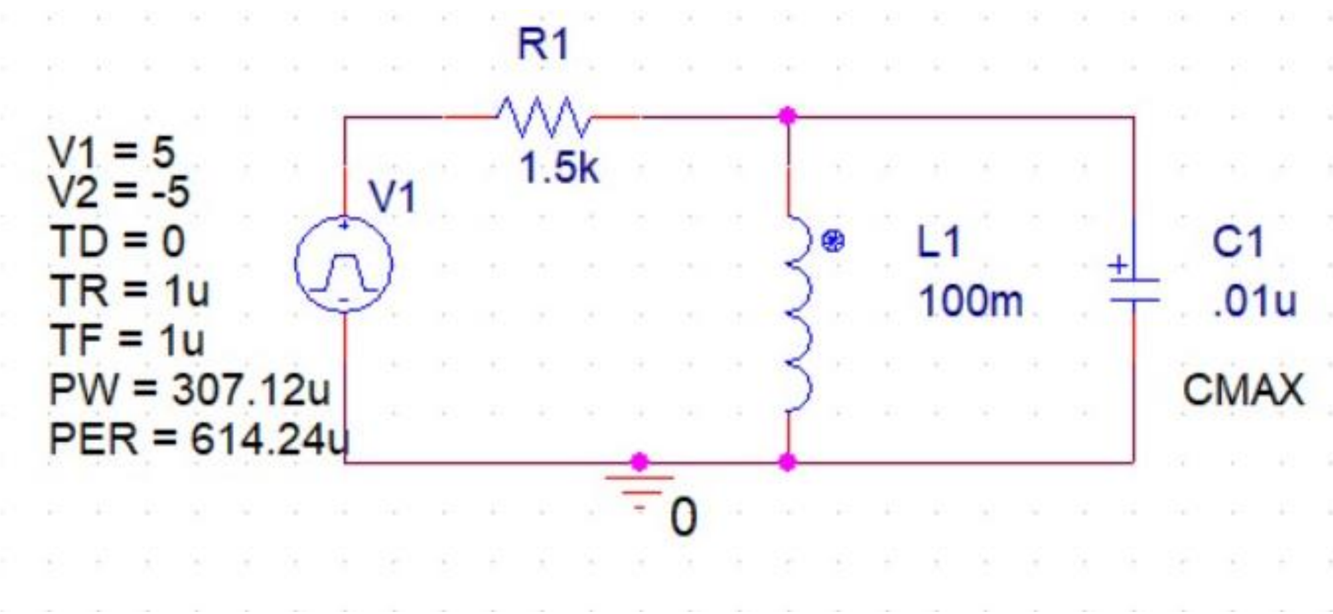
ECE 2101L Lab #4

Transient Response Of 2nd Order

Overdamped and Critically Damped Circuits



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Objective

The objective of the lab is for students to be able to calculate and identify the transient responses of critically damped and overdamped circuits. Students must be able to analyze collected or simulated data to determine if they agree with the hand calculated responses. Finally, we must be able to use periodic waveforms to produce multiple sets of results we can analyze.

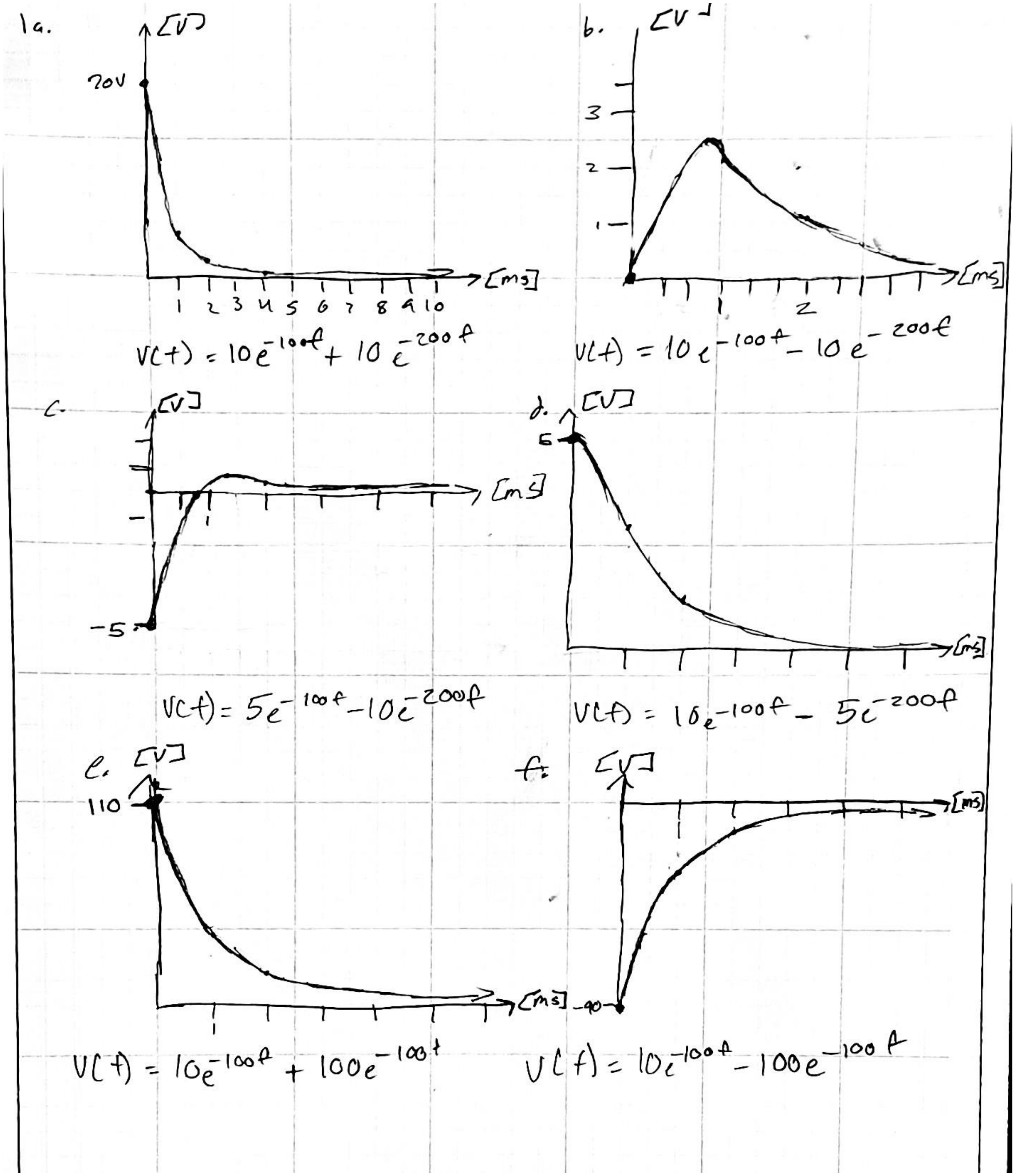
Materials

The necessary equipment needed for the lab are as follows

1. Breadboard
2. 4 BNC Clip Connectors
3. Clip Leads
4. 1 μF Capacitor & .01 μF Capacitor
5. 1.5KΩ & Two 10k Ω Resistors
6. 100mH inductor
7. Op Amp
8. LCR Meter
9. Digital Multimeter
10. Oscilloscope
11. Function Generator

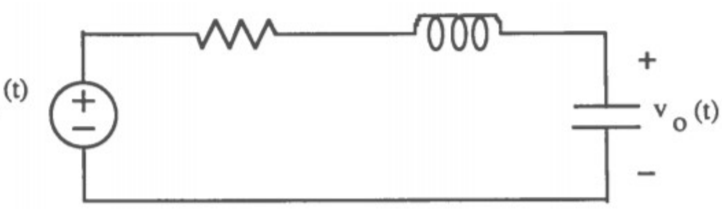
Pre-Lab

1.



1a.
[V]
20V
[ms]
1 2 3 4 5 6 7 8 9 10
$V(t) = 10e^{-100t} + 10e^{-200t}$
b.
[V]
3
2
1
[ms]
1
2
$V(t) = 10e^{-100t} - 10e^{-200t}$
c.
[V]
[ms]
1
-5
$V(t) = 5e^{-100t} - 10e^{-200t}$
d.
[V]
5
[ms]
$V(t) = 10e^{-100t} - 5e^{-200t}$
e.
[V]
110
[ms]
1
$V(t) = 10e^{-100t} + 100e^{-100t}$
f.
[V]
[ms]
-90
$V(t) = 10e^{-100t} - 100e^{-100t}$

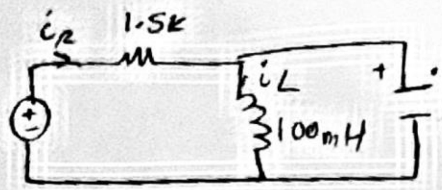
2.



$$v_o(t) = 10\, e^{-1000t} - 10\, e^{-2000t}$$

It will take approximately 5ms for the complete transient response to decay, this was determined by using five time constants, based on e^(-1000t). However, seven time constants would also be appropriate. A frequency of 100Hz would be fitting for a square wave to allow for five time constants in the negative and positive region for the source.

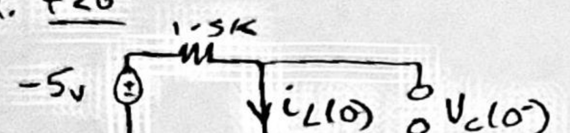
3.



For $t > 0$ $V_{in} = +5V$

For $t < 0$ $V_{in} = -5V$

a. $t < 0$



$$\boxed{\begin{aligned} i_L(0^-) &= i_L(0^+) = \frac{-5}{1.5k} = -3.\overline{333}\ mA \\ V_c(0^-) &= 0\,V = V_c(0^+) = V_c(0) \end{aligned}}$$

b. $\dfrac{V_c - V_{in}}{1.5k} + i_L + i_c = 0$, $\displaystyle\int \frac{di_L}{dt} = \int \frac{V_c}{L}\, dt$

$$\frac{V_c - V_{in}}{1.5k} + \int \frac{V_c}{L}\, dt + C\frac{dV_c}{dt} = 0$$

$$\left.\frac{di_L}{dt}\right|_{t=0} = \frac{V_c(0)}{L} = 0\ A/s$$

$$\frac{V_c'}{1.5k} + \frac{V_c}{L} + C V_c'' = 0$$

$$V_c'' + \frac{V_c'}{(1.5k)(.01\mu)} + \frac{V_c}{(100m)(.01\mu)} = 0$$

$$\boxed{V_c'' + 66.667k\, V_c' + 10^9\, V_c = 0}$$

$$D^2 + 66.667k\,D + 10^9 = 0$$

$$s_{1,2} = -22792.37\ ,\ -43874.33 \quad \text{Overdamped}$$

C. $$\frac{V_c(0) - 5}{1.5k} + i_L(0) + C\,v'(0) = 0$$

$$\frac{-5}{1.5k} - 3.\overline{333}\times10^{-3} + (.01\times10^{-6})\,v'(0) = 0$$

$$\boxed{v'(0) = 6.667\times10^{5}\ V/s}$$

D. $t \to \infty$

$i_L(\infty) = 3.\overline{333}\ mA$

$V_c(\infty) = 0V$

$$V_c(t) = A_1 e^{-22792.37t} + A_2 e^{-43874.33t} + 0$$

$$V_c(0) = A_1 + A_2 = 0\ ,\ A_1 = -A_2$$

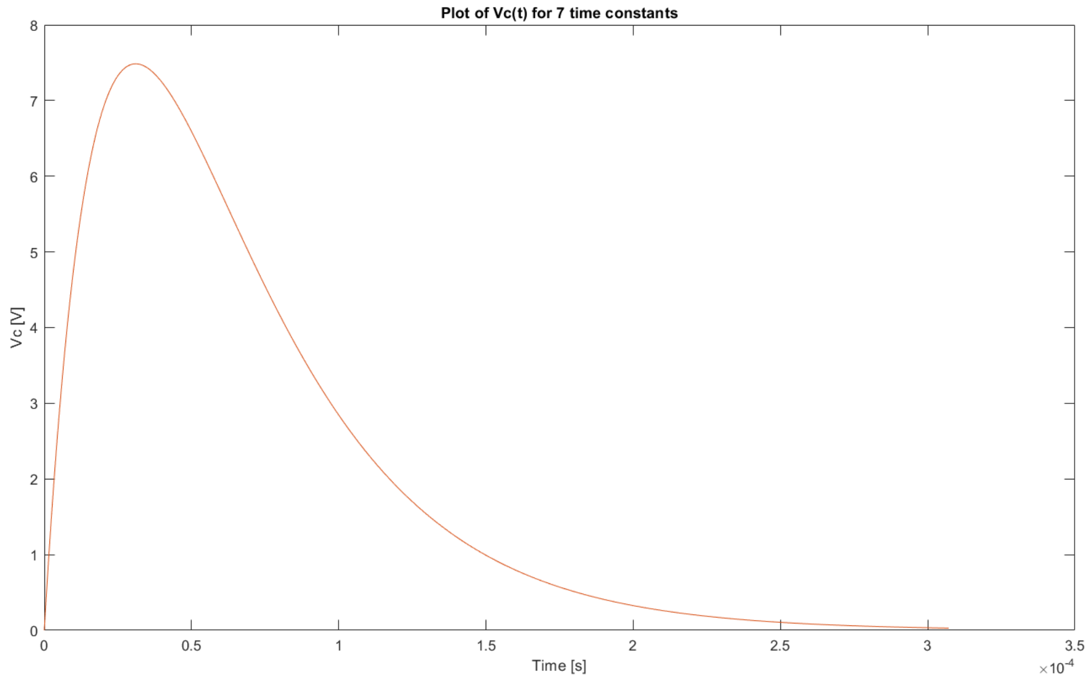
$$V_c'(t) = (-22792)A_1 e^{-22792t} + (A_2)(-43874.33)\,e^{-43874.33t}$$

$$V_c'(0) = (-22792)A_1 + A_1(43874.33) = 6.667\times10^{5}$$

$$A_1 = 31.623 \qquad A_2 = -31.623$$

$$\boxed{V_c(t) = 31.623\left(e^{-22792.37t} - e^{-43874.33t}\right)[V] \text{ for } t>0}$$

e.



f.

$V_C(t)$ first increases and then decreases because the voltage is proportional to the rate of change of the current of the inductor. Initially the current is negative and increases to the steady state current which results in the increasing portion of the voltage and then the current begins to change less as the current reaches the steady state voltage until the current through the inductor no longer changes and the voltage of the capacitor is then zero.

g.

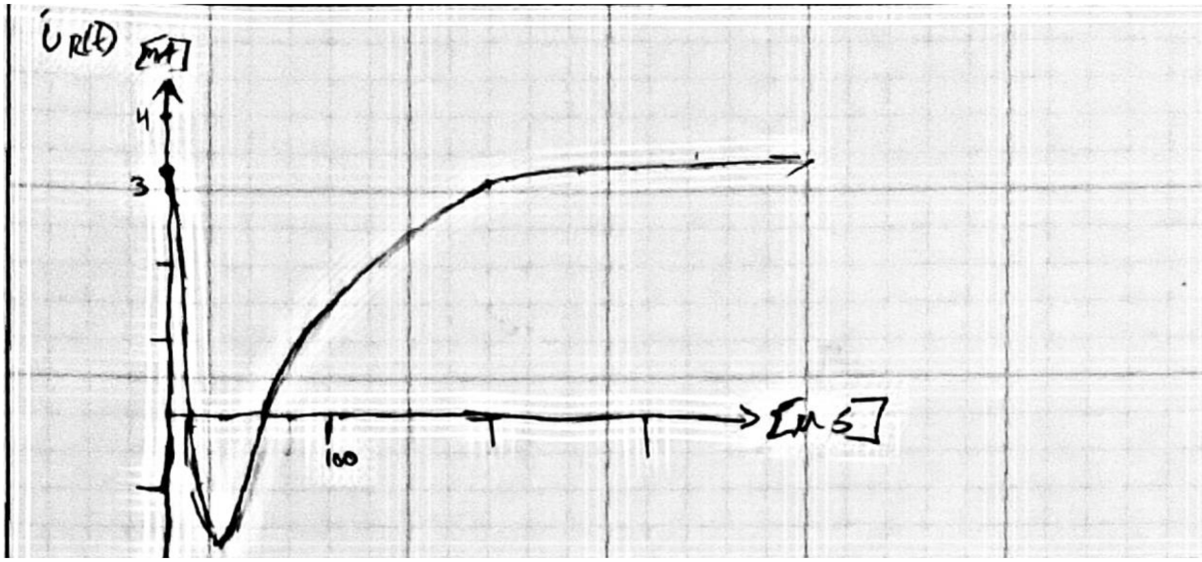
For practical purposes, the voltage of the capacitor takes 7τ to decay to zero, which is 307.12 is. Tau was based on the smaller damping factor of 22792.37 Np/s, was found by 1/(22792.37). The smaller value was chosen since the smaller factor decays the fastest.

h.

To observe the transient response of $V_C(t)$ with a square wave, we would want to use a period of 14τ to allow for 7τ for the charging down portion of the wave and 7τ for the charging up portion of the wave. The period would be 614.24µs and the frequency would be 1.628 mHz.

i. $i_R(t) = \frac{5 - V_C(t)}{1.5k}$

$$\boxed{i_R(t) = 3.333 - 21.082(e^{-22792.37t} - e^{-43874.33t}) \text{ [mA] } t>0}$$



j. $i_R = i_C + i_L$, $i_C = C\frac{dv}{dt}$ [A]

$i_L = i_R - i_C$

$i_C = C \cdot (-22792)(31.623)\left(e^{-22792.37t} - \frac{4387.33}{22792.37} e^{-43874.3t}\right)$

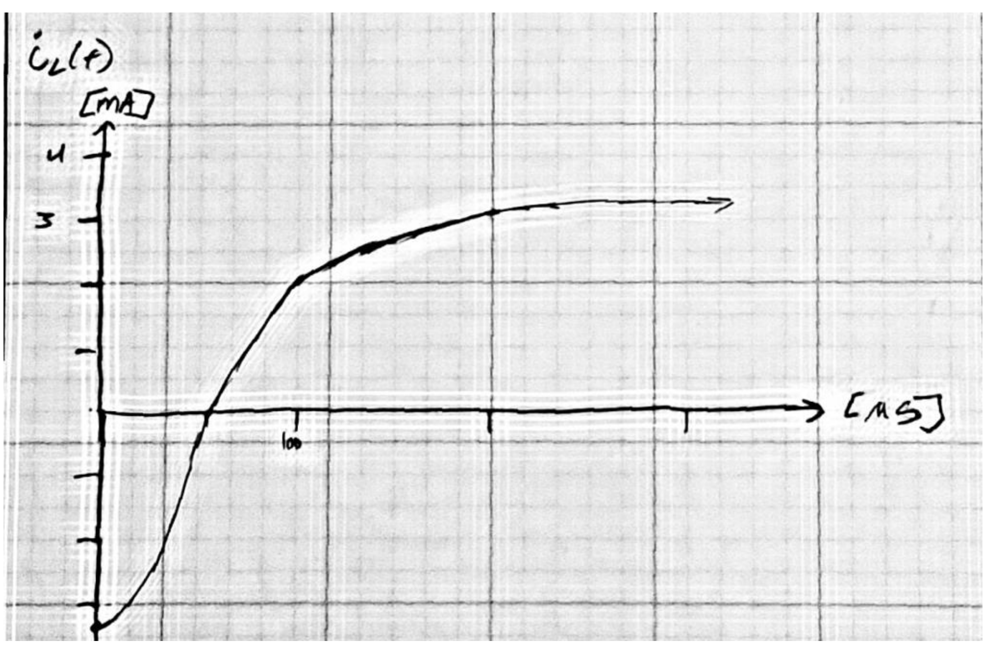
$i_C = -7.208\left(e^{-22792.37t} - 1.925 e^{-43874.3t}\right)$ [mA]

$7.208e^{-22792t} - 13.874e^{-43874.3t}$ [mA]

$i_L = 3.333 - 21.082\left(e^{-22792.37t} - e^{-43874.33t}\right)$

$i_L(t) = 3.333 - 13.874e^{-22792.37t} + 7.208e^{-43874.33t}$ [mA]

$$\boxed{i_L(t) = 3.333 - 13.874e^{-22792t} + 7.208e^{-43874t} \text{ [mA]}}$$



Procedure

To begin the lab we constucted the circuit seen in figure 1 and then simualted the circuit in PSPICE. We then took data points from the voltage response and then added the points to our graph from the pre-lab in order to confrim our therortical calculations. We then used a variavble resitance and chose various resistance values to see how changing the reistance affects the response, figure 3. Finally we build the active circuit in figure 3 and then simulated it and checked the output to see what waveform type the repsonse resemebled.

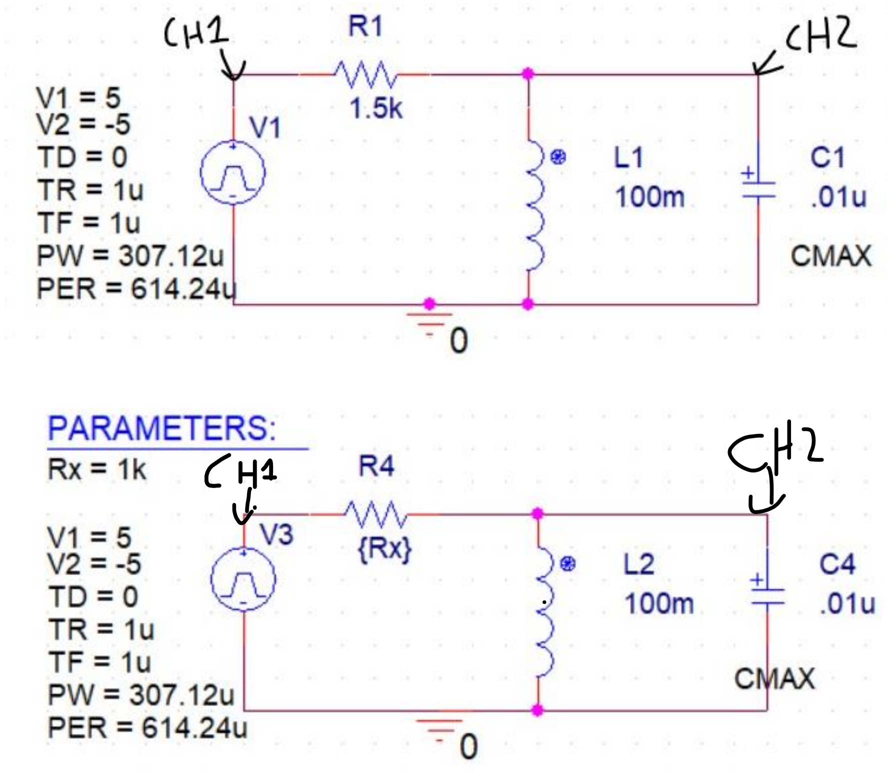


Figure 1: Parallel RLC Circuit

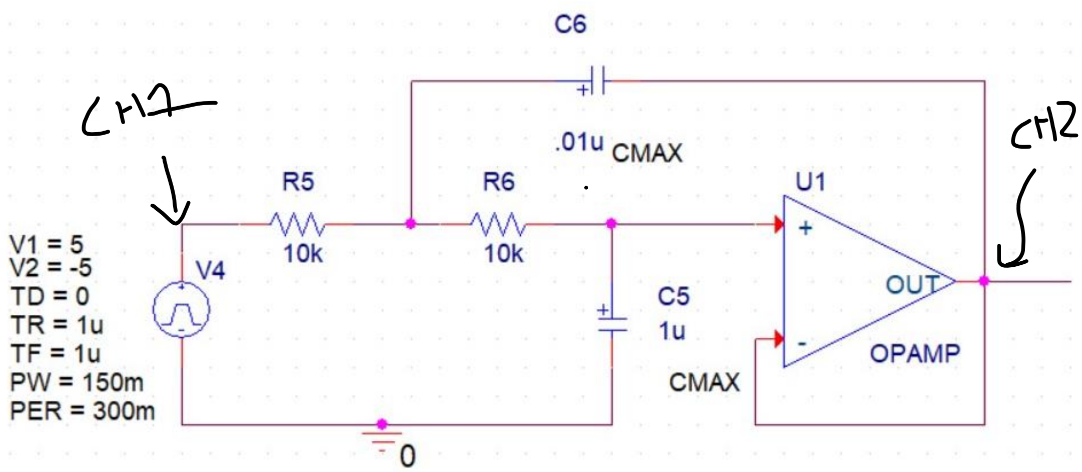


Figure 2: Active Circuit

Results

Note: The circuit in figure 1 is considered parallel RLC since source transformation can alter it to be a current source with a resistor, inductor, and capacitor all in parallel.

Parallel RLC

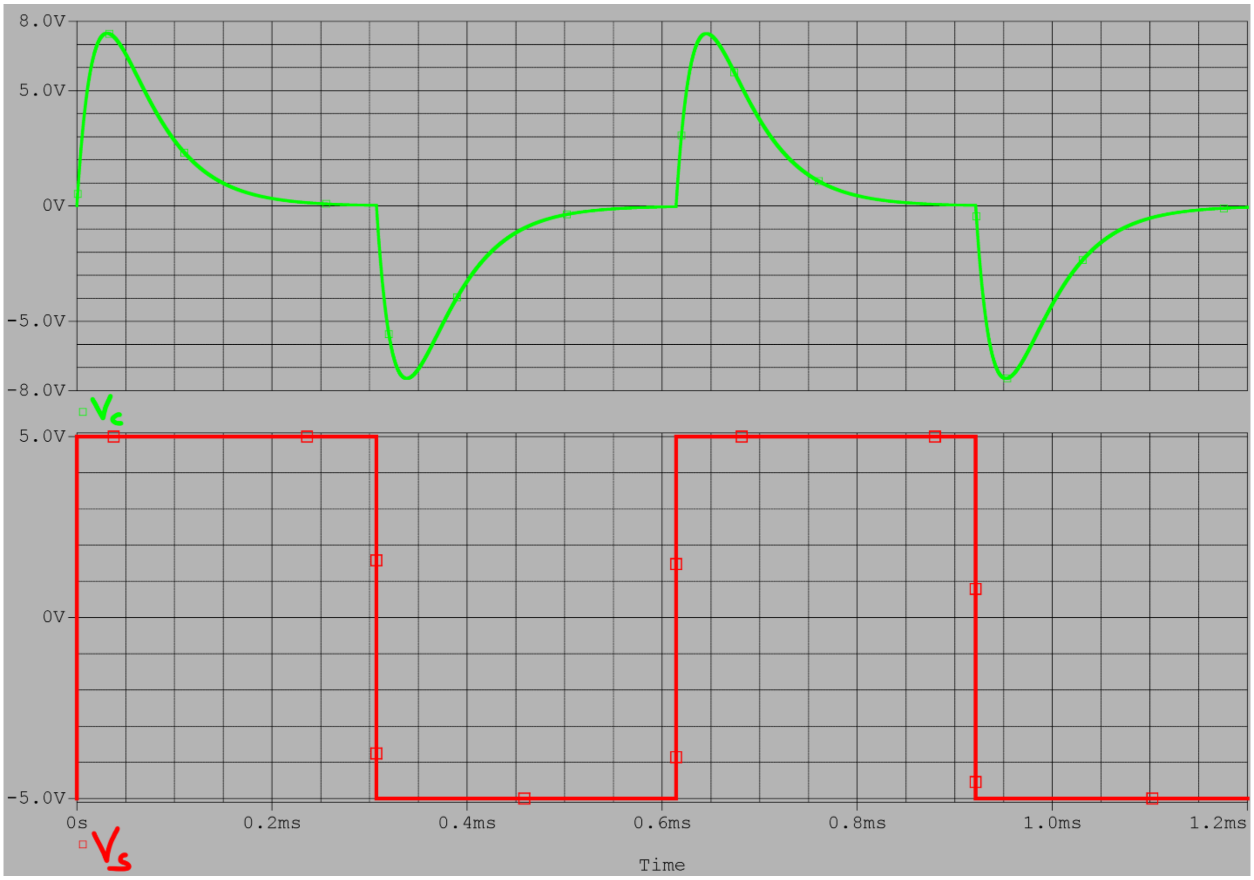


Figure 3: Response of Vs and Vc For Parallel RLC

Time [µs]	Voltage [V]
0	0
10.345	4.8946
17.241	6.5053
31.065	7.4852
51.035	6.5121
113.104	2.1800
163.449	.737946
285.518	.047068

Table 1: Table Containing Sample Times & Voltages To Verify With Calculations

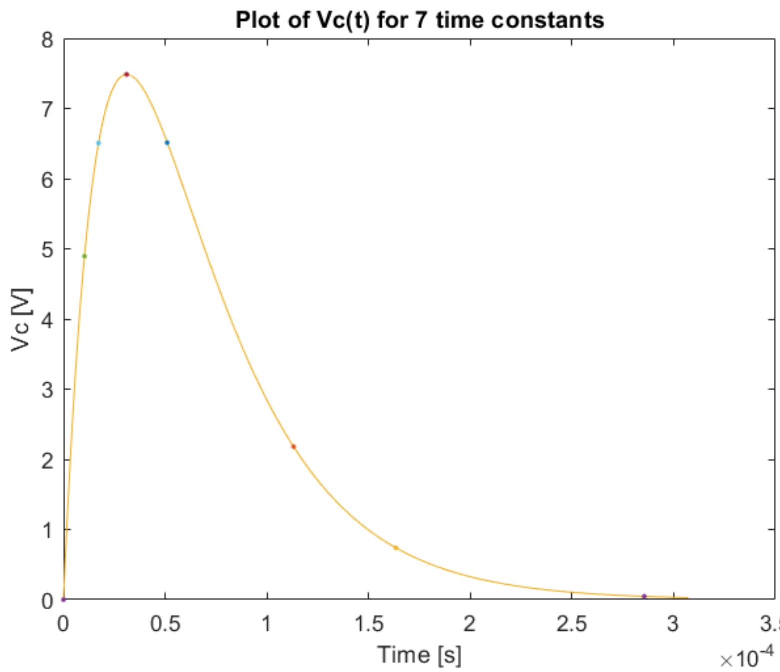


Figure 4: MATLAB Graph of Vc(t) With Points From Simulation

Parallel RLC W/ Variable Resistance

To incorporate various levels of resistance the values of R used were 1kΩ, 1.58kΩ, 5kΩ

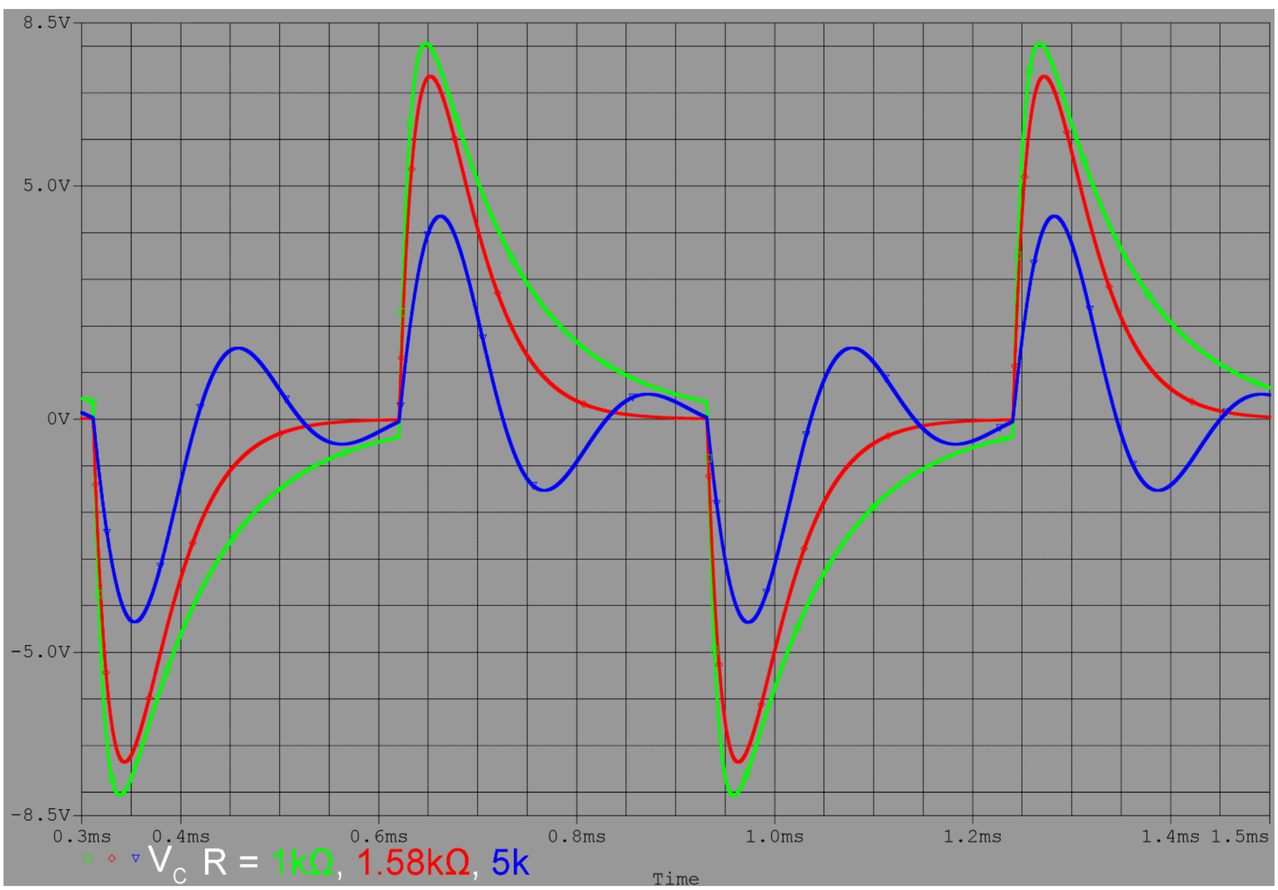


Figure 5: Various Responses Of The Parallel RLC Circuit With Different Resistances

Active Circuit

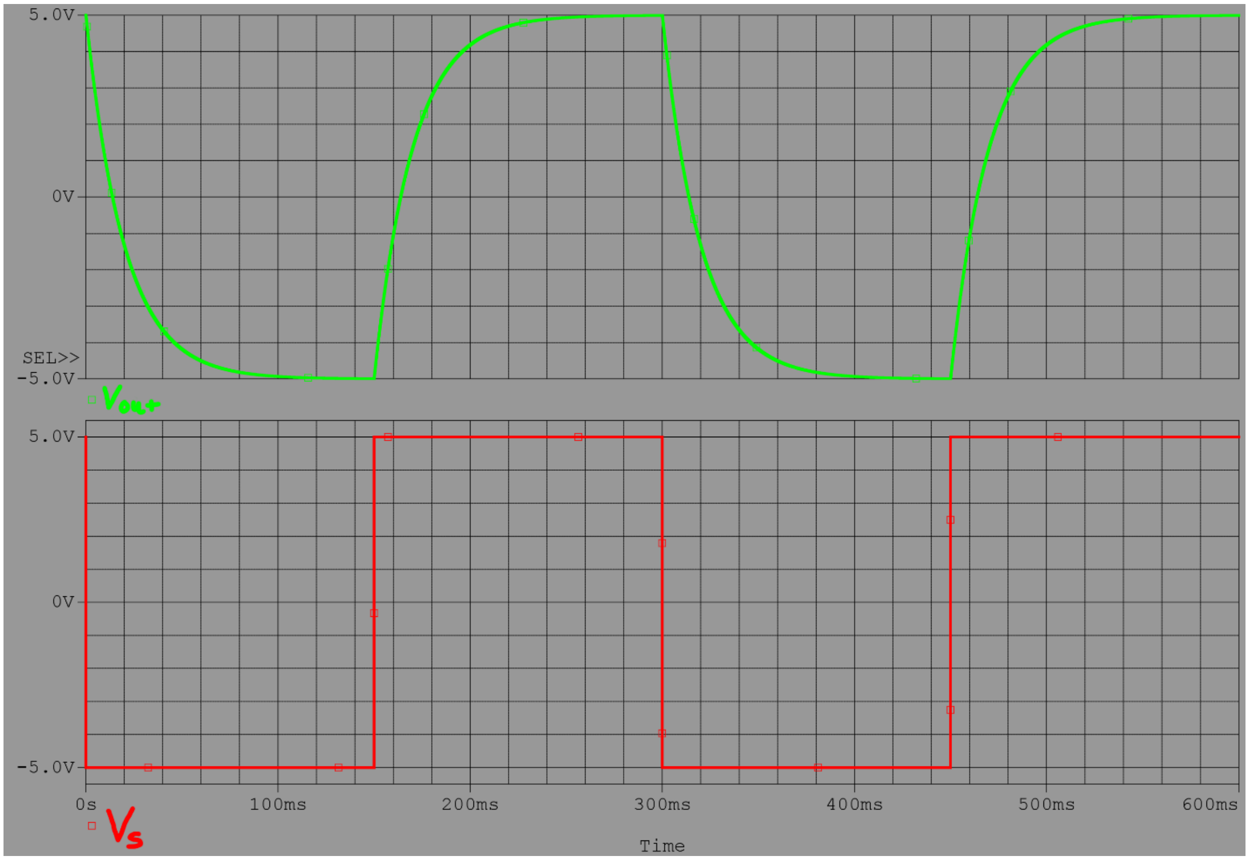


Figure 6: Response of Active Circuit

General Analysis

The response of the parallel RLC circuit is seen as overdamped with the characteristic of the singular hump, for some responses. We can also see that 7τ is adequate in showing the complete transient response of the circuit. As we altered the resistance, we could see that for lesser values of R, the response will be overdamped and as the resistance increases, the response becomes underdamped. After simulating the active circuit in figure 2, we were able to verify that the response is overdamped because the response takes a larger portion of the period to decay to 0 volts.

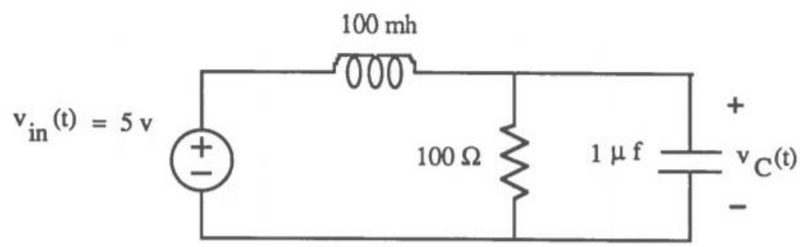
Problems

1. Compare the theoretical and simulated values for the lab.

Voltage From Simulation [V]	Voltage From Calculations [V]	% Difference
0	0	0%
4.8946	4.8949	0.0065%
6.5053	6.5054	0.0019
7.4852	7.4853	$8.3959*10^{-4}$
6.5121	6.5121	$5.6701*10^{-4}$
2.1800	2.1800	0.0013
.737946	0.7379	0.0018
.047068	0.0471	0.0020

The percent difference between the calculated and the simulated responses is incredibly small and which the direct result to the simulation being configured to be as close to ideal results as possible. The small percent error that is present is due to the precision of PSPICE.

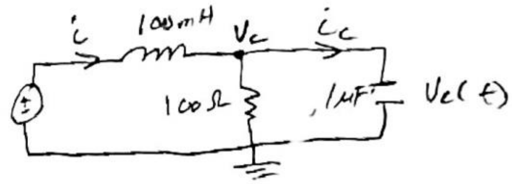
2.



$v_C(0-) = 0$ $i_L(0-) = 0$

a. Write the differential equation for $v_C(t)$
b. Solve for $v_C(t)$
c. Sketch $v_C(t)$
d. Find the forced response of $v_C(t)$ if $v_{in}(t) = 5 \cos 1000t$

2.



$V_c(0^-) = V_c(0^+) = 0\,V$

$i_L = i\,,\ i_L(0^-) = 0\,A$

a. $-i + \frac{V_c}{100} + C V_c' = 0\,,\quad i = \frac{V_c}{100} + C V_c'$

$L \frac{di}{dt} + V_c = 5$

$i_0 = \frac{V_c(0)}{100} + C\,V_c'(0)$

$V_c'(0) = 0\ V/s$

$L\left(\frac{V_c'}{100} + C V_c''\right) + V_c = 5$

$V_c(\infty) = 5\,V$

$$\boxed{V_c'' + 10k\,V_c' + 10^7 V_c = 5 \cdot 10^7}$$

$D^2 + 10k\,D + 10 \cdot 10^6 = 0$

$S_{1,2} = \frac{-10k \pm \sqrt{(10k)^2 - 4(10 \cdot 10^6)}}{2} = -1127.02\,,\ -8872.98$

$V_c(t) = A_1 e^{-1127.02t} + A_2 e^{-8872.98t} + V(\infty)$

$V_c(0) = A_1 + A_2 + 5 = 0$

$A_1 + A_2 = -5$

$V_c'(t) = -1227.02(A_1) - 8872.98 A_2 = 0$

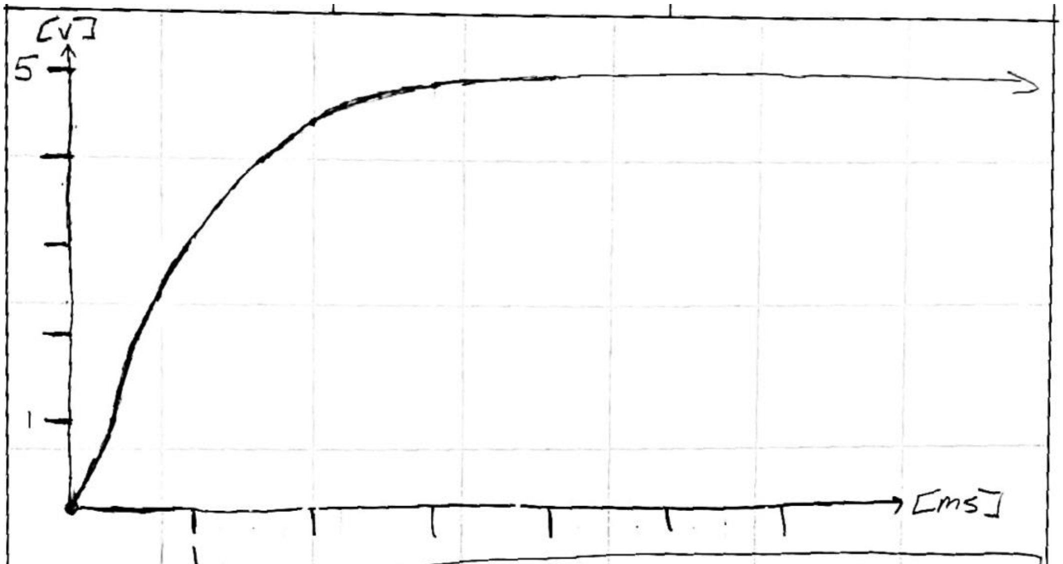
$A_1 = -7.873 A_2$

$-7.873 A_2 + A_2 = -5$

$A_2 = .7275$

$A_1 = -5.7275$

$$\boxed{V_c(t) = -5.728 e^{-1127.02t} + .7275 e^{-8872.98t} + 5\ [V]\quad t > 0}$$



$$\boxed{V_c(t) = -5.728e^{-1127.02t} + .7275e^{-8872.98t} + 5\,[V]\ t>0}$$

d. Using the diff eq from part a, but altered to $V_s = 5\cos(1kt)$

$$V_c'' + 10k\,V_c' + 10^7 V_c = 5\cdot 10^7 \cdot \cos(1000t)$$

The forced response will be of the form $A\cos(1kt+\phi)$

$$-A(1K)^2\cos(1Kt+\phi) + 10k(-(A)(1K)(\sin(1Kt+\phi))) + A\,10^7\cos(1Kt+\phi) = 5\cdot 10^7\cos(1Kt)$$

$$A(10^7-10^6)\cos(1Kt+\phi) - 10^7(A)\sin(1Kt+\phi) = 5\cdot 10^7\cos(1Kt)$$

Using the trig identity $C\sin(x) + B\cos(x) = \sqrt{B^2+C^2}\cos(x+\tan^{-1}(-\frac{C}{B}))$

$$\sqrt{(A(10^7-10^6))^2+(10^7(A))^2}\cos\left(1kt+\phi+\tan^{-1}\left(\frac{10^7 A}{A(10^7-10^6)}\right)\right) = 5\cdot 10^7\cos(1kt)$$

$$A\sqrt{(10^7-10^6)^2+10^{14}} = 5\cdot 10^7 \qquad 1kt+\phi+\tan^{-1}\left(\frac{10^7}{10^7-10^6}\right) = 1kt$$

$A = 3.7165$

$$\phi = -\tan^{-1}\left(\frac{10^7}{10^7-10^6}\right)$$

$$\phi = -48.018^\circ$$

$$\boxed{V_c(t) = 3.7165\cos(1000t - 48.018^\circ)\ [V]}$$