\right) + \phi\right) = 5\cdot 10^7\cos(10^3 t)$$

$$A\sqrt{(10^7-10^6)^2 + (10^7)^2} = 5\cdot 10^7 \qquad 10^3 t + \tan^{-1}\left(\frac{10^7}{10^7-10^6}\right) + \phi = 10^3 t$$

$$A = 3.7165 \qquad \phi = -\tan^{-1}\left(\frac{10^7}{10^7-10^6}\right)$$

$$\phi = -48.0128^\circ$$

$$\boxed{V_c(t) = 3.717\cos(10^3 t - 48.01^\circ)\ [V]}$$